Chapter - 1
Real Number

Exercise No: 1.1

Question 1:

For some integer m, every even integer is of the form:

(A) m (B) m +1 (C) 2m (D) 2m+1

Solution:

(C) 2m

Integers which are divisible by 2 are called even integers.

So, every even integer must be a multiple of 2. So, we can conclude that, for any integer m,

Where, m = 1, 2, 3, 4….n, even integer is in the form of 2 × m = 2m.

Question 2:

For some integer q, every odd integer is of the form

(A) q (B) q + 1 (C) 2q (D) 2q + 1

Solution:

(D) 2q+1

Integers which are not divisible by 2 are called odd integers.

And integer which is a multiple of 2 is an even integer, if 1 is added to any integer which is multiplied by 2, it gives an odd integer.

So, we can say that, for any integer ‘q’, every odd integer is of the form 2q+1.

Question 3:

$n^2 - 1$ is divisible by 8, if n is

(A) an integer (B) a natural number

(C) an odd integer (D) an even integer

Solution:

(C) an odd integer

Let $a = n^2 - 1$

In the above equation, n can be even or n can be odd.

Let us say that n is even integer.

So, If n = even i.e., n = 2x, where x is an integer,

We get,

$a = (2x)^2 - 1$

$a = 4x^2 - 1$

Let, x =0,

$a = 4(0)^2 - 1$

$= 0 - 1$

$= -1$

a = -1 is not divisible by 8.

Let us say that n = odd:

When n = odd i.e.

n = 2x + 1, where x is an integer,

We have,

$a = (2x+1)^2 - 1$

$a = 4x^2 + 4x + 1 - 1$

$a = 4x^2 + 4x$

$a = 4x(x+1)$

At x = 0,

$a = 4(0)(0+1)$

a = 0 which is divisible by 8.

So, we can conclude that, n^2-1 is divisible by 8 when n is an odd integer.

Question 4:

If the HCF of 65 and 117 is expressible in the form 65m – 117, then the value of m is

(A) 4 (B) 2 (C) 1 (D) 3

Solution:

(B) 2

If we find the HCF of 65 and 117, we get,

117 = 1×65 + 52

65 = 1× 52 + 13

52 = 4 ×13 + 0

The HCF of 65 and 117 is 13.

Putting value in the question, we get

65m – 117 = 13

65m = 117+13

65m = 130

$\therefore m = \frac{130}{65}$

m = 2

Question 5:

The largest number which divides 70 and 125, leaving remainders 5 and 8, respectively, is

(A) 13 (B) 65 (C) 875 (D) 1750

Solution:

(A) 13

We have to find the largest number which divides 70 and 125, leaving remainders 5 and 8.

So, we can write it as,

The largest number which divides (70 – 5), and (125 – 8), as 5 and 8 are remainder,

Ultimately we have to find the Highest Common Factor of 65 and 117

On solving,

Multiples of 65 = 1, 5, 13, 65

Multiples of 117 = 1, 3, 9, 13, 39, 117

Common multiple = 1, 13

So, HCF= 13

Therefore, 13 is the number which divides 70 and 125, leaving remainders 5 and 8.

Question 6:

If two positive integers a and b are written as $a = x^3y^2$ and $b = xy^3$; x, y are prime numbers, then HCF (a, b) is

(A) xy (B) xy^2 (C) x^3y^3 (D) x^2y^2

Solution:

(B) xy^2

To find the HCF of a and b, we have to see the common terms in a and b , on solving, we get,

HCF= xy^2

Question7:

If two positive integers p and q can be expressed as $p = ab^2$ and $q = a^3b$; a, b being prime numbers, then LCM (p, q) is

(A)ab (B) a^2b^2 (C) a^3b^2 (D) a^3b^3

Solution:

(C) a^3b^2

$p = ab^2$

$q = a^3b$

To find LCM we see the highest powers of each number.

So, LCM = a^3b^2

Question 8:

The product of a non-zero rational and an irrational number is

(A) always irrational (B) always rational

(C) rational or irrational (D) one

Solution:

(A) always irrational

Product of a non-zero rational and an irrational number is irrational.

Question 9:

The least number that is divisible by all the numbers from 1 to 10 (both inclusive) is

(A) 10 (B) 100 (C) 504 (D) 2520

Solution:

(D) 2520

We have to find the LCM of numbers from 1 to 10.

We will first see the multiples of numbers from 1 – 10,

$1 = 1, 2 = 2, 3 = 3$

$4 = 2 \times 2, 5 = 5, 6 = 2 \times 3, 7 = 7$

$8 = 2 \times 2 \times 2, 9 = 3 \times 3, 10 = 2 \times 5$

LCM of all numbers 1 to 10 $= 1 \times 2 \times 3 \times 2 \times 5 \times 7 \times 2 \times 3$

$= 2520$

Question10:

The decimal expansion of the rational number $\frac{14587}{1250}$ will terminate after:

(A) One decimal place (B) two decimal places

(C) Three decimal places (D) four decimal places

Solution:

(D) Four decimal places

On dividing we get,

$$\frac{14587}{1250} = 11.6696$$

Exercise No: 1.2

Question 1:

Write whether every positive integer can be of the from (4q + 2), where q is an integer. Justify your answer.

Solution 1:

'No'

We know that,

Dividend = divisor × quotient + remainder

According to Euclid's division lemma,

$a = bq + r$

If, $b = 4$ then,

$a = 4q + r$

So,

$r = 0, 1, 2, 3$ (as r is positive and less than 4)

We get,

$4q$, $4q + 1$, $4q + 2$ and $4q + 3$ respectively.

So, every positive integer cannot be only in the form of $4q+2$.

Question 2:

"The product of two consecutive positive integers is divisible by 2". Is this statement true or false? Give reasons.

Solution 2:

This statement is true.

For any two consecutive numbers one will be even and the other will be odd. Let $n, (n+1)$. So, their product is always multiple of 2.

Hence, the product of two consecutive positive integers is divisible by 2.

Question 3.

"The product of three consecutive positive integers is divisible by 6". Is this

statement true or false? Justify your answer.

Solution:

The statement is true.

Three consecutive positive integers are n, (n + 1), (n + 2).

In those 3 consecutive integers, one will be even and other will be divisible by 3.

Therefore, the product of three will be divisible by 6,

For e.g. 16, 17, 18

Here 16 is even, and18 is divisible by 3.

So, $16 \times 17 \times 18$ is divisible by 6.

Question 4.

Write whether the square of any positive integer can be of the form of (3m + 2), where m is a natural number. Justify your answer.

Solution:

No, the square of any positive integer cannot be written in the form 3m + 2 where m is a natural number

Explanation:

By Euclid's division lemma, we have

a = bq + r, where b, q and r are positive integers,

For b = 3

a = 3(q) + r,

For r = 0, 1, 2, 3

So, 3q + 0, 3q + 1, 3q + 2, 3q + 3 are positive integers,

According to question, the square of any positive integer can be written as,

$(3q)^2 = 9q^2$

$= 3(3q^2) = 3m$ (where $3q^2 = m$)

$(3q+1)^2 = (3q+1)^2$

$= 9q^2+1+6q$

$= 3(3q^2+2q) +1$

$= 3m + 1$ (Where, $m = 3q^2+2q$)

$(3q+2)^2 = (3q+2)^2$

$= 9q^2+4+12q$

$= 3(3q^2+4q) +4$

$= 3m + 4$ (Where, $m = 3q^2+2q$)

$(3q+3)^2 = (3q+3)^2$

$= 9q^2+9+18q$

$= 3(3q^2+6q) +9$

$= 3m + 9$ (Where, $m = 3q^2+2q$)

So the square of any positive integer cannot be written in the form $3m + 2$.

Question 5 :

A positive integer is of the form (3q + 1), q being a natural number. Can you write its square in any form other than (3m + 1) i.e., 3m or (3m + 2) for some integer m? Justify your answer.

Solution 5:

No. $(3q + 1)^2$ cannot be expressed in any other form other than $3m + 1$.

Explanation:

Consider the positive integer $3q + 1$, so,

$(3q + 1)^2 = 9q^2 + 6q + 1$

$= 3(3q^2 + 2q) + 1$

$= 3m + 1$, (where m is an integer $= 3q^2 + 2q$)

Hence,$(3q + 1)^2$ cannot be expressed in any other form other than $3m + 1$.

Question 6.

The numbers 525 and 3000 are both divisible only by 3, 5, 15, 25 and 75, what is HCF of (3000, 525)? Justify your answer.

Solution 6 :

The numbers 525 and 3000 both are divisible only by 3, 5, 15, 25 and 75, So, highest common factors out of 3, 5, 15, 25 and 75 is 75

So, HCF of (525, 3000) is 75.

Justification:

525 $= 5 \times 5 \times 3 \times 7$

$= 3 \times 5^2 \times 7^1$

3000 $= 2^3 \times 5^3 \times 3^1$

$= 2^3 \times 3^1 \times 5^3$

Therefore,

HCF $= 3^1 \times 5^2$

$= 75$

Hence, justified.

Question 7.

Explain why 3 × 5 × 7 + 7 is a composite number.

Solution 7:

A number which is not prime is composite.

$3 \times 5 \times 7 + 7 = 7[3 \times 5 + 1]$

$= 7[15 + 1]$

$= 7 \times 16$

So, it have factors $= 7 \times 2 \times 2 \times 2 \times 2$

As the factors are other than one and itself, so, the number is not prime.

Hence, the number $(3 \times 5 \times 7 + 7)$ is composite.

Question 8.

Can two numbers have 18 as their HCF and 380 as their LCM? Give reasons.

Solution 8:

380 and 18 are not the LCM and HCF of any two numbers.

We know that,

HCF $(x,y) \times$ LCM $(x, y) = (x \times y)$

So, 18 must be factor of 380.

But, 380 is not divisible by 18.

Hence, 380 and 18 are not the LCM and HCF of any two numbers.

Question 9.

Without actually performing the long division, find if $\frac{987}{10500}$ will have terminating or non-terminating (repeating) decimal expansion. Give reasons for your answer.

Solution 9:

The denominator has prime factors only in 2 and 5 so, the number is terminating decimal. On solving we get,

$$\frac{987}{10500} = 0.094$$

Question 10.

A rational number in its decimal expansion is 327.7081. What can you say about the prime factors of q, when this number is expressed in the form p/q? Give reasons.

Solution:

327.7081 is terminating decimal in the form of

$$\frac{p}{q} = \frac{3277081}{10000}$$

$q = 2^4 \times 5^4$

As, q has only multiples of 2 and 5 so it is terminating decimal.

Exercise No: 1.3

Question 1.

Show that the square of any positive integer is either of the form 4q or 4q + 1 for some integer q.

Solution:

By Euclid's division lemma,

$a = 4m + r$ ... (i)

Where $r = 0 \leq r < 4$

or

$r = 0, 1, 2, 3$

When $r = 0$, $a = 4m$ [From (i)]

When $r = 0$, we get,

$a = 4k$

$a^2 = 16m^2$

$= 4(4m^2)$

$= 4q$, where $q = 4m^2$

When $r = 1$, we get,

$a = 4m + 1$

$a^2 = (4m + 1)^2$

$= 16m^2 + 1 + 8m$

$= 4(4m + 2) + 1$

$= 4q + 1$, where $q = m(4m + 2)$

When $r = 2$, we get, $a = 4m + 2$

$a^2 = (4m + 2)^2$

$= 16m^2 + 4 + 16m$

$= 4(4m^2 + 4m + 1)$

$= 4q$, where $q = 4m^2 + 4m + 1$

When r = 3, we get,

a = 4m + 3

$a^2 = (4m + 3)^2$

= 16m2 + 9 + 24m

= 4(4m2 + 6m + 2) + 1

= 4q + 1, where q = 4k2 + 6k + 2

Hence, the square of any positive integer is either of the form 4q or 4q + 1 for some integer q.

Question 2.

Show that the cube of any positive integer is of the form 4m, 4m + 1 or 4m + 3 for some integer m.

Solution:

Let x be any positive integer and b = 4.

Using Euclid Algorithm,

x = bq + r

x = 4q + r

The possible values of r are,

r = 0,1,2,3

If r = 0,

x = 4q + 0

x = 4q

Taking cubes on LHS and RHS,

$x^3 = (4q)^3$

$x^3 = 4 (16q^3)$

$x^3 = 4m$ [where $m = 16q^3$]

If r = 1,

x = 4q + 1

Taking cubes on LHS and RHS,

$x^3 = (4q + 1)^3$

$x^3 = 64q^3 + 1^3 + 3 \times 4q \times 1 (4q + 1)$

$x^3 = 64q^3 + 1 + 48q^2 + 12q$

$x^3 = 4 (16q^3 + 12q^2 + 3q) + 1$

$x^3 = 4m + 1$ [where $m = 16q^3 + 12q^2 + 3q$]

If r = 2,

$x = 4q + 2$

Taking cubes on LHS and RHS,

We have,

$x^3 = (4q + 2)^3$

$x^3 = 64q^3 + 2^3 + 3 \times 4q \times 2 (4q + 2)$

$x^3 = 64q^3 + 8 + 96q^2 + 48q$

$x^3 = 4 (16q^3 + 2 + 24q^2 + 12q)$

$x^3 = 4m$ [here $m =16q^3 + 2 + 24q^2 + 12q$]

If r = 3,

$x = 4q + 3$

Taking cubes on LHS and RHS,

We have,

$x^3 = (4q + 3)^3$

$x^3 = 64q^3 + 27 + 3 \times 4q \times 3 (4q + 3)$

$x^3 = 64q^3 + 24 + 3 + 144q^2 + 108q$

$x^3 = 4 (16q^3 + 36q^2 + 27q + 6) + 3$

$x^3 = 4m + 3$ [where $m =16q^3 + 36q^2 + 27q + 6$]

So, the cube of any positive integer is in the form of 4m, 4m+1 or 4m+3.

Question 3.

Show that the square of any positive integer cannot be of the form 5q + 2 or 5q + 3 for any integer q.

Solution.

Let the positive integer = x

Using Euclid algorithm,

$x = bm + r$

According to the question, b = 5

$x = 5m + r$

So, r = 0, 1, 2, 3, 4

For r = 0, x = 5m.

For r= 1, x = 5m + 1.

For r = 2, x = 5m + 2.

For r = 3, x = 5m + 3.

For r = 4, x = 5m + 4.

Now, When x = 5m

$x^2 = (5m)^2$

$= 25m^2$

$x^2 = 5(5m^2)$

$= 5q$, where $q = 5m^2$

When x = 5m + 1

$x^2 = (5m + 1)^2$

$= 25m^2 + 10 m + 1$

$x^2 = 5 (5m^2 + 2m) + 1$

$= 5q + 1$, where $q = 5m^2 + 2m$

When x = 5m + 2

$x^2 = (5m + 2)^2$

$x^2 = 25m^2 + 20m + 4$

$x^2 = 5 (5m^2 + 4m) + 4$

$x^2 = 5q + 4$ where $q = 5m^2 + 4m$

When $x = 5m + 3$

$x^2 = (5m + 3)^2$

$= 25m^2 + 30m + 9$

$x^2 = 5 (5m^2 + 6m + 1) + 4$

$x^2 = 5q + 4$ where $q = 5m^2 + 6m + 1$

When $x = 5m + 4$

$x^2 = (5m + 4)^2$

$= 25m^2 + 40m + 16$

$x^2 = 5 (5m^2 + 8m + 3) + 1$

$x^2 = 5q + 1$ where $q = 5m^2 + 8m + 3$

So, square of any positive integer cannot be of the form $5q + 2$ or $5q + 3$.

Question 4.

Show that the square of any positive integer cannot be of the form 6m + 2 or 6m + 5 for any integer m.

Solution:

Let the positive integer = x

Using Euclid's algorithm,

$x = 6q + r$, where $0 \leq r < 6$

$x^2 = (6q + r)^2$

$= 36q^2 + r^2 + 12qr$

$x^2 = 6(6q^2 + 2qr) + r^2$

Here, $0< r< 6$

If $r = 0$, we get

$x^2 = 6 (6q^2)$

$= 6m,$

(Here, $m = 6q^2$).

When $r = 1$, we get

$x^2 = 6 (6q^2 + 2q) + 1$

$= 6m + 1,$

Here, $m = (6q^2 + 2q)$

If $r = 2$, we get

$x^2 = 6(6q^2 + 4q) + 4$

$= 6m + 4,$

Here, $m = (6q^2 + 4q)$.

If $r = 3$, we get

$x^2 = 6(6q^2 + 6q) + 9$

$= 6(6q^2 + 6a) + 6 + 3$

$x^2 = 6(6q^2 + 6q + 1) + 3$

$= 6m + 3,$

Here, $m = (6q + 6q + 1)$

If $r = 4$, we get

$x^2 = 6(6q^2 + 8q) + 16$

$= 6(6q^2 + 8q) + 12 + 4$

$x^2 = 6(6q^2 + 8q + 2) + 4$

$= 6m + 4,$

Here, $m = (6q^2 + 8q + 2)$

If $r = 5$, we get

$x^2 = 6 (6q^2 + 10q) + 25$

$= 6(6q^2 + 10q) + 24 + 1$

$x^2 = 6(6q^2 + 10q + 4) + 1$

$= 6m + 1,$

Here, $m = (6q^2 + 10q + 1)$.

So, the square of any positive integer cannot be of the form $6m + 2$ or $6m + 5$ for any integer m.

Question 5.

Show that the square of any odd integer is of the form 4q + 1, for some integer q.

Solution:

Let c be any odd integer and d = 4.

Using Euclid's algorithm,

$c = 4m + r$

and

$r = 0,1,2,3$ as, $0 \leq r < 4$.

Therefore,

$c = 4m, 4m + 1, 4m + 2, 4m + 3$

So, c can be $4m + 1$ or $4m + 3$

c cannot be $4m$ or $4m + 2$, as they are even integer

Now,

$(4m + 1)^2 = 16m^2 + 8m + 1$

$= 4(4m^2 + 2m) + 1$

$= 4q + 1$, where $q = 4m^2 + 2m$.

$(4m + 3)^2 = 16m^2 + 24m + 9$

$= 4(4m^2 + 6m + 2) + 1$

$= 4q + 1$, where $q = 4m^2 + 6m + 2$

Thus we can conclude that square of any odd integer is of the form 4q + 1, for some integer q.

Question 6.

If n is an odd integer, then show that $n^2 – 1$ is divisible by 8.

Solution:

Let

$x = n^2 – 1$ ... (i)

As n is odd number so,

n = 1, 3, 5, 7

If n = 1

$x = 1^2 - 1$

$= 0$, which is divisible by 8.

If n = 3

$x = 3^2 - 1$

$= 9 - 1$

$= 8$, which is also divisible by 8.

If n = 5,

$x = 5^2 - 1$

$= 25 - 1$

$= 24$

$= 8 \times 3$, which is divisible by 8.

Therefore, $n^2 - 1$ is divisible by 8 when n is odd.

Question 7.

Prove that, if x and y, both are odd positive integers, then $(x^2 + y^2)$ is even but not divisible by 4.

Solution:

Let the two odd positive numbers x and y be 2m + 1 and 2n + 1 respectively.

So,

$x^2 + y^2 = (2m + 1)^2 + (2n + 1)^2$

$= 4m^2 + 4m + 1 + 4n^2 + 4n + 1$

$= 4m^2 + 4n^2 + 4m + 4n + 2$

$= 4(m^2 + n^2 + m + n) + 2$

In the above expression we see that the sum of square is even, but the number is not divisible by 4. Therefore, we can say that if x and y are odd positive integer, then $x^2 + y^2$ is even but not divisible by four.

Question 8.

Use Euclid's division algorithm to find HCF of 441, 567 and 693.

Solution.

Let p = 693 and q = 567

Using Euclid's division lemma,

p = qm+ r

693 = 567 × 1 + 126

567 = 126 × 4 + 63

126 = 63 × 2 + 0

So, HCF (693 and 567) = 63.

Further,

Taking 441 and HCF = 63

Using Euclid's algorithm,

c = dm + r

c = 441 and,

d = 63

441 = 63 × 7 + 0

So, HCF of (693, 567, 441) = 63.

Question 9.

Using Euclid's division algorithm, find the largest number that divides 1251, 9377 and 1562 8 leaving remainders, 1, 2, and 3 respectively.

Solution.

According to question 1, 2, and 3 are the remainders when the largest number divides 1251, 9377 and 15628 respectively.

So, we have to find HCF of (1251 – 1), (9377 – 2) and (15628 – 3)

That are,

1250, 9375, 15625

For HCF of 1250, 9375, 15625

Let p = 15625,

q = 9375

Using Euclid's algorithm, p= qm+ r

15625 = 9375 × 1 + 6250

9375 = 6250 × 1 + 3125

6250 = 3125 × 2 + 0

Therefore,

HCF (15625, 9375) = 3125

Let a= 1250 and

b= 3125

By Euclid's division algorithm, b= am + r

3125 = 1250 × 2 + 625

1250 = 625 × 2 + 0

Therefore, the HCF of (15625, 1250 and 9375) is 625.

Question 10.

Prove that $\left(\sqrt{3}+\sqrt{5}\right)$ is irrational.

Solution:

Let us consider $\left(\sqrt{3}+\sqrt{5}\right)$ is a rational number that can be written as

$\left(\sqrt{3}+\sqrt{5}\right)$= x

$$\left(\sqrt{3}+\sqrt{5}\right)=x$$

$$\sqrt{5}=x-\sqrt{3}$$

Squaring both sides,

$$\left(\sqrt{5}\right)^2=\left(x-\sqrt{3}\right)^2$$

$$5=\left(x^2-2x\sqrt{3}+3\right)$$

$$\sqrt{3}=\frac{x^2+3-5}{2x}$$

$$\sqrt{3}=\frac{x^2-2}{2x}$$

From the above expression, RHS comes out to be rational but we know that LHS=$\sqrt{3}$ is irrational, which contradicts our fact. So, $\left(\sqrt{3}+\sqrt{5}\right)$ is irrational.

Question 11.

Show that 12^n cannot end with the digit 0 or 5 for any natural number n.

Solution:

Number ending at 0 or 5 must be divisible by 5.

So,

$(12)^n = (2 \times 2 \times 3)^n$

$= 2^{2n} \times 3^n$

$(12)^n$ does not have 5 as its factor. So, 12^n can never end with 5 and zero.

Question 12.

On a morning walk, three persons, step off together and their steps measure 40cm,42 cm and 45cm respectively. What is the minimum distance each should walk, so that each can cover the same distance in complete steps?

Solution:

The minimum distance = LCM of covered steps.

$40 = 2^3 \times 5$

$42 = 2 \times 3 \times 7$

$45 = 3^2 \times 5$

LCM of (40, 42, 45) $= 2^3 \times 3^2 \times 5 \times 7$

$= 2520$ cm

Hence, the minimum distance each should walk is 2520 cm.

Question 13.

Write the denominator of rational number $\frac{257}{5000}$ in the form of $2^m \times 5^n$, where m, n are non-negative integers. Hence, write its decimal expansion, without actual division.

Solution:

We have, denominator of the rational number $\frac{257}{5000}$ =5000.

Therefore,

$5000 = 2^3 \times 5^4$

It is of the form $2^m \times 5^n$

Here m = 3 and n = 4

So,

$$\frac{257}{5000} = \frac{257 \times 2}{5000 \times 2}$$

$$= 0.0514$$

Question 14.

Prove that $\left(\sqrt{p} + \sqrt{q}\right)$ is irrational, where p and q are primes.

Solution:

Let us take $\left(\sqrt{p} + \sqrt{q}\right)$ = rational and can be represented as $\left(\sqrt{p} + \sqrt{q}\right)$ = x

$$\left(\sqrt{p} + \sqrt{q}\right) = x$$

$$\sqrt{p} = x - \sqrt{q}$$

Squaring both sides,

$$\left(\sqrt{p}\right)^2 = \left(x - \sqrt{q}\right)^2$$

$$p = \left(x^2 - 2x\sqrt{q} + q\right)$$

$$\sqrt{q} = \frac{x^2 + q - p}{2x}$$

As q is prime so $\sqrt{q}$ is no rational but in above solution we had $\sqrt{q}$ = rational because a, p, q are non-zero integers which contradicts our fact.

So, $\left(\sqrt{p} + \sqrt{q}\right)$ is irrational.

Exercise No: 1.4

Question 1.

Show that the cube of a positive integer of the form 6q + r, q is an integer and r = 0, 1, 2, 3, 4, 5 is also of the form 6m + r.

Solution:

We have,

6q + r is a positive integer, where q is an integer and r = 0, 1, 2, 3, 4, 5

So, the positive integers are of the form 6q, 6q+1, 6q+2, 6q+3, 6q+4 and 6q+5.

Cube of these integers will be:

Taking 6q,

$(6q)^3 = 216\ q^3$

$= 6(36q)^3 + 0$

$= 6m + 0, (m = (36q)^3)$

Taking 6q+1,

$(6q+1)^3 = 216q^3 + 108q^2 + 18q + 1$

$= 6(36q^3 + 18q^2 + 3q) + 1$

$= 6m + 1, (m = 36q^3 + 18q^2 + 3q)$

Taking 6q+2,

$(6q+2)^3 = 216q^3 + 216q^2 + 72q + 8$

$= 6(36q^3 + 36q^2 + 12q + 1) +2$

$= 6m + 2, (m= 36q^3 + 36q^2 + 12q + 1)$

Taking 6q+3,

$(6q+3)^3 = 216q^3 + 324q^2 + 162q + 27$

$= 6(36q^3 + 54q^2 + 27q + 4) + 3$

$= 6m + 3, (m = 36q^3 + 54q^2 + 27q + 4)$

Taking 6q+4,

$(6q+4)^3 = 216q^3 + 432q^2 + 288q + 64$

$= 6(36q^3 + 72q^2 + 48q + 10) + 4$

$= 6m + 4$, (where m is an integer $= 36q^3 + 72q^2 + 48q + 10$)

Taking 6q+5,

$(6q+5)^3 = 216q^3 + 540q^2 + 450q + 125$

$= 6(36q^3 + 90q^2 + 75q + 20) + 5$

$= 6m + 5$ ($m = 36q^3 + 90q^2 + 75q + 20$)

Therefore, the cube of a positive integer of the form 6q + r, q is an integer and r = 0, 1, 2, 3, 4, 5 is also of the form 6m + r.

2. Prove that one and only one out of n, n + 2 and n + 4 is divisible by 3, where n is any positive integer.

Solution:

Using Euclid's Algorithm,

Let the positive integer = n and b =3.

n =3q+r, where q is the quotient and r is the remainder.

Remainders can be 0, 1 and 2

So, n can be 3q, 3q+1, 3q+2

For n = 3q

n+2 = 3q+2

n+4 = 3q+4

In this case n is only divisible by 3.

For n = 3q+1

n+2 = 3q+3

In this case n+2 is divisible by 3.

For n = 3q+2

n+2 = 3q+4

n+4 = 3q+2+4

=3q+6

In this case n+4 is divisible by 3.

Therefore, we can say that one and only one out of n, n + 2 and n + 4 is divisible by 3.

Question 3.

Prove that one of any three consecutive positive integers must be divisible by 3.

Solution:

Let the three consecutive number be n, n+1, n+2.

By Euclid's algorithm, we have

n = 3q+r

Where q is quotient and r is remainder

The values of r can be = 0, 1, 2.

For r = 0,

n = 3q

n+1=3q+1

n+2 = 3q+2

So, here n is divisible by 3.

Now, For r = 1,

n = 3q+1

n+1=3q+2

n+2 = 3q+3

So, here n+2 is divisible by 3.

For r = 2,

n = 3q+2

n+1=3q+3

n+2 = 3q+4

So, here n+1 is divisible by 3.

Therefore one of any three consecutive positive integers must be divisible by 3.

Question 4.

For any positive integer n, prove that $n^3 – n$ is divisible by 6.

Solution:

Let x = $n^3 – n$

$x = n(n^2 - 1)$

$x = n(n - 1)(n + 1)$

$(n - 1)$, n, $(n + 1)$ are consecutive integers so out of three consecutive numbers at least one will be even.

So, we can say that x is divisible by 2.

Sum of number = $(n - 1) + n + (n + 1)$

$= n - 1 + n + n + 1$

$= 3n$

Here, the sum of three consecutive numbers is divisible by 3, so one of them is divisible by 3.

So, out of n, $(n - 1)$, $(n + 1)$, one is divisible by 2 and one is divisible by 3 and

$x = (n - 1) \times n \times (n + 1)$

Hence, out of three factors of x, one is divisible by 2 and one is divisible by 3.

Therefore, x is divisible by 6 or $n^3 - n$ is divisible by 6.

Question 5.

Show that one and only one out of n, (n + 4), (n + 8), (n + 12), (n + 16) is divisible by 5, where n is any positive integer.

[Hint: Any positive integer can be written in the form 5q, (5q + 1), (5q + 2), (5q + 3), (5q +4)]

Solution:

If a number n is divided by 5 then let the quotient is q and remainder is r. Then by Euclid's algorithm,

$n = 5q + r$, where n, q, r are positive integers and $0 \leq r < 5$.

For, $r = 0$,

$n = 5q + 0$

$= 5q$

So, n is divisible by 5.

For, $r = 1$,

$n = 5q + 1$

$n + 4 = (5q + 1) + 4$

$= 5q + 5$

$= 5(q + 1)$ divisible by 5.

We can say that $(n + 4)$ is divisible by 5.

For $r = 2$,

$n = 5q + 2$

$(n + 8) = (5q + 2) + 8$

$= 5q + 10 = 5(q + 2)$

$= 5m$ is divisible by 5.

We can say that, $(n + 8)$ is divisible by 5.

For, $r = 3$,

$n = 5q + 3$

$n + 12 = (5q + 3) + 12$

$= 5q + 15$

$= 5(q + 3) = 5m$ is divisible by 5.

We can say that, $(n + 12)$ is divisible by 5.

For, $r = 4$,

$n = 5q + 4$

$n + 16 = (5q + 4) + 16$

$= 5q + 20 = 5(q + 4)$

$(n + 16) = 5m$ is divisible by 5.

Therefore, n, $(n + 4)$, $(n + 8)$, $(n + 12)$ and $(n + 16)$ are divisible by 5.