

Chapter - 8
Introduction to Trigonometry and its Equation
Exercise No. 8.1

Multiple Choice Questions:

Choose the correct answer from the given four options:

1. If $\cos A = \frac{4}{5}$, then the value of $\tan A$ is

(A) $\frac{3}{5}$
(B) $\frac{3}{4}$
(C) $\frac{4}{3}$
(D) $\frac{5}{3}$

Solution:

As given in the question,

$$\cos A = \frac{4}{5} \quad \dots(1)$$

Also,

$$\tan A = \frac{\sin A}{\cos A}$$

We have,

$$\sin^2 \theta + \cos^2 \theta = 1$$

So,

$$\sin \theta = \sqrt{(1 - \cos^2 \theta)}$$

Then,

$$\sin A = \sqrt{(1 - \cos^2 A)} \quad \dots(2)$$

$$\sin^2 A = 1 - \cos^2 A$$

$$\sin A = \sqrt{(1 - \cos^2 A)}$$

Putting equation (1) in (2),

We get,

$$\begin{aligned}\sin A &= \sqrt{1 - \left(\frac{4}{5}\right)^2} \\ &= \left(\frac{3}{5}\right)\end{aligned}$$

So,

$$\begin{aligned}\tan A &= \left(\frac{3}{5}\right) \times \left(\frac{5}{4}\right) \\ &= \left(\frac{3}{4}\right)\end{aligned}$$

2. If $\sin A = \frac{1}{2}$, then the value of $\cot A$ is

(A) $\sqrt{3}$

(B) $\frac{1}{\sqrt{3}}$

(C) $\frac{\sqrt{3}}{2}$

(D) 1

Solution:

As given in the question,

$$\sin A = \frac{1}{2} \quad \dots (1)$$

We have,

$$\cot A = \frac{\cos A}{\sin A} \quad \dots (2)$$

Now to find the value of $\cos A$,

We have,

$$\sin^2 \theta + \cos^2 \theta = 1$$

So,

$$\cos \theta = \sqrt{(1 - \sin^2 \theta)}$$

Then,

$$\cos A = \sqrt{(1 - \sin^2 A)} \quad \dots (3)$$

$$\cos^2 A = 1 - \sin^2 A$$

$$\cos A = \sqrt{1 - \sin^2 A}$$

Putting equation 1 in 3, we get,

$$\begin{aligned}\cos A &= \sqrt{1 - \frac{1}{4}} \\ &= \sqrt{\frac{3}{4}} \\ &= \frac{\sqrt{3}}{2}\end{aligned}$$

Putting values of $\sin A$ and $\cos A$ in equation 2,

$$\cot A = \left(\frac{\sqrt{3}}{2}\right) \times 2 = \sqrt{3}$$

3. The value of the expression $[\cosec(75^\circ + \theta) - \sec(15^\circ - \theta) - \tan(55^\circ + \theta) + \cot(35^\circ - \theta)]$ is

- (A) -1
- (B) 0
- (C) 1
- (D) 3 2

Solution:

As given in the question,

The value of the equation,

$$\begin{aligned}\cosec(75^\circ + \theta) - \sec(15^\circ - \theta) - \tan(55^\circ + \theta) + \cot(35^\circ - \theta) \\ = \cosec[90^\circ - (15^\circ - \theta)] - \sec(15^\circ - \theta) - \tan(55^\circ + \theta) + \cot[90^\circ - (55^\circ + \theta)]\end{aligned}$$

As,

$$\cosec(90^\circ - \theta) = \sec \theta$$

Also,

$$\cot(90^\circ - \theta) = \tan \theta$$

So,

$$\begin{aligned}\cosec[90^\circ - (15^\circ - \theta)] - \sec(15^\circ - \theta) - \tan(55^\circ + \theta) + \cot[90^\circ - (55^\circ + \theta)] \\ = \sec(15^\circ - \theta) - \sec(15^\circ - \theta) - \tan(55^\circ + \theta) + \tan(55^\circ + \theta) \\ = 0\end{aligned}$$

4. Given that $\sin \theta = \frac{a}{b}$, then $\cos \theta$ is equal to

(A) $\frac{b}{\sqrt{b^2 - a^2}}$

(B) $\frac{b}{a}$

(C) $\frac{\sqrt{b^2 - a^2}}{b}$

(D) $\frac{a}{\sqrt{b^2 - a^2}}$

Solution:

As given in the question,

$$\sin \theta = \frac{a}{b}$$

We have,

$$\begin{aligned}\sin^2 \theta + \cos^2 \theta &= 1 \\ \sin^2 A &= 1 - \cos^2 A \\ \sin A &= \sqrt{1 - \cos^2 A}\end{aligned}$$

So,

$$\cos \theta = \sqrt{1 - \frac{a^2}{b^2}}$$

$$= \left(\sqrt{\frac{b^2 - a^2}{b^2}} \right)$$

$$= \frac{\sqrt{b^2 - a^2}}{b}$$

Therefore,

$$\cos \theta = \frac{\sqrt{b^2 - a^2}}{b}$$

5. If $\cos(\alpha + \beta) = 0$, then $\sin(\alpha - \beta)$ can be reduced to

(A) $\cos \beta$

(B) $\cos 2\beta$

(C) $\sin \alpha$
 (D) $\sin 2\alpha$

Solution:

As given in the question,

$$\cos(\alpha+\beta) = 0$$

we have,

$$\cos 90^\circ = 0$$

Also, we can write,

$$\cos(\alpha+\beta) = \cos 90^\circ$$

On comparing cosine equation on L.H.S and R.H.S,

We get,

$$\begin{aligned} \alpha+\beta &= 90^\circ \\ \alpha &= 90^\circ - \beta \end{aligned}$$

Now we need to reduce $\sin(\alpha - \beta)$,

So, we take,

$$\begin{aligned} \sin(\alpha - \beta) &= \sin(90^\circ - \beta - \beta) \\ &= \sin(90^\circ - 2\beta) \end{aligned}$$

Also,

$$\sin(90^\circ - \theta) = \cos \theta$$

So,

$$\sin(90^\circ - 2\beta) = \cos 2\beta$$

Hence, $\sin(\alpha - \beta) = \cos 2\beta$

6. The value of $(\tan 1^\circ \tan 2^\circ \tan 3^\circ \dots \tan 89^\circ)$ is

(A) 0
 (B) 1
 (C) 2
 (D) $\frac{1}{2}$

Solution:

According to question,

$$\tan 1^\circ \cdot \tan 2^\circ \cdot \tan 3^\circ \dots \tan 89^\circ$$

$$= \tan 1^\circ \cdot \tan 2^\circ \cdot \tan 3^\circ \dots \tan 43^\circ \cdot \tan 44^\circ \cdot \tan 45^\circ \cdot \tan 46^\circ \cdot \tan 47^\circ \dots \tan 87^\circ \cdot \tan 88^\circ \cdot \tan 89^\circ$$

Also,

$$\tan 45^\circ = 1,$$

$$= \tan 1^\circ \cdot \tan 2^\circ \cdot \tan 3^\circ \dots \tan 43^\circ \cdot \tan 44^\circ \cdot 1 \cdot \tan 46^\circ \cdot \tan 47^\circ \dots \tan 87^\circ \cdot \tan 88^\circ \cdot \tan 89^\circ$$

Now, we can write,

$$= \tan 1^\circ \cdot \tan 2^\circ \cdot \tan 3^\circ \dots \tan 43^\circ \cdot \tan 44^\circ \cdot 1 \cdot \tan(90^\circ - 44^\circ) \cdot \tan(90^\circ - 43^\circ) \dots \tan(90^\circ - 3^\circ) \cdot \tan(90^\circ - 2^\circ) \cdot \tan(90^\circ - 1^\circ)$$

As,

$$\tan(90^\circ - \theta) = \cot \theta,$$

$$= \tan 1^\circ \cdot \tan 2^\circ \cdot \tan 3^\circ \dots \tan 43^\circ \cdot \tan 44^\circ \cdot 1 \cdot \cot 44^\circ \cdot \cot 43^\circ \dots \cot 3^\circ \cdot \cot 2^\circ \cdot \cot 1^\circ$$

And also,

$$\tan \theta = \left(\frac{1}{\cot \theta} \right)$$

$$= \tan 1^\circ \cdot \tan 2^\circ \cdot \tan 3^\circ \dots \tan 43^\circ \cdot \tan 44^\circ \cdot 1 \cdot (1/\tan 44^\circ) \cdot (1/\tan 43^\circ) \dots (1/\tan 2^\circ) \cdot (1/\tan 1^\circ) \\ = 1$$

$$\text{Therefore, } \tan 1^\circ \cdot \tan 2^\circ \cdot \tan 3^\circ \dots \tan 89^\circ = 1$$

7. If $\cos 9\alpha = \sin \alpha$ and $9\alpha < 90^\circ$, then the value of $\tan 5\alpha$ is

(A) $\frac{1}{\sqrt{3}}$

(B) $\sqrt{3}$

(C) 1

(D) 0

Solution:

As given in the question,

$$\cos 9\alpha = \sin \alpha$$

and

$$9\alpha < 90^\circ$$

Which means,

9α is an acute angle

We know that,
 $\sin(90^\circ - \theta) = \cos \theta$
So,
 $\cos 9\alpha = \sin (90^\circ - \alpha)$

Also,
 $\cos 9\alpha = \sin(90^\circ - 9\alpha)$
and
 $\sin(90^\circ - \alpha) = \sin \alpha$

So,

$$\begin{aligned}\sin (90^\circ - 9\alpha) &= \sin \alpha \\ 90^\circ - 9\alpha &= \alpha \\ 10\alpha &= 90^\circ \\ \alpha &= 9^\circ\end{aligned}$$

Putting $\alpha = 9^\circ$ in $\tan 5\alpha$,
we get,

$$\begin{aligned}\tan 5\alpha &= \tan (5 \times 9) \\ &= \tan 45^\circ \\ &= 1\end{aligned}$$

So,
 $\tan 5\alpha = 1$

8. If ΔABC is right angled at C , then the value of $\cos (A+B)$ is

(A) 0
(B) 1
(C) $\frac{1}{2}$
(D) $\frac{\sqrt{3}}{2}$

Solution:

(A)

We have, in ΔABC ,

$$\begin{aligned}\text{Sum of three angles} &= 180^\circ \\ \angle A + \angle B + \angle C &= 180^\circ\end{aligned}$$

Also the triangle is right angled at C , so,
 $\angle C = 90^\circ$
 $\angle A + \angle B + 90^\circ = 180^\circ$

$$\begin{aligned}
 A + B &= 90^\circ \\
 \cos(A + B) &= \cos 90^\circ \\
 &= 0
 \end{aligned}$$

9. If $\sin A + \sin^2 A = 1$, then the value of the expression $(\cos^2 A + \cos^4 A)$ is

- (A) 1
- (B) $\frac{1}{2}$
- (C) 2
- (D) 3

Solution:

(A)

We have,
 $\sin A + \sin^2 A = 1$
 $\sin A = 1 - \sin^2 A$
 $= \cos^2 A$

[as, $\sin^2 \theta + \cos^2 \theta = 1$]

$$\sin A = \cos^2 A$$

On squaring both sides, we get

$$\begin{aligned}
 \sin^2 A &= \cos^4 A \\
 1 - \cos^2 A &= \cos^4 A \\
 \cos^2 A + \cos^4 A &= 1
 \end{aligned}$$

10. Given that $\sin \alpha = \frac{1}{2}$ and $\cos \beta = \frac{1}{2}$, then the value of $(\alpha + \beta)$ is

- (A) 0°
- (B) 30°
- (C) 60°
- (D) 90°

Solution:

$$\sin \alpha = \frac{1}{2} = \sin 30^\circ \quad [\text{and, } \sin 30^\circ = \frac{1}{2}]$$

So,
 $\alpha = 30^\circ$

and

$$\begin{aligned}
 \cos \beta &= \frac{1}{2} \\
 &= \cos 60^\circ
 \end{aligned}$$

$$\beta = 60^\circ$$

$$\alpha + \beta = 30^\circ + 60^\circ$$

$$= 90^\circ$$

11. The value of the expression $\left[\frac{\sin^2 22^\circ + \sin^2 68^\circ}{\cos^2 22^\circ + \cos^2 68^\circ} + \sin^2 63^\circ + \cos 63^\circ \sin 27^\circ \right]$ **is**

- (A) 3
- (B) 2
- (C) 1
- (D) 0

Solution:

(B)

$$\begin{aligned} \left[\frac{\sin^2 22^\circ + \sin^2 68^\circ}{\cos^2 22^\circ + \cos^2 68^\circ} + \sin^2 63^\circ + \cos 63^\circ \sin 27^\circ \right] &= \left[\frac{\sin^2 22^\circ + \sin^2 68^\circ}{\cos^2 22^\circ + \cos^2 68^\circ} + \sin^2 63^\circ + \cos 63^\circ \sin 27^\circ \right] \\ &= \left[\frac{\sin^2 22^\circ + \sin^2 (90 - 22^\circ)}{\cos^2 22^\circ + \cos^2 (90 - 22^\circ)} + \sin^2 63^\circ + \cos 63^\circ \sin (90 - 63^\circ) \right] \\ &= \left[\frac{\sin^2 22^\circ + \sin^2 (90 - 22^\circ)}{\cos^2 22^\circ + \cos^2 (90 - 22^\circ)} + \sin^2 63^\circ + \cos 63^\circ \cos 63^\circ \right] \\ &= \frac{\sin^2 22^\circ + \cos^2 22^\circ}{\cos^2 22^\circ + \sin^2 22^\circ} + \sin^2 63^\circ + \cos^2 63^\circ \\ &= 1 + 1 \\ &= 2 \end{aligned}$$

12. If $4 \tan \theta = 3$, **then** $\left(\frac{4 \sin \theta - \cos \theta}{4 \sin \theta + \cos \theta} \right)$ **is equal to**

- (A) $\frac{2}{3}$
- (B) $\frac{1}{3}$
- (C) $\frac{1}{2}$
- (D) $\frac{3}{4}$

Solution:

(C)

We have,

$$4\tan\theta = 3$$

$$\tan\theta = \frac{3}{4}$$

we have,

$$\begin{aligned} \left(\frac{4\sin\theta - \cos\theta}{4\sin\theta + \cos\theta} \right) &= \left(\frac{\frac{4\sin\theta}{\cos\theta} - 1}{\frac{\sin\theta}{\cos\theta} + 1} \right) && \text{(dividing by } \cos\theta\text{)} \\ &= \left(\frac{4\tan\theta - 1}{4\tan\theta + 1} \right) \\ &= \left(\frac{4\left(\frac{3}{4}\right) - 1}{4\left(\frac{3}{4}\right) + 1} \right) \\ &= \frac{1}{2} \end{aligned}$$

13. If $\sin\theta - \cos\theta = 0$, then the value of $(\sin^4\theta + \cos^4\theta)$ is

(A) 1

(B) $\frac{3}{4}$

(C) $\frac{1}{2}$

(D) $\frac{1}{4}$

Solution:

We have,

$$\sin \theta - \cos \theta = 0$$

$$\sin \theta = \cos \theta$$

$$\frac{\sin \theta}{\cos \theta} = 1$$

$$\tan \theta = 1$$

also,

$$\tan 45^\circ = 1$$

$$\tan \theta = \tan 45^\circ$$

$$\theta = 45^\circ$$

So,

$$(\sin^4 \theta + \cos^4 \theta) = (\sin^4 45^\circ + \cos^4 45^\circ)$$

$$= \frac{1}{2}$$

14. $\sin(45^\circ + \theta) - \cos(45^\circ - \theta)$ is equal to

- (A) $2\cos \theta$
- (B) **0**
- (C) $2\sin \theta$
- (D) **1**

Solution:

(B)

$$\begin{aligned}\sin(45^\circ + \theta) - \cos(45^\circ - \theta) &= \cos[90^\circ - (45^\circ + \theta)] - \cos(45^\circ - \theta) \\ [\text{as, } \cos(90^\circ - \theta) &= \sin \theta] \\ &= \cos(45^\circ - \theta) - \cos(45^\circ - \theta) \\ &= 0\end{aligned}$$

15. A pole 6 m high casts a shadow $2\sqrt{3}$ m long on the ground, then the Sun's elevation is

- (A) **60°**
- (B) **45°**
- (C) **30°**
- (D) **90°**

Solution:

Taking BC = 6 m be the height of the pole and AB = $2\sqrt{3}$ m be the length of the shadow on the ground.

And let the Sun's elevation be θ .

In ABC,

$$\tan \theta = \frac{BC}{AB}$$

$$\tan \theta = \frac{6}{2\sqrt{3}}$$

$$\tan \theta = \sqrt{3} = \tan 60^\circ$$

$$\tan \theta = \tan 60^\circ$$

$$\theta = 60^\circ$$

So, the Sun's elevation is 60° .

Exercise No. 8.2

Short Answer Questions with Reasoning:

Write 'True' or 'False' and justify your answer in each of the following:

1.
$$\frac{\tan 47^\circ}{\cot 43^\circ} = 1$$

Solution:

True

As, $\tan(90^\circ - \theta) = \cot \theta$

$$\begin{aligned}\frac{\tan 47^\circ}{\cot 43^\circ} &= \frac{\tan(90^\circ - 43^\circ)}{\cot 43^\circ} \\ &= \frac{\cot 43^\circ}{\cot 43^\circ} \\ &= 1\end{aligned}$$

2. The value of the expression $(\cos^2 23^\circ - \sin^2 67^\circ)$ is positive.

Solution:

False

As,

$$(a^2 - b^2) = (a+b)(a-b)$$

$$\cos^2 23^\circ - \sin^2 67^\circ = (\cos 23^\circ + \sin 67^\circ)(\cos 23^\circ - \sin 67^\circ)$$

$$\begin{aligned}&= [\cos 23^\circ + \sin(90^\circ - 23^\circ)] [\cos 23^\circ - \sin(90^\circ - 23^\circ)] \\ &= (\cos 23^\circ + \cos 23^\circ)(\cos 23^\circ - \cos 23^\circ) \\ &\quad (\text{as, } \sin(90^\circ - \theta) = \cos \theta) \\ &= (\cos 23^\circ + \cos 23^\circ) \cdot 0 \\ &= 0,\end{aligned}$$

And 0 is neither positive nor negative

3. The value of the expression $(\sin 80^\circ - \cos 80^\circ)$ is negative.

Solution:

False

As,

$\sin \theta$ increases when $0^\circ \leq \theta \leq 90^\circ$

$\cos \theta$ decreases when $0^\circ \leq \theta \leq 90^\circ$

Therefore,

$$\begin{aligned}
 (\sin 80^\circ - \cos 80^\circ) &= (\text{increasing value} - \text{decreasing value}) \\
 &= \text{a positive value.}
 \end{aligned}$$

So,

$$(\sin 80^\circ - \cos 80^\circ) > 0.$$

$$4. \sqrt{(1 - \cos^2 \theta) \sec^2 \theta} = \tan \theta$$

Solution:

True

$$\begin{aligned}
 LHS &= \sqrt{(1 - \cos^2 \theta) \sec^2 \theta} \\
 &= \sqrt{\sin^2 \theta \sec^2 \theta} \\
 &= \sqrt{\frac{\sin^2 \theta}{\cos^2 \theta}} \\
 &= \sqrt{\tan^2 \theta} \\
 &= \tan \theta
 \end{aligned}$$

$$5. \text{ If } \cos A + \cos^2 A = 1, \text{ then } \sin^2 A + \sin^4 A = 1$$

Solution:

True

As given in the question,

$$\cos A + \cos^2 A = 1$$

$$\cos A = 1 - \cos^2 A$$

Also,

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\sin^2 \theta = 1 - \cos^2 \theta$$

So,

$$\cos A = \sin^2 A \quad \dots(1)$$

Squaring L.H.S and R.H.S,

$$\cos^2 A = \sin^4 A \quad \dots(2)$$

Adding equations (1) and (2),

We get

$$\sin^2 A + \sin^4 A = \cos A + \cos^2 A$$

So,

$$\sin^2 A + \sin^4 A = 1$$

$$6. (\tan \theta + 2)(2 \tan \theta + 1) = 5 \tan \theta + \sec^2 \theta$$

Solution:

False

L.H.S

$$\begin{aligned} &= (\tan \theta + 2)(2 \tan \theta + 1) \\ &= 2 \tan^2 \theta + \tan \theta + 4 \tan \theta + 2 \\ &= 2 \tan^2 \theta + 5 \tan \theta + 2 \end{aligned}$$

As,

$$\sec^2 \theta - \tan^2 \theta = 1,$$

we get,

$$\begin{aligned} \tan^2 \theta &= \sec^2 \theta - 1 \\ &= 2(\sec^2 \theta - 1) + 5 \tan \theta + 2 \\ &= 2 \sec^2 \theta - 2 + 5 \tan \theta + 2 \\ &= 5 \tan \theta + 2 \sec^2 \theta \end{aligned}$$

So,

L.H.S \neq R.H.S

7. If the length of the shadow of a tower is increasing, then the angle of elevation of the sun is also increasing.

Solution:

False

To understand the fact of this question, consider the following example

(i) A tower $2\sqrt{3}$ m high casts a shadow 2 m long on the ground, then the Sun's elevation is 60°

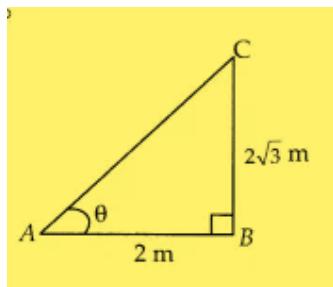
In ΔABC ,

$$\tan \theta = \frac{BC}{AB}$$

$$\tan \theta = \frac{2\sqrt{3}}{2}$$

$$\tan \theta = \sqrt{3}$$

$$\tan \theta = 60^\circ$$



(ii) The same height of tower casts a shadow 4 m more from preceding point, then the Sun's elevation is 30°

In ΔPBC ,

$$\tan \theta = \frac{CB}{PB}$$

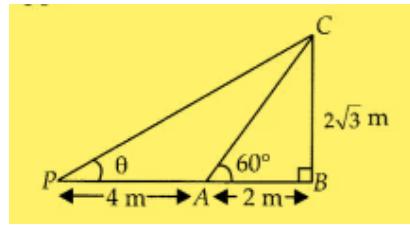
$$\tan \theta = \frac{CB}{PA + AB}$$

$$\tan \theta = \frac{2\sqrt{3}}{4 + 2}$$

$$\tan \theta = \frac{2\sqrt{3}}{6}$$

$$\tan \theta = \frac{1}{\sqrt{3}}$$

$$\tan \theta = 30^\circ$$



Therefore, we conclude from above two examples that if the length of the shadow of a tower is increasing, then the angle of elevation of the Sun is decreasing

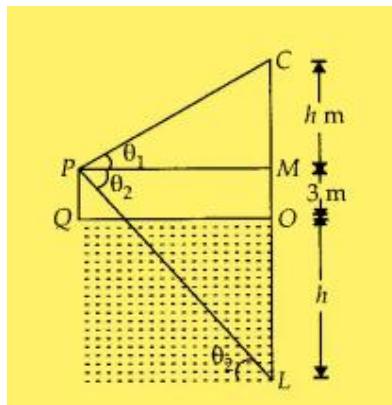
8. If a man standing on a platform 3 metres above the surface of a lake observes a cloud and its reflection in the lake, then the angle of elevation of the cloud is equal to the angle of depression of its reflection.

Solution:

False

We can observe that, a man standing on a platform at point P, 3 m above the surface of a lake observes a cloud at point C.

Let the height of the cloud from the surface of the platform is h .



In MPC ,

$$\tan \theta_1 = \frac{CM}{PM}$$

$$\tan \theta_1 = \frac{h}{PM}$$

$$\frac{\tan \theta_1}{h} = \frac{1}{PM} \quad \dots(i)$$

In LPM,

$$\tan \theta_2 = \frac{LM}{PM}$$

$$\tan \theta_2 = \frac{OL+OM}{PM}$$

$$\tan \theta_2 = \frac{h+3}{PM}$$

$$\frac{\tan \theta_2}{h+3} = \frac{1}{PM} \quad \dots(ii)$$

From, (i) and (ii),

$$\frac{\tan \theta_2}{h+3} = \frac{\tan \theta_1}{h}$$

$$\tan \theta_2 = \frac{(h+3)}{h} \tan \theta_1$$

Therefore,

$$\theta_1 \neq \theta_2$$

9. The value of $2\sin \theta$ can be $a + \frac{1}{a}$, where a is a positive number, and $a \neq 1$.

Solution:

False

Let us take $a = 2$,

Then,

$$\begin{aligned} a + \frac{1}{a} &= 2 + \frac{1}{2} \\ &= \frac{5}{2} \end{aligned}$$

If $2\sin \theta = a + \frac{1}{a}$, then ,

$$\begin{aligned} 2\sin \theta &= \frac{5}{2} \\ \sin \theta &= \frac{5}{4} \\ &= 1.25 \end{aligned}$$

Which is not possible.

[as, $\sin \theta \leq 1$]

10. $\cos \theta = \frac{a^2 + b^2}{2ab}$, where a and b are two distinct numbers such that $ab > 0$.

Solution:

False

$$(a+b)^2 > 0$$

$$a^2 + b^2 > 2ab$$

$$\frac{a^2 + b^2}{2ab} > 1$$

$$\cos\theta = \cos\theta > 1$$

$$[\text{as, } \cos\theta = \frac{a^2 + b^2}{2ab}]$$

Which is not possible.

$$[\text{as, } -1 \leq \cos\theta \leq 1]$$

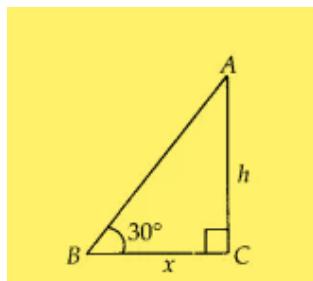
$$\text{So, } \cos\theta \neq \frac{a^2 + b^2}{2ab}$$

11. The angle of elevation of the top of a tower is 30° . If the height of the tower is doubled, then the angle of elevation of its top will also be doubled.

Solution:

False

Case (i): Let us take the height of the tower is h In ΔABC , let $BC = x$ m



In ΔABC ,

$$\tan 30^\circ = \frac{AC}{BC}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{x} \quad \dots(i)$$

Case (ii): By condition, the height of the tower is doubled, $PR = 2h$,

I

in ΔPQR ,

$$\tan \theta = \frac{PQ}{QR}$$

$$\tan \theta = \frac{2h}{x}$$

$$\tan \theta = \frac{2}{x} \times \frac{x}{\sqrt{3}} \quad \dots \text{(from (i))}$$

$$\tan \theta = \frac{2}{1} \times \frac{1}{\sqrt{3}}$$

$$\tan \theta = 1.15$$

$$\theta = \tan^{-1}(1.15)$$

which is less than 60°

So, the required angle is not doubled.

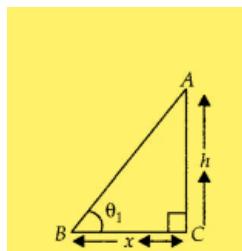
12. If the height of a tower and the distance of the point of observation from its foot, both, are increased by 10%, then the angle of elevation of its top remains unchanged.

Solution:

True

Case(i):

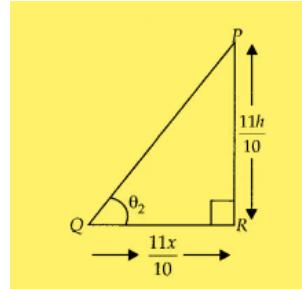
Let us take the height of a tower be h and the distance of the point of observation from its foot is x .



In ABC ,

$$\begin{aligned} \tan \theta_1 &= \frac{AC}{BC} \\ &= \frac{h}{x} \quad \dots (i) \end{aligned}$$

Case (ii):



Now, the height of a tower increased by 10%

$$\begin{aligned}
 &= h + 10\% \text{ of } h \\
 &= h + h \times 10/100 \\
 &= 11h/10
 \end{aligned}$$

and the distance of the point of observation from its foot

$$= x + 10\% \text{ of } x$$

$$= \frac{11x}{10}$$

In $\triangle PQR$,

$$\begin{aligned}
 \tan \theta_2 &= \frac{PR}{QR} \\
 &= \frac{\frac{11h}{10}}{\frac{11x}{10}} \\
 &= \frac{h}{x} \quad \dots\dots(ii)
 \end{aligned}$$

From (i) and (ii),

$$\tan \theta_1 = \tan \theta_2$$

$$\theta_1 = \theta_2$$

So, the required angle of elevation of its top remains unchanged.

Exercise No. 8.3

Short Answer Questions:

Prove the following (from Q.1 to Q.7):

$$1. \frac{\sin \theta}{1+\cos \theta} + \frac{1+\cos \theta}{\sin \theta} = 2\operatorname{cosec} \theta$$

Solution:

L.H.S=

$$\begin{aligned} \frac{\sin \theta}{1+\cos \theta} + \frac{1+\cos \theta}{\sin \theta} &= \frac{\sin^2 \theta + (1+\cos \theta)^2}{(1+\cos \theta)\sin \theta} \\ &= \frac{\sin^2 \theta + 1 + \cos^2 \theta + 2\cos \theta}{(1+\cos \theta)\sin \theta} \\ &= \frac{1+1+2\cos \theta}{(1+\cos \theta)\sin \theta} \\ &= \frac{2+2\cos \theta}{(1+\cos \theta)\sin \theta} \\ &= \frac{2(1+\cos \theta)}{(1+\cos \theta)\sin \theta} \\ &= \frac{2}{\sin \theta} \\ &= 2\operatorname{cosec} \theta \end{aligned}$$

=R.H.S

Hence proved.

$$2. \frac{\tan A}{1+\sec A} - \frac{\tan A}{1-\sec A} = 2\operatorname{cosec} A$$

Solution:

L.H.S:

$$\begin{aligned}
\frac{\tan A}{1 + \sec A} - \frac{\tan A}{1 - \sec A} &= \frac{\tan A(1 - \sec A) - \tan A(1 + \sec A)}{(1 + \sec A)(1 - \sec A)} \\
&= \frac{\tan A(1 - \sec A - 1 - \sec A)}{(1^2 - \sec^2 A)} \\
&= \frac{\tan A(-2\sec A)}{(1^2 - \sec^2 A)} \\
&= \frac{2\tan A \sec A}{(\sec^2 A - 1)} \\
&= \frac{2\tan A \sec A}{(\tan^2 A)} \\
&= \frac{2\sec A}{(\tan A)} \\
&= \frac{2\cos A}{\cos A \sin A} \\
&= \frac{2}{\sin A} \\
&= 2\csc A
\end{aligned}$$

(As,

$$\sec^2 A - \tan^2 A = 1$$

$$\sec^2 A - 1 = \tan^2 A$$

= R.H.S

Hence proved.

3. If $\tan A = \frac{3}{4}$, **then** $\sin A \cos A = \frac{12}{25}$

Solution:

As given in the question,

$$\tan A = \frac{3}{4}$$

Also,

$$\tan A = \frac{\text{Perpendicular}}{\text{Base}}$$

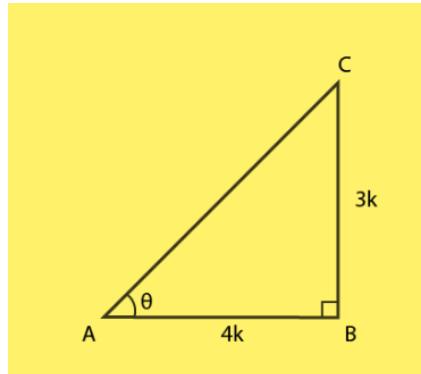
So,

$$\tan A = \frac{3k}{4k}$$

Where,

Perpendicular = 3k

Base = 4k



Using Pythagoras Theorem,

$$(\text{hypotenuse})^2 = (\text{perpendicular})^2 + (\text{base})^2$$

$$(\text{hypotenuse})^2 = (3k)^2 + (4k)^2$$

$$= 9k^2 + 16k^2$$

$$= 25k^2$$

$$\text{hypotenuse} = 5k$$

Now find $\sin A$ and $\cos A$,

$$\sin A = \frac{3k}{5k}$$

$$= \frac{3}{5}$$

$$\cos A = \frac{4k}{5k}$$

$$= \frac{4}{5}$$

So,

$$\begin{aligned} \sin A \cos A &= \frac{3}{5} \times \frac{4}{5} \\ &= \frac{12}{25} \end{aligned}$$

Hence, proved.

$$4. (\sin \alpha + \cos \alpha)(\tan \alpha + \cot \alpha) = \sec \alpha + \operatorname{cosec} \alpha$$

Solution:

L.H.S:

$$(\sin \alpha + \cos \alpha)$$

So,

$$\begin{aligned} & (\sin \alpha + \cos \alpha)(\tan \alpha + \cot \alpha) \\ & (\sin \alpha + \cos \alpha) \left(\frac{\sin \alpha}{\cos \alpha} + \frac{\cos \alpha}{\sin \alpha} \right) \\ & (\sin \alpha + \cos \alpha) \left(\frac{\sin^2 \alpha + \cos^2 \alpha}{\cos \alpha \sin \alpha} \right) \\ & (\sin \alpha + \cos \alpha) \left(\frac{1}{\cos \alpha \sin \alpha} \right) \\ & \left(\frac{\sin \alpha}{\cos \alpha \sin \alpha} + \frac{\cos \alpha}{\cos \alpha \sin \alpha} \right) \\ & \frac{1}{\cos \alpha} + \frac{1}{\sin \alpha} \\ & \sec \alpha + \csc \alpha \end{aligned}$$

= R.H.S

Hence, proved.

$$5. (\sqrt{3} + 1)(3 - \cot 30^\circ) = \tan^3 60^\circ - 2 \sin 60^\circ$$

Solution:

$$\begin{aligned} & (\sqrt{3} + 1)(3 - \cot 30^\circ) = (\sqrt{3} + 1)(3 - \sqrt{3}) \\ & \quad [\text{as, } \cot 30^\circ = \sqrt{3}] \\ & = (\sqrt{3} + 1)\sqrt{3}(\sqrt{3} - 1) \\ & \quad [\text{as, } (3 - \sqrt{3}) = \sqrt{3}(\sqrt{3} - 1)] \\ & = ((\sqrt{3})2 - 1)\sqrt{3} \\ & \quad [\text{as, } (\sqrt{3} + 1)(\sqrt{3} - 1) = ((\sqrt{3})^2 - 1)] \\ & = (3 - 1)\sqrt{3} \\ & = 2\sqrt{3} \end{aligned}$$

Also, solving

R.H.S:

$$\tan^3 60^\circ - 2 \sin 60^\circ$$

As,

$$\tan 60^\circ = \sqrt{3} \text{ and}$$
$$\sin 60^\circ = \sqrt{3}/2,$$

We get,

$$(\sqrt{3})^3 - 2 \cdot (\sqrt{3}/2) = 3\sqrt{3} - \sqrt{3}$$
$$= 2\sqrt{3}$$

So,

$$\text{L.H.S} = \text{R.H.S}$$

Hence, proved.

6. $1 + \frac{\cot^2 \alpha}{1 + \operatorname{cosec} \alpha} = \operatorname{cosec} \alpha$

Solution:

Taking LHS,

$$1 + \frac{\cot^2 \alpha}{1 + \operatorname{cosec} \alpha}$$

$$1 + \frac{\cot^2 \alpha}{1 + \operatorname{cosec} \alpha} = 1 + \frac{\cot^2 \alpha}{1 + \operatorname{cosec} \alpha}$$
$$= 1 + \frac{\frac{\cos^2 \alpha}{\sin^2 \alpha}}{1 + \frac{1}{\sin \alpha}}$$
$$= 1 + \frac{\cos^2 \alpha}{\sin \alpha (1 + \sin \alpha)}$$
$$= \frac{\sin \alpha + \sin^2 \alpha + \cos^2 \alpha}{\sin \alpha (1 + \sin \alpha)}$$
$$= \frac{1 + \sin \alpha}{\sin \alpha (1 + \sin \alpha)}$$
$$= \operatorname{cosec} \alpha$$

$$7. \tan \theta + \tan(90^\circ - \theta) = \sec \theta \sec(90^\circ - \theta)$$

Solution:

L.H.S=

As,

$$\tan(90^\circ - \theta) = \cot \theta$$

$$\begin{aligned}
 \tan \theta + \tan(90^\circ - \theta) &= \tan \theta + \cot(\theta) \\
 &= \frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta} \\
 &= \frac{\sin^2 \theta + \cos^2 \theta}{\sin \theta \cos \theta} \\
 &= \frac{1}{\sin \theta \cos \theta} \\
 &= \sec \theta \cosec \theta \\
 &= \sec \theta \sec(90^\circ - \theta)
 \end{aligned}$$

Hence proved!!!

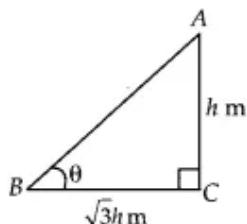
8. Find the angle of elevation of the sun when the shadow of a pole h metres high is $\sqrt{3}h$ metres long.

Solution:

Let the angle of elevation of the Sun is θ .

Given, height of pole = h m

Now, in $\triangle ABC$,



$$\begin{aligned}
 \tan \theta &= \frac{1}{\sqrt{3}} \\
 &= \tan 30^\circ \\
 \theta &= 30^\circ
 \end{aligned}$$

So, the angle of elevation of the Sun is 30° .

9. If $\sqrt{3} \tan \theta = 1$, then find the value of $\sin^2 \theta - \cos^2 \theta$.

Solution:

Given,

$$\sqrt{3} \tan \theta = 1$$

$$\tan \theta = \frac{1}{\sqrt{3}}$$

$$\tan \theta = \tan 30^\circ$$

$$\theta = 30^\circ$$

so,

$$\sin^2 \theta - \cos^2 \theta = \sin^2 30^\circ - \cos^2 30^\circ$$

$$= \left(\frac{1}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2$$

$$= -\frac{1}{2}$$

10. A ladder 15 metres long just reaches the top of a vertical wall. If the ladder makes an angle of 60° with the wall, find the height of the wall.

Solution:

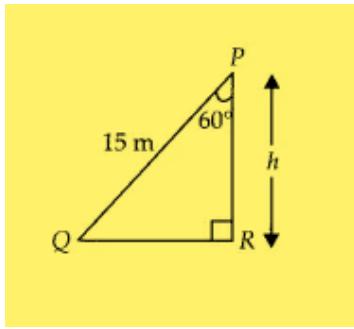
Given,

height of the ladder = 15 m

Let the height of the vertical wall = h

Because, and the ladder makes an angle of 60° with the wall i.e., $\theta = 60^\circ$

In ΔQPR ,



$$\cos 60^\circ = \frac{PR}{PQ}$$

$$\frac{1}{2} = \frac{h}{15}$$

$$h = \frac{15}{2} m$$

11. Simplify $(1 + \tan^2 \theta)(1 - \sin \theta)(1 + \sin \theta)$

Solution:

$$(1 + \tan^2 \theta)(1 - \sin \theta)(1 + \sin \theta)$$

$$= (1 + \tan^2 \theta)(1 - \sin^2 \theta)$$

$$[\text{as, } (a - b)(a + b) = a^2 - b^2]$$

$$= \sec^2 \theta \cdot \cos^2 \theta$$

$$[\text{as, } 1 + \tan^2 \theta = \sec^2 \theta \text{ and } \cos^2 \theta + \sin^2 \theta = 1]$$

$$= \frac{1}{\cos^2 \theta} \cdot \cos^2 \theta$$

$$= 1$$

$$[\text{as, } \sec \theta = \frac{1}{\cos \theta}]$$

12. If $2\sin^2 \theta - \cos^2 \theta = 2$, then find the value of θ .

Solution:

Given,

$$2\sin^2 \theta - \cos^2 \theta = 2$$

$$2\sin^2 \theta - (1 - \sin^2 \theta) = 2$$

$$[\text{as, } \sin^2 \theta + \cos^2 \theta = 1]$$

$$2\sin^2 \theta + \sin^2 \theta - 1 = 2$$

$$3\sin^2 \theta = 3$$

$$\sin^2 \theta = 1$$

$$\sin \theta = 1 = \sin 90^\circ$$

[as, $\sin 90^\circ = 1$]

$$\theta = 90^\circ$$

13. Show that
$$\frac{\cos^2(45^\circ + \theta) + \cos^2(45^\circ - \theta)}{\tan(60^\circ + \theta) \tan(30^\circ - \theta)} = 1$$

Solution:

$$LHS = \frac{\cos^2(45^\circ + \theta) + \cos^2(45^\circ - \theta)}{\tan(60^\circ + \theta) \tan(30^\circ - \theta)}$$

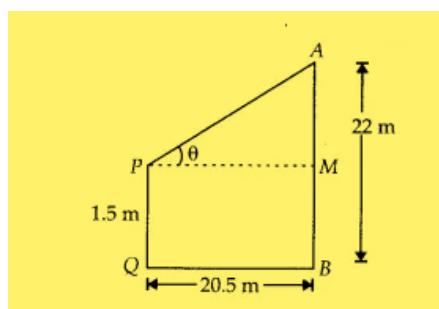
$$\begin{aligned} \frac{\cos^2(45^\circ + \theta) + \cos^2(45^\circ - \theta)}{\tan(60^\circ + \theta) \tan(30^\circ - \theta)} &= \frac{\cos^2(45^\circ + \theta) + \sin[90 - (45^\circ - \theta)]^2}{\tan(60^\circ + \theta) \cot[90 - (30^\circ - \theta)]} \\ &= \frac{\cos^2(45^\circ + \theta) + \sin^2(45^\circ - \theta)}{\tan(60^\circ + \theta) \frac{1}{\tan(60^\circ + \theta)}} \quad [\text{as, } \cot \theta = \frac{1}{\tan \theta}] \\ &= 1 \\ &= RHS \end{aligned}$$

14. An observer 1.5 metres tall is 20.5 metres away from a tower 22 metres high. Determine the angle of elevation of the top of the tower from the eye of the observer.

Solution:

Let the angle of elevation of the top of the tower from the eye of the observer is θ

Given that,



$$AB = 22\text{m},$$

$$PQ = 1.5 \text{ m} = MB$$

and

$$QB = PM = 20.5 \text{ m}$$

Also,

$$\begin{aligned} AM &= AB - MB \\ 22 - 1.5 &= 20.5 \text{ m} \end{aligned}$$

Now, in ΔAPM ,

$$\begin{aligned} \tan\theta &= AM/PM \\ &= 20.5/20.5 \\ &= 1 \\ &= \tan 45^\circ \end{aligned}$$

$$\begin{aligned} \tan\theta &= \tan 45^\circ \\ \theta &= 45^\circ \end{aligned}$$

Hence, the required angle of elevation of the top of the tower from the eye of the observer is 45° .

15. Show that $\tan^4\theta + \tan^2\theta = \sec^4\theta - \sec^2\theta$.

Solution:

$$\begin{aligned} LHS &= \tan^4\theta + \tan^2\theta \\ &= \tan^2\theta(\tan^2\theta + 1) \\ &= \tan^2\theta \cdot \sec^2\theta && [\text{as, } \sec^2\theta = \tan^2\theta + 1] \\ &= (\sec^2\theta - 1) \cdot \sec^2\theta && [\text{as, } \tan^2\theta = \sec^2\theta - 1] \\ &= \sec^4\theta - \sec^2\theta \\ &= RHS \end{aligned}$$

Exercise No. 8.4

Long Answer Questions:

Question:

1. If $\csc\theta + \cot\theta = p$, then prove that $\cos\theta = \frac{p^2 - 1}{p^2 + 1}$.

Solution:

As,

$$\csc\theta + \cot\theta = p$$

$$\frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta} = p$$

$$\frac{1 + \cos\theta}{\sin\theta} = p$$

Squaring both sides,

$$\left(\frac{1 + \cos\theta}{\sin\theta}\right)^2 = p^2$$

$$\frac{1 + \cos^2\theta + 2\cos\theta}{\sin^2\theta} = p^2$$

Now,

Using componendo dividendo rule,

$$\frac{1 + \cos^2\theta + 2\cos\theta - \sin^2\theta}{1 + \cos^2\theta + 2\cos\theta + \sin^2\theta} = \frac{p^2 - 1}{p^2 + 1}$$

As,

$$\cos^2\theta + \sin^2\theta = 1$$

So,

$$\frac{1 + \cos^2\theta + 2\cos\theta - (1 - \cos^2\theta)}{1 + 2\cos\theta + (\cos^2\theta + \sin^2\theta)} = \frac{p^2 - 1}{p^2 + 1}$$

$$\frac{2\cos^2\theta + 2\cos\theta}{2 + 2\cos\theta} = \frac{p^2 - 1}{p^2 + 1}$$

$$\frac{2\cos\theta(1 + \cos\theta)}{2(1 + \cos\theta)} = \frac{p^2 - 1}{p^2 + 1}$$

$$\cos\theta = \frac{p^2 - 1}{p^2 + 1}$$

2. Prove that $\sqrt{\sec^2 \theta + \operatorname{cosec}^2 \theta} = \tan \theta + \cot \theta$.

Solution:

Taking,

$$\text{L.H.S} = \sqrt{(\sec^2 \theta + \operatorname{cosec}^2 \theta)}$$

Since,

$$\sqrt{\sec^2 \theta + \operatorname{cosec}^2 \theta}$$

$$\sec^2 \theta = \frac{1}{\cos^2 \theta}$$

$$\operatorname{cosec}^2 \theta = \frac{1}{\sin^2 \theta}$$

So,

$$\begin{aligned}\sqrt{\sec^2 \theta + \operatorname{cosec}^2 \theta} &= \sqrt{\frac{1}{\cos^2 \theta} + \frac{1}{\sin^2 \theta}} \\ &= \sqrt{\frac{\cos^2 \theta + \sin^2 \theta}{\cos^2 \theta \sin^2 \theta}} \\ &= \frac{1}{\cos \theta \sin \theta}\end{aligned}$$

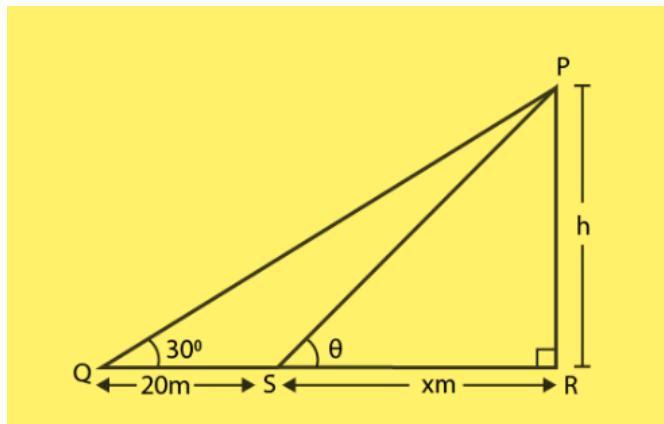
Now,

$$1 = \cos^2 \theta + \sin^2 \theta$$

$$\begin{aligned}\text{So, } \frac{1}{\cos \theta \sin \theta} &= \frac{\cos^2 \theta + \sin^2 \theta}{\cos \theta \sin \theta} \\ &= \frac{\cos^2 \theta}{\cos \theta \sin \theta} + \frac{\sin^2 \theta}{\cos \theta \sin \theta} \\ &= \frac{\cos \theta}{\sin \theta} + \frac{\sin \theta}{\cos \theta} \\ &= \tan \theta + \cot \theta\end{aligned}$$

3. The angle of elevation of the top of a tower from certain point is 30° . If the observer moves 20 metres towards the tower, the angle of elevation of the top increases by 15° . Find the height of the tower.

Solution:



Let us take $PR = h$ meter, the height of the tower.

The distance between the observer and tower is $QR = (20+x)$ m,

where

$$QR = QS + SR$$

$$= 20 + x$$

$$\angle PQR = 30^\circ$$

$$\angle PSR = \theta$$

In $\triangle PQR$,

$$\tan 30^\circ = \frac{h}{20+x}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{20+x}$$

On cross multiplication,

We get,

$$20+x = \sqrt{3}h$$

$$x = \sqrt{3}h - 20 \quad \dots \text{eq(i)}$$

In $\triangle PSR$,

$$\tan \theta = \frac{h}{x}$$

As, angle of elevation increases by 15° when the observer moves 20 m towards the tower.

So,

$$\begin{aligned}\theta &= 30^\circ + 15^\circ \\ &= 45^\circ\end{aligned}$$

Therefore,

$$\begin{aligned}\tan 45^\circ &= \frac{h}{x} \\ 1 &= \frac{h}{x} \\ h &= x\end{aligned}$$

Putting $x=h$ in eq. (i),

$$\begin{aligned}h &= \sqrt{3}h - 20 \\ \sqrt{3}h - h &= 20 \\ h(\sqrt{3} - 1) &= 20\end{aligned}$$

On rationalizing we get,

$$h = 10(\sqrt{3} + 1)$$

The required height of the tower is $10(\sqrt{3} + 1)$ meter.

4. If $1 + \sin^2 \theta = 3 \sin \theta \cos \theta$, then prove that $\tan \theta = 1$ or $\frac{1}{2}$.

Solution:

We have,

$$1 + \sin^2 \theta = 3 \sin \theta \cos \theta$$

On dividing L.H.S and R.H.S equations with $\sin^2 \theta$,

We get,

$$\begin{aligned}\frac{1 + \sin^2 \theta}{\sin^2 \theta} &= \frac{3 \sin \theta \cos \theta}{\sin^2 \theta} \\ \frac{1}{\sin^2 \theta} + 1 &= \frac{3 \cos \theta}{\sin \theta}\end{aligned}$$

$$\csc^2 \theta + 1 = 3 \cot \theta$$

Also,

$$\begin{aligned}\operatorname{cosec}^2 \theta - \cot^2 \theta &= 1 \\ \operatorname{cosec}^2 \theta &= \cot^2 \theta + 1 \\ \cot^2 \theta + 1 + 1 &= 3 \cot \theta \\ \cot^2 \theta + 2 &= 3 \cot \theta \\ \cot^2 \theta - 3 \cot \theta + 2 &= 0\end{aligned}$$

We split the middle term and then solve the equation,

$$\begin{aligned}\cot^2 \theta - \cot \theta - 2 \cot \theta + 2 &= 0 \\ \cot \theta(\cot \theta - 1) - 2(\cot \theta + 1) &= 0 \\ (\cot \theta - 1)(\cot \theta - 2) &= 0 \\ \cot \theta &= 1, 2\end{aligned}$$

We have,

$$\begin{aligned}\tan \theta &= \frac{1}{\cot \theta} \\ \tan \theta &= 1, \frac{1}{2}\end{aligned}$$

Proved!!

5. Given that $\sin \theta + 2 \cos \theta = 1$, **then prove that** $2 \sin \theta - \cos \theta = 2$.

Solution:

We have,

$$\sin \theta + 2 \cos \theta = 1$$

Squaring on both sides,

$$\begin{aligned}(\sin \theta + 2 \cos \theta)^2 &= 1 \\ \sin^2 \theta + 4 \cos^2 \theta + 4 \sin \theta \cos \theta &= 1\end{aligned}$$

Also,

$$\sin^2 \theta = 1 - \cos^2 \theta$$

and

$$\cos^2 \theta = 1 - \sin^2 \theta$$

$$\begin{aligned}(1 - \cos^2 \theta) + 4(1 - \sin^2 \theta) + 4 \sin \theta \cos \theta &= 1 \\ 1 - \cos^2 \theta + 4 - 4 \sin^2 \theta + 4 \sin \theta \cos \theta &= 1 \\ -4 \sin^2 \theta - \cos^2 \theta + 4 \sin \theta \cos \theta &= -4 \\ 4 \sin^2 \theta + \cos^2 \theta - 4 \sin \theta \cos \theta &= 4\end{aligned}$$

Also,

$$a^2 + b^2 - 2ab = (a - b)^2$$

So,

$$(2\sin \theta - \cos \theta)^2 = 4$$

$$2\sin \theta - \cos \theta = 2$$

Proved!!

6. The angle of elevation of the top of a tower from two points distant s and t from its foot are complementary. Prove that the height of the tower is \sqrt{st} .

Solution:

Let BC = s;

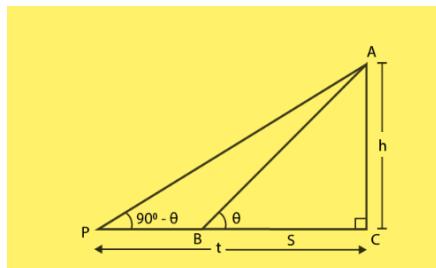
$$PC = t$$

Let height of the tower be AB = h.

$$\angle ABC = \theta \text{ and}$$

$$\angle APC = 90^\circ - \theta$$

(\because the angle of elevation of the top of the tower from two points P and B are complementary)



In $\triangle ABC$,

$$\begin{aligned} \tan \theta &= \frac{AC}{BC} \\ &= \frac{h}{s} \end{aligned} \quad \dots(i)$$

In $\triangle APC$,

$$\begin{aligned} \tan(90^\circ - \theta) &= \frac{AC}{PC} \\ &= \frac{h}{t} \\ \cot \theta &= \frac{h}{t} \end{aligned} \quad \dots(ii)$$

So, multiplying (i) and (ii),

$$\tan \theta \times \cot \theta = \frac{h}{t} \times \frac{h}{s}$$

$$1 = \frac{h^2}{st}$$

$$h^2 = st$$

$$h = \sqrt{st}$$

Therefore, the height of the tower is \sqrt{st} .

7. The shadow of a tower standing on a level plane is found to be 50 m longer when Sun's elevation is 30° than when it is 60° . Find the height of the tower.

Solution:

Let $SQ = h$ be the tower.

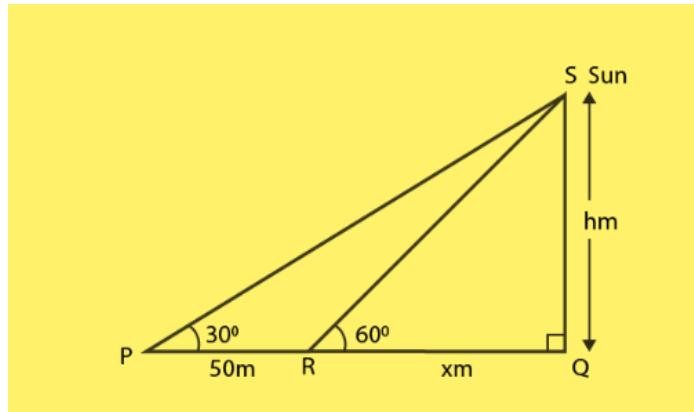
$$\angle SPQ = 30^\circ \text{ and}$$

$$\angle SRQ = 60^\circ$$

As given in the question, the length of shadow is 50 m long when angle of elevation of the sun is 30° than when it was 60° . So,

$$PR = 50 \text{ m and}$$

$$RQ = x \text{ m}$$



In SRQ,

$$\tan 60^\circ = \frac{h}{x}$$

$$\sqrt{3} = \frac{h}{x}$$

$$x = \frac{h}{\sqrt{3}}$$

In SPQ,

$$\tan 30^\circ = \frac{h}{50+x}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{50+x}$$

$$50+x = \sqrt{3}h$$

Putting value of x ,

$$50 + \frac{h}{\sqrt{3}} = \sqrt{3}h$$

$$50\sqrt{3} + h = 3h$$

$$50\sqrt{3} = 3h - h$$

$$3h - h = 50\sqrt{3}$$

$$2h = 50\sqrt{3}$$

$$h = 25\sqrt{3}$$

So, the required height is $25\sqrt{3}$ m.

8. A vertical tower stands on a horizontal plane and is surmounted by a vertical flag staff of height h . At a point on the plane, the angles of elevation of the bottom and the top of the flag staff are α and β , respectively. Prove that the height of the tower is $\left(\frac{h \tan \alpha}{\tan \beta - \tan \alpha} \right)$.

Solution:

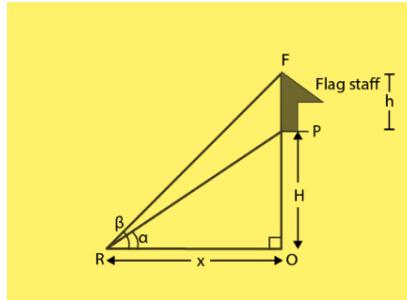
We have, a vertical flag staff of height h is surmounted on a vertical tower of height let (H) ,

so that,

$FP = h$ and

$FO = H$.

The angle of elevation of the bottom and top of the flag staff on the plane is $\angle PRO = \alpha$ and $\angle FRO = \beta$ respectively.



In $\triangle PRO$, we have,

$$\begin{aligned}\tan \alpha &= \frac{PO}{RO} \\ &= \frac{h}{x} \\ x &= \frac{h}{\tan \alpha} \quad \dots(i)\end{aligned}$$

In $\triangle FRO$,

$$\begin{aligned}\tan \beta &= \frac{FO}{RO} \\ \tan \beta &= \frac{FP + PO}{RO} \\ \tan \beta &= \frac{H + h}{x} \\ x &= \frac{H + h}{\tan \beta} \quad \dots(ii)\end{aligned}$$

From (i) and (ii),

$$\frac{H + h}{\tan \beta} = \frac{h}{\tan \alpha}$$

On solving,

$$H = \frac{h \tan \alpha}{\tan \alpha + \tan \beta}$$

Hence, proved.

9. If $\tan \theta + \sec \theta = l$, then prove that $\sec \theta = \frac{l^2 + 1}{2l}$.

Solution:

Given:

$$\tan \theta + \sec \theta = l \quad \dots \text{eq.(i)}$$

Multiplying and dividing by $(\sec \theta - \tan \theta)$ on numerator and denominator of L.H.S,

$$\frac{(\sec \theta + \tan \theta)(\sec \theta - \tan \theta)}{(\sec \theta - \tan \theta)} = l$$

$$\frac{(\sec^2 \theta - \tan^2 \theta)}{(\sec \theta - \tan \theta)} = l$$

$$\frac{(1)}{(\sec \theta - \tan \theta)} = l$$

So,

$$\sec \theta - \tan \theta = \frac{1}{l} \quad \dots \text{eq.(ii)}$$

Adding eq. (i) and eq. (ii), we get

$$(\tan \theta + \sec \theta) + (\sec \theta - \tan \theta) = l + \frac{1}{l}$$

So,

$$\sec \theta = \frac{l^2 + 1}{2l}$$

Hence, proved.

10. If $\sin \theta + \cos \theta = p$ and $\sec \theta + \operatorname{cosec} \theta = q$, then prove that $q(p^2 - 1) = 2p$.

Solution:

Given that,

$$\sin \theta + \cos \theta = p \dots \text{(i)}$$

and

$$\sec \theta + \operatorname{cosec} \theta = q$$

$$\frac{1}{\sin \theta} + \frac{1}{\cos \theta} = q$$

$$\frac{\sin \theta + \cos \theta}{\sin \theta \cos \theta} = q$$

$$\frac{p}{\sin \theta \cos \theta} = q$$

$$\sin \theta \cos \theta = \frac{p}{q}$$

Since, $\sin \theta + \cos \theta = p$

On squaring both sides, we get

$$(\sin \theta + \cos \theta)^2 = p^2$$

$$(\sin^2 \theta + \cos^2 \theta) + 2\sin \theta \cos \theta = p^2$$

$$[\text{as, } (a + b)^2 = a^2 + 2ab + b^2]$$

$$1 + 2\sin \theta \cos \theta = p^2$$

$$1 + 2\left(\frac{p}{q}\right) = p^2 \quad [\text{as, } \sin \theta + \cos \theta = p]$$

$$q + 2p = p^2 q$$

$$2p = p^2 q - q$$

$$q(p^2 - 1) = 2p$$

11. If $a \sin \theta + b \cos \theta = c$, then prove that $a \cos \theta - b \sin \theta = \sqrt{a^2 + b^2 - c^2}$.

Solution:

Given that,

$$a \sin \theta + b \cos \theta = c$$

On squaring both sides,

$$(a \sin \theta + b \cos \theta)^2 = c^2$$

$$a^2 \sin^2 \theta + b^2 \cos^2 \theta + 2ab \sin \theta \cos \theta = c^2$$

$$a^2(1 - \cos^2 \theta) + b^2(1 - \sin^2 \theta) + 2ab \sin \theta \cos \theta = c^2$$

$$a^2 - a^2 \cos^2 \theta + b^2 - b^2 \sin^2 \theta + 2ab \sin \theta \cos \theta = c^2$$

$$\begin{aligned} a^2 + b^2 - c^2 &= a^2 \cos^2 \theta + b^2 \sin^2 \theta - 2ab \sin \theta \cdot \cos \theta \\ a^2 + b^2 - c^2 &= (a \cos \theta - b \sin \theta)^2 \end{aligned}$$

$$[\text{as, } a^2 + b^2 - 2ab = (a - b)^2]$$

$$a \cos \theta - b \sin \theta = \sqrt{a^2 + b^2 - c^2}$$

$$12. \text{ Prove that } \frac{1 + \sec \theta - \tan \theta}{1 + \sec \theta + \tan \theta} = \frac{1 - \sin \theta}{\cos \theta}.$$

Solution:

Taking LHS,

On multiplying by $(\sec \theta - \tan \theta)$ on numerator and denominator,

$$\begin{aligned} & \frac{1 + \sec \theta - \tan \theta}{1 + \sec \theta + \tan \theta} \\ &= \frac{(1 + \sec \theta - \tan \theta)(\sec \theta - \tan \theta)}{(1 + \sec \theta + \tan \theta)(\sec \theta - \tan \theta)} \\ &= \frac{(1 + \sec \theta - \tan \theta)(\sec \theta - \tan \theta)}{(\sec \theta - \tan \theta) + (\sec \theta + \tan \theta)(\sec \theta - \tan \theta)} \\ &= \frac{(1 + \sec \theta - \tan \theta)(\sec \theta - \tan \theta)}{(\sec \theta - \tan \theta) + (\sec^2 \theta - \tan^2 \theta)} \\ &= \frac{(1 + \sec \theta - \tan \theta)(\sec \theta - \tan \theta)}{(\sec \theta - \tan \theta + 1)} \quad [\text{as, } (\sec^2 \theta - \tan^2 \theta) = 1] \\ &= \sec \theta - \tan \theta \\ &= \frac{1}{\cos \theta} - \frac{\sin \theta}{\cos \theta} \\ &= \frac{1 - \sin \theta}{\cos \theta} = RHS \end{aligned}$$

13. The angle of elevation of the top of a tower 30 m high from the foot of another tower in the same plane is 60° and the angle of elevation of the top of the second tower from the foot of the first tower is 30° . Find the distance between the two towers and also the height of the other tower.

Solution:

Let us take the distance between the two towers = AB

$$= x \text{ m}$$

and height of the other tower = PA

$$= h \text{ m}$$

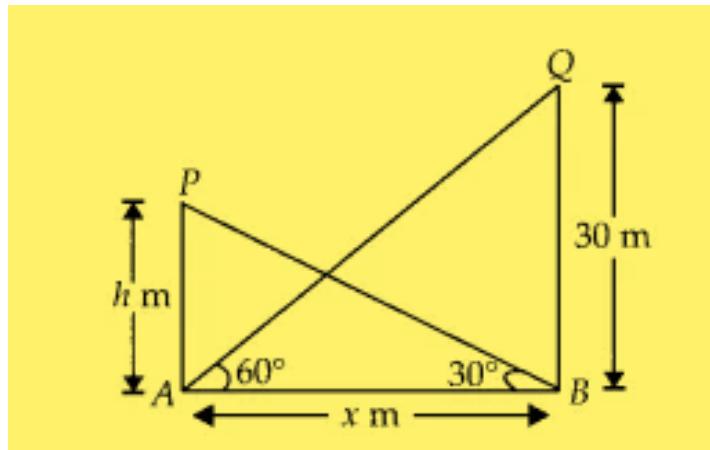
Given,

height of the tower = QB

$$= 30 \text{ m,}$$

$\angle QAB = 60^\circ$ and

$\angle PBA = 30^\circ$



Now, in $\triangle QAB$,

$$\tan 60 = \frac{QB}{AB}$$

$$= \frac{30}{x}$$

$$\sqrt{3} = \frac{30}{x}$$

$$x = \frac{30}{\sqrt{3}}$$

$$x = 10\sqrt{3}$$

In PBA,

$$\tan 30 = \frac{PA}{AB}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{x}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{10\sqrt{3}}$$

$$h = 10$$

14. From the top of a tower h m high, the angles of depression of two objects, which are in line with the foot of the tower are α and β ($\beta > \alpha$). Find the distance between the two objects.

Solution:

Let the distance between the two objects is x m and $CD = y$ m

Given,

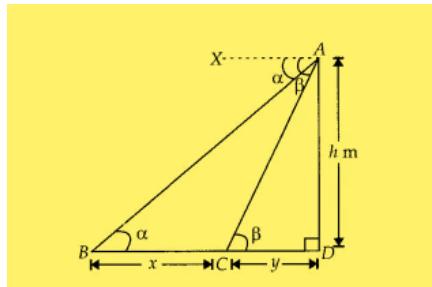
$$\angle BAX = \alpha = \angle ABD$$

and

$$\angle CAX = \beta = \angle ACD \quad [\text{Alternate angle}]$$

and

height of tower, $AD = h$ m



In ACD ,

$$\tan \beta = \frac{AD}{CD}$$

$$\tan \beta = \frac{h}{y}$$

$$y = \frac{h}{\tan \beta} \quad \dots(i)$$

In ABD ,

$$\tan \alpha = \frac{AD}{BD}$$

$$\tan \alpha = \frac{AD}{BC + CD}$$

$$\tan \alpha = \frac{h}{x + y}$$

$$y = \frac{h}{\tan \alpha} - x \quad \dots(ii)$$

From (i) and (ii),

$$\frac{h}{\tan \alpha} - x = \frac{h}{\tan \beta}$$

$$x = \frac{h}{\tan \alpha} - \frac{h}{\tan \beta}$$

$$x = h(\cot \alpha - \cot \beta)$$

Which is the required distance between the two objects.

15. A ladder rests against a vertical wall at an inclination α to the horizontal. Its foot is pulled away from the wall through a distance p so that its upper end slides a distance q down the wall and then the ladder makes an angle β to the horizontal. Show that

$$\frac{p}{q} = \frac{\cos \beta - \cos \alpha}{\sin \alpha - \sin \beta}$$

Solution:

Let,

$$OQ = x$$

$$OA = y$$

Given,

$$BQ = q,$$

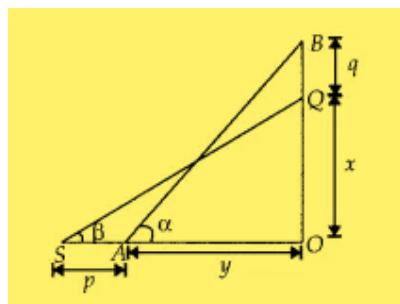
$$SA = p$$

and

$$AB = SQ = \text{Length of ladder}$$

Also,

$$\angle BAO = \alpha \text{ and } \angle QSO = \beta$$



$$\cos \alpha = \frac{OA}{AB}$$

$$\cos \alpha = \frac{y}{AB}$$

$$y = AB \cos \alpha = OA \quad \dots (i)$$

also,

$$\sin \alpha = \frac{OB}{AB}$$

$$OB = AB \sin \alpha$$

In QSO ,

$$\cos \beta = \frac{OS}{SQ}$$

$$OS = SQ \cos \beta = AB \cos \beta$$

and,

$$\sin \beta = \frac{OQ}{SQ}$$

$$OQ = SQ \sin \beta = AB \sin \beta$$

Now,

$$SA = OS - AO$$

$$p = AB(\cos \beta - \cos \alpha)$$

Also,

$$BQ = BO - QO$$

$$q = AB(\sin \alpha - \sin \beta)$$

So,

$$\frac{p}{q} = \frac{(\cos \beta - \cos \alpha)}{(\sin \alpha - \sin \beta)}$$

16. The angle of elevation of the top of a vertical tower from a point on the ground is 60° . From another point 10 m vertically above the first, its angle of elevation is 45° . Find the height of the tower.

Solution:

Let the height the vertical tower be,

$$OT = H \text{ m}$$

and

$$OP = AB = xm$$

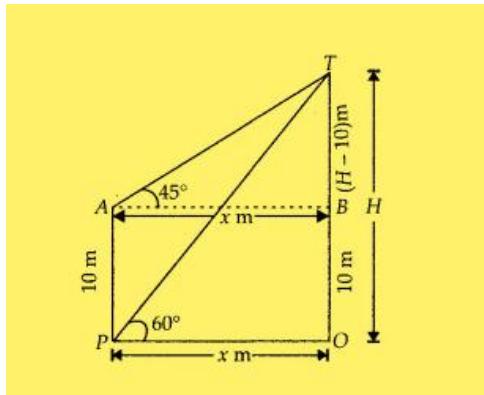
Given that,

$$AP = 10 \text{ m}$$

and

$$\angle TPO = 60^\circ,$$

$$\angle TAB = 45^\circ$$



Now, in $\triangle TPO$,

$$\tan 60^\circ = \frac{OT}{OP}$$

$$\tan 60^\circ = \frac{H}{x}$$

$$x = \frac{H}{\sqrt{3}}$$

In $\triangle TAB$,

$$\tan 45^\circ = \frac{TB}{AB}$$

$$1 = \frac{H - 10}{x}$$

$$x = H - 10$$

Putting value of x ,

$$\frac{H}{\sqrt{3}} = H - 10$$

On solving,

$$H = 5(\sqrt{3} + 3)m$$

17. A window of a house is h metres above the ground. From the window, the angles of elevation and depression of the top and the bottom of another house situated on the opposite side of the lane are found to be α and β , respectively. Prove that the height of the other house is $h(1 + \tan \alpha \cot \beta)$ metres.

Solution:

Let the height of the other house ,

$OQ = Hm$ and

$OB = MW = xm$

Given,

height of the first house $= WB = h = MO$

and $\angle QWM = \alpha$,

$\angle OWM = \beta = \angle WOB$

[Alternate angle]

Now, In ΔWOB ,

$$\begin{aligned}\tan \beta &= \frac{WB}{OB} \\ \tan \beta &= \frac{h}{x} \\ x &= \frac{h}{\tan \beta} \quad \dots(i)\end{aligned}$$

In QWM ,

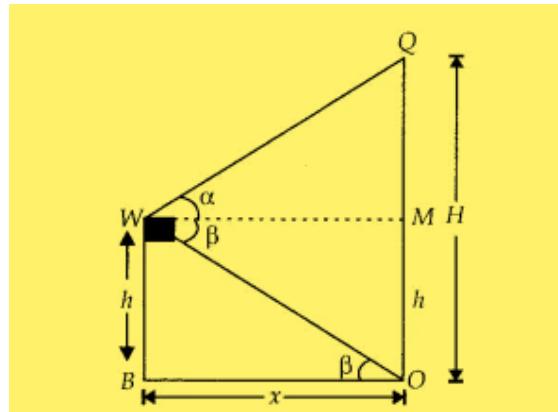
$$\begin{aligned}\tan \alpha &= \frac{QM}{WM} \\ \tan \alpha &= \frac{OQ - MO}{WM} \\ \tan \alpha &= \frac{H - h}{x} \\ x &= \frac{H - h}{\tan \alpha} \quad \dots(ii)\end{aligned}$$

From (i) and (ii),

$$\frac{H - h}{\tan \alpha} = \frac{h}{\tan \beta}$$

On solving,

$$H = h \left(1 + \tan \alpha \cdot \frac{1}{\tan \beta} \right)$$



18. The lower window of a house is at a height of 2 m above the ground and its upper window is 4 m vertically above the lower window. At certain instant the angles of elevation of a balloon from these windows are observed to be 60° and 30° , respectively. Find the height of the balloon above the ground.

Solution:

Let the height of the balloon from above the ground is H m

and

$$OP = W_2 R$$

$$= W_1 Q$$

$$= x \text{ m}$$

Given, height of lower window from the ground = $W_2 P$

$$= 2 \text{ m} = OR$$

Height of upper window from the lower window = $W_1 W_2$

$$= 4 \text{ m} = QR$$

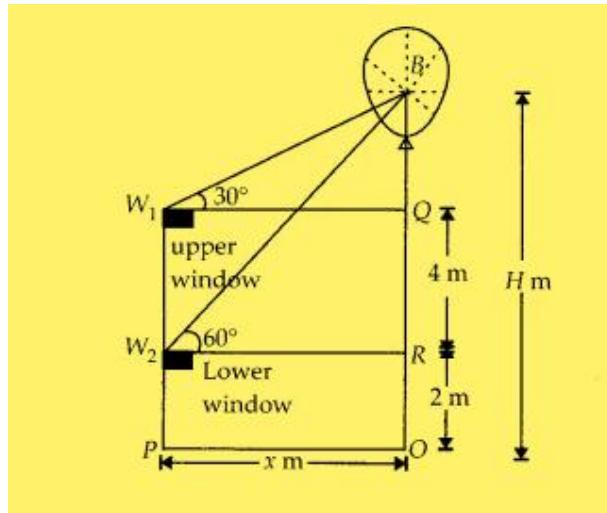
$$BQ = OB - (QR + RO)$$

$$= H - (4 + 2)$$

$$= H - 6$$

$$\angle BW_1 Q = 30^\circ \text{ and}$$

$$\angle BW_2 R = 60^\circ$$



In BW_2R ,

$$\begin{aligned}\tan 60^\circ &= \frac{BR}{W_2R} \\ &= \frac{BQ + QR}{x} \\ \sqrt{3} &= \frac{(H-6) + 4}{x} \\ x &= \frac{(H-2)}{\sqrt{3}} \quad \dots(i)\end{aligned}$$

In BW_1R ,

$$\begin{aligned}\tan 30^\circ &= \frac{BR}{W_1R} \\ \frac{1}{\sqrt{3}} &= \frac{(H-6)}{x} \\ x &= \sqrt{3}(H-6) \quad \dots(ii)\end{aligned}$$

From (i) and (ii),

$$\sqrt{3}(H-6) = \frac{(H-2)}{\sqrt{3}}$$

On solving,

$$H = 8 \text{ m}$$

The required height is 8 m.

Therefore, the required height of the balloon from the ground is 8 m.