Chapter - 10

Construction

Exercise No. 10.1

Multiple Choice Questions:

Choose the correct answer from the given four options:

1. To divide a line segment AB in the ratio 5:7, first a ray AX is drawn so that $\angle BAX$ **is an acute angle and then at equal distances points are marked on the ray AX such that the minimum number of these points is**

(A) 8

(B) 10

(C) 11

(D) 12

Solution:

(D) 12

As given in the question,

A line segment AB in the ratio 5:7

So,

A:B = 5:7

We draw a ray AX making an acute angle ∠BAX,

And mark A+B points at equal distance.

A=5 and B=7

Therefore,

Minimum number of these points = A+B

= 5+7 =12

2. To divide a line segment AB in the ratio 4:7, a ray AX is drawn first such that BAX is an acute angle and then points A_1, A_2, A_3,…. are located at equal distances on the ray AX and the point B is joined to

(A) A_{12}

(B) A_{11}

(C) A_{10}

(D) A_9

Solution:

(B) A_{11}

As given in the question,

A line segment AB in the ratio 4:7

So,
A:B = 4:7
Now,
Draw a ray AX making an acute angle BAX

Minimum number of points located at equal distances on the ray,
AX = A+B
= 4+7
= 11

A_1, A_2, A_3… are located at equal distances on the ray AX.

Point B is joined to the last point is A_{11}.

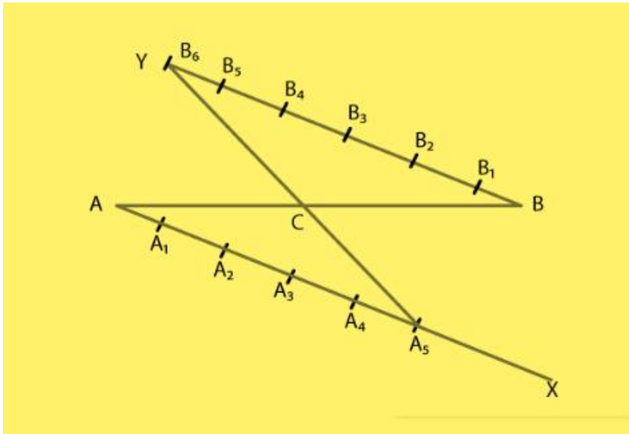
3. To divide a line segment AB in the ratio 5 : 6, draw a ray AX such that ∠BAX is an acute angle, then draw a ray BY parallel to AX and the points A_1, A_2, A_3, … and B_1, B_2, B_3, … are located at equal distances on ray AX and BY, respectively. Then the points joined are
(A) A_5 and B_6
(B) A_6 and B_5
(C) A_4 and B_5
(D) A_5 and B_4

Solution:
(A)
A_5 and B_6

As given in the question,
A line segment AB in the ratio 5:7
So,
A:B = 5:7

Steps of construction:
1. Draw a ray AX, an acute angle BAX.
2. Draw a ray BY ||AX, angle ABY = angle BAX.
3. Now, locate the points A_1,A_2,A_3,A_4 and A_5 on AX and B_1,B_2,B_3,B_4,B_5 and B_6
(Because A: B = 5:7)
4. Join A_5B_6.

A_5B_6 intersect AB at a point C.
AC: BC= 5:6



4. To construct a triangle similar to a given ΔABC with its sides $\frac{3}{7}$ of the corresponding sides of ΔABC, first draw a ray BX such that ∠CBX is an acute angle and X lies on the opposite side of A with respect to BC. Then locate points $B_1, B_2, B_3, \ldots$ on BX at equal distances and next step is to join

(A) B_{10} **to C**

(B) B_3 **to C**

(C) B_7 **to C**

(D) B_4 **to C**

Solution:

(C)

In this, we locate points B_1, B_2, B_3, B_4, B_5, B_6 and B_7 on BX at equal distance and in next step join the last point B_7 to C.

5. To construct a triangle similar to a given ΔABC with its sides $\frac{8}{5}$ of the corresponding sides of ΔABC draw a ray BX such that $\angle CBX$ is an acute angle and X is on the opposite side of A with respect to BC. The minimum number of points to be located at equal distances on ray BX is

(A) 5

(B) 8

(C) 13

(D) 3

Solution:

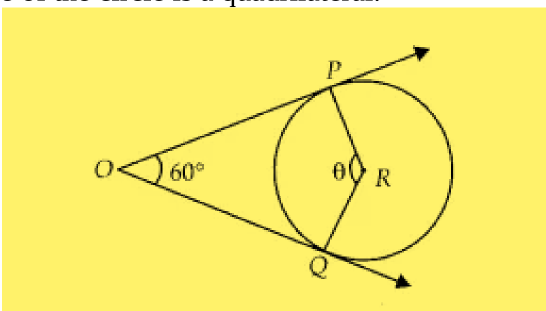
(B)

To construct a triangle similar to a given triangle, with its sides $\frac{m}{n}$ of the n corresponding sides of given triangle the minimum number of points to be located at equal distance is equal to the greater of m and n in $\frac{m}{n}$. Here, $\frac{m}{n} = \frac{8}{5}$ So, the minimum number of point to be located at equal distance on ray BX is 8.

6. To draw a pair of tangents to a circle which are inclined to each other at an angle of 60°, it is required to draw tangents at end points of those two radii of the circle, the angle between them should be
(A) 135°
(B) 90°
(C) 60°
(D) 120°

Solution:
(D)

The angle between them should be 120° because in that case the figure formed by the intersection point of pair of tangent, the two end points of those two radii (at which tangents are drawn) and the centre of the circle is a quadrilateral.



From figure POQR is a quadrilateral,

$\angle POQ + \angle PRQ = 180°$ [as, sum of opposite angles are 180°]

$60° + \theta = 180°$

$\theta = 120°$

Therefore, the required angle between them is 120.

Exercise No. 10.2

Short Answer Questions with Reasoning:

Write 'True' or 'False' and justify your answer in each of the following:

1. By geometrical construction, it is possible to divide a line segment in the ratio $\sqrt{3} : \frac{1}{\sqrt{3}}$.

Solution:
True

Explanation:
As given in the question,
Ratio= $\sqrt{3} : \frac{1}{\sqrt{3}}$

On solving,
$\sqrt{3} : \frac{1}{\sqrt{3}}$ = 3:1
Required ratio = 3:1

Therefore, geometrical construction is possible to divide a line segment in the ratio 3:1.

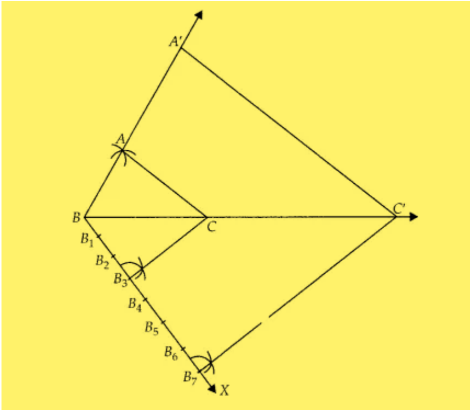
2. To construct a triangle similar to a given ΔABC with its sides $\frac{7}{3}$ of the corresponding sides of ΔABC, draw a ray BX making acute angle with BC and X lies on the opposite side of A with respect to BC. The points $B_1, B_2, ..., B_7$ are located at equal distances on BX, B_3 is joined to C and then a line segment B_6C' is drawn parallel to B_3C where C' lies on BC produced. Finally, line segment $A'C'$ is drawn parallel to AC.

Solution:

False

Steps of construction:
1. Draw a line segment BC with suitable length.
2. Taking B and C as centers draw two arcs of suitable radii intersecting each other at A.
3. Join BA and CA. ΔABC is the required triangle.
4. From B draw any ray BX downwards making an acute angle CBX.

5. Locate seven points B_1, B_2, B_3, …. B_7 on BX such that $BB_1 = B_1B_2 = B_1B_3 = B_3B_4 = B_4B_5 = B_5B_6 = B_6B_7$.
6. Join B_3C and from B7 draw a line B7C' ∥ B3C intersecting the extended line segment BC at C'.
7. From point C' draw C'A' ∥ CA intersecting the extended line segment BA at A'.



Then
ΔA'BC' is the required triangle whose sides are $\frac{7}{3}$ of the corresponding sides of ΔABC.
Given,
Segment B_6C' is drawn parallel to B_3C.
But from our construction is never possible that segment B_6C' is parallel to B_3C because the similar triangle A'BC' has its sides $\frac{7}{3}$ of the corresponding sides of triangle ABC.
Therefore, B_7C' is parallel to B_3C.

3. A pair of tangents can be constructed from a point P to a circle of radius3.5 cm situated at a distance of 3 cm from the centre.

Solution:

False

As, the radius of the circle is 3.5 cm
r = 3.5 cm and a point P situated at a distance of 3 cm from the centre
So,
d = 3 cm.
We can see that $r > d$
Therefore, a point P lies inside the circle.
And, no tangent can be drawn to a circle from a point lying inside it.

4. A pair of tangents can be constructed to a circle inclined at an angle of 170°.

Solution:

True

As, the angle between the pair of tangents is always greater than 0 but less than 180°. Therefore, we can draw a pair of tangents to a circle inclined at an angle at 170°.

Exercise No. 10.3

Short Answer Questions:

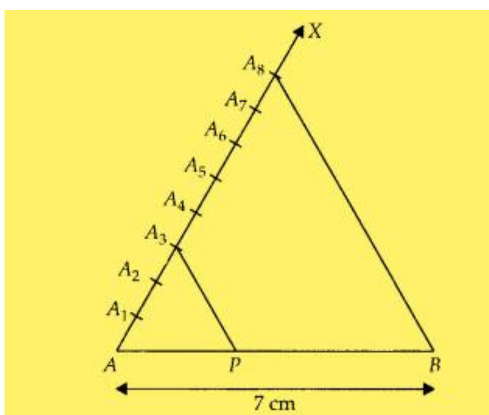
Question:

1. Draw a line segment of length 7 cm. Find a point P on it which divides it in the ratio 3:5.

Solution:

Steps of construction:
1. Draw a line segment AB = 7 cm.
2. Draw a ray AX, making an acute ∠BAX.
3. Along AX, mark 3 + 5 = 8 points
   $A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8$
   Such that $AA_1 = A_1A_2 = A_2A_3 = A_3A_4 = A_4A_5 = A_6A_7 = A_7A_8$
4. Join A_8B.
5. From A_3, draw $A_3P \parallel A_8B$ meeting AB at P.
   [by making an angle equal to $\angle BA_8A$ at A_3]

So, P is the point on AB which divides it in the ratio 3 : 5.



Explanation:
Let
$AA_1 = A_1A_2 = A_2A_3 = A_3A_4 = \ldots\ldots\ldots = A_7A_8 = x$

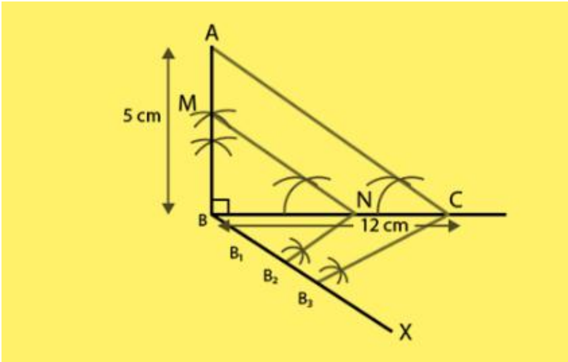
In ΔABA_8, we have
$A_3P \parallel A_8B$

$$\frac{AP}{PB} = \frac{AA_3}{A_3A_8} = \frac{3x}{5x}$$

Therefore, AP: PB = 3 : 5

2. Draw a right triangle ABC in which BC = 12 cm, AB = 5 cm and ∠B = 90°. Construct a triangle similar to it and of scale factor $\frac{2}{3}$. Is the new triangle also a right triangle?

Solution:



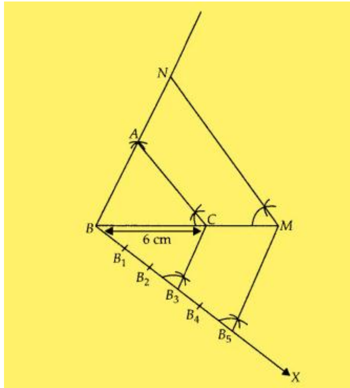
Steps of construction:

1. Draw a line segment BC = 12 cm.
2. From B draw a line AB = 5 cm which makes right angle at B.
3. Join AC, ΔABC is the given right triangle.
4. From B draw an acute ∠CBX downwards.
5. On ray BX, mark three points B_1, B_2 and B_3, such that $BB_1 = B_1B_2 = B_2B_3$.
6. Join B_3C.
7. From point B_2 draw $B_2N \parallel B_3C$ intersect BC at N.
8. From point N draw NM ∥ CA intersect BA at M. ΔMBN is the required triangle. ΔMBN is also a right angled triangle at B.

3. Draw a triangle ABC in which BC = 6 cm, CA = 5 cm and AB = 4 cm. Construct a triangle similar to it and of scale factor $\frac{5}{3}$.

Solution:

Steps of construction:
1. Draw a line segment BC = 6 cm.
2. Taking B and C as centers, draw two arcs of radii 4 cm and 5 cm intersecting each other at A.
3. Join BA and CA. ΔABC is the required triangle.
4. From B, draw any ray BX downwards making at acute angle ∠CBX
5. Mark five points B_1, B_2, B_3, B_4 and B_5 on BX, such that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4 = B_4B_5$.

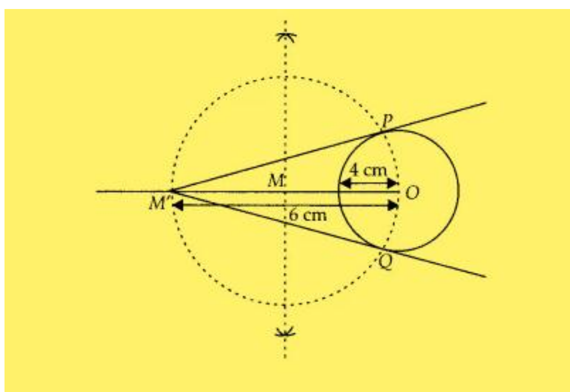


6. Join B_3C and from B_5 draw $B_5M \parallel B_3C$ intersecting the extended line segment BC at M.
7. From point M draw MN ∥ CA intersecting the extended line segment BA at N.

Therefore, ΔNBM is the required triangle whose sides is equal to $\frac{5}{3}$ of the corresponding sides of the ΔABC.

4. Construct a tangent to a circle of radius 4 cm from a point which is at a distance of 6 cm from its center.

Solution:



We have, a point M' is at a distance of 6 cm from the centre of a circle of radius 4 cm.
Steps of construction:
1. Draw a circle of radius 4 cm. Let the centre of this circle be O.
2. Join OM' and bisect it. Let M be mid-point of OM'.
3. Taking M as centre and MO as radius draw a circle to intersect circle (0, 4) at two points, P and Q.
4. Join PM' and QM'. PM' and QM' are the required tangents from M' to circle C(0, 4).

Exercise No. 10.4

Long Answer Questions:

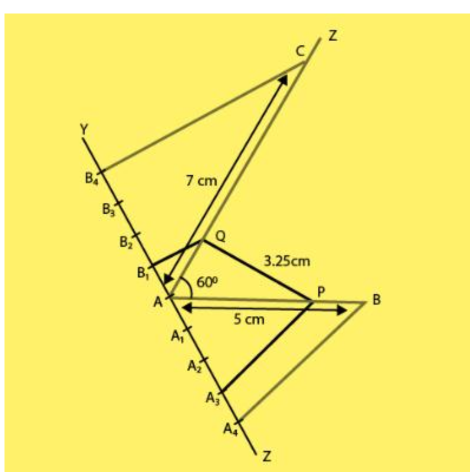
Question:

1. Two line segments AB and AC include an angle of 60° where AB = 5 cm and AC = 7 cm. Locate points P and Q on AB and AC, respectively such that $AP = \frac{3}{4}AB$ and $AQ = \frac{1}{4}AC$. Join P and Q and measure the length PQ.

Solution:

Steps of construction:

1. Draw a line segment AB = 5 cm.
2. Also, make ∠BAZ = 60°.
3. With center A and radius 7 cm, draw an arc cutting the line AZ at C.
4. Draw a ray AX, making an acute ∠BAX.
5. Divide AX into four equal parts, namely $AA_1 = A_1A_2 = A_2A_3 = A_3A_4$.
6. Join A_4B.
7. Draw $A_3P \parallel A_4B$ meeting AB at P.
8. Therefore, P is the point on AB such that AP = $\frac{3}{4}$ AB.
9. Now, draw a ray AY, such that it makes an acute ∠CAY.
10. Divide AY into four parts, namely $AB_1 = B_1B_2 = B_2B_3 = B_3B_4$.
11. Join B_4C.
12. Draw $B_1Q \parallel B_4C$ meeting AC at Q. We get, Q is the point on AC such that AQ = $\frac{1}{4}$ AC.
13. Join PQ and measure it.
14. PQ = 3.25 cm.

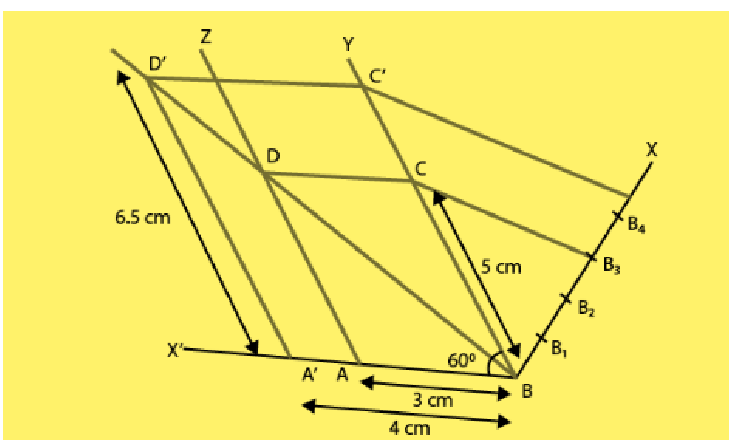


2. Draw a parallelogram ABCD in which BC = 5 cm, AB = 3 cm and angle $\angle ABC = 60°$, divide it into triangles BCD and ABD by the diagonal BD. Construct the triangle $BD'C'$ similar to triangle BDC with scale factor $\frac{4}{3}$. Draw the line segment $D'A'$ parallel to DA where A' lies on extended side BA. Is $A'BC'D'$ a parallelogram?

Solution:
Steps of constructions:

1. Draw a line AB=3 cm.
2. Now draw a ray BY making an acute ∠ABY=60°.
3. With centre B and radius 5 cm, draw an arc cutting the point C on BY.
4. Draw a ray AZ making an acute ∠ZAX'=60°
   (BY||AZ, as, ∠YBX'=ZAX'=60°)
5. With centre A and radius 5 cm, draw an arc cutting the point D on AZ.
6. Join CD
7. We obtain a parallelogram ABCD.
8. Join BD, the diagonal of parallelogram ABCD.
9. Draw a ray BX downwards making an acute ∠CBX.
10. Locate 4 points B_1, B_2, B_3, B_4 on BX, such that $BB_1=B_1B_2=B_2B_3=B_3B_4$.
11. Join B_4C and from B_3C draw a line $B_4C' \| B_3C$ intersecting the extended line segment BC at C'.
12. Draw C'D'|| CD intersecting the extended line segment BD at D'. Then, ΔD'BC' is the required triangle whose sides are $\frac{4}{3}$ of the corresponding sides of ΔDBC.
13. Now we draw a line segment D'A'|| DA, where A' lies on the extended side BA.

14. We observe that A'BC'D' is a parallelogram in which A'D'=6.5 cm A'B = 4 cm and ∠A'BD'= 60° divide it into triangles BC'D' and A'BD' by the diagonal BD'.

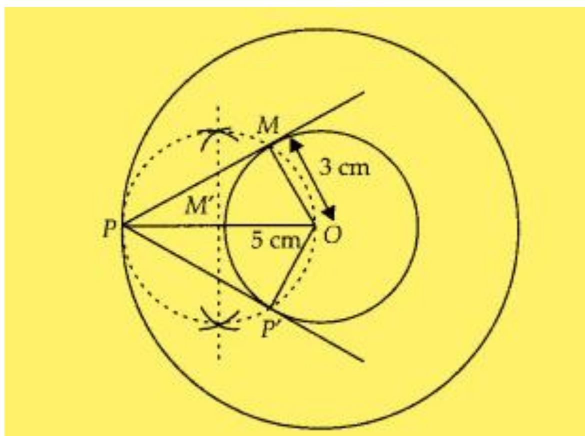


3. Draw two concentric circles of radii 3 cm and 5 cm. Taking a point on outer circle construct the pair of tangents to the other. Measure the length of a tangent and verify it by actual calculation.

Solution:

We have, two concentric circles of radii 3 cm and 5 cm with centre O.
We draw pair of tangents from point P on outer circle to the other.

1. Draw two concentric circles with centre O and radii 3 cm and 5 cm.
2. Taking any point P on outer circle. Join OP.
3. Bisect OP, let M' be the mid-point of OP taking M' as centre and OM' as radius draw a circle dotted which cuts the inner circle at M and P'.
4. Join PM and PP'. Thus, PM and PP' are the required tangents.
5. On measuring PM and PP', we find that PM = PP' = 4 cm.



Now actual calculation:
In right angle ΔOMP,
∠PMO = 90°
$PM^2 = OP^2 - OM^2$ [by Pythagoras theorem]
$PM^2 = (5)^2 - (3)^2$
$25 - 9 = 16$
PM = 4 cm

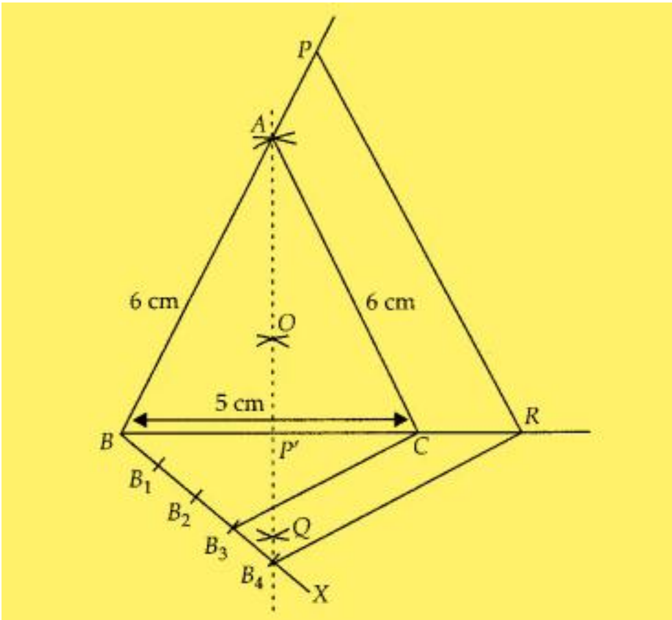
Therefore, the length of both tangents is 4 cm.

4. Draw an isosceles triangle ABC in which AB = AC = 6 cm and BC = 5 cm. Construct a triangle PQR similar to triangle ABC in which PQ = 8 cm. Also justify the construction.

Solution:

Let ΔPQR and ΔABC are similar triangles, then its scale factor between the corresponding sides is $\frac{PQ}{AB} = \frac{8}{6} = \frac{4}{3}$

Steps of construction:
1. Draw a line segment BC = 5 cm.
2. Construct OQ the perpendicular bisector of line segment BC meeting BC at P'.
3. Taking B and C as centre we draw two arcs of equal radius 6 cm intersecting each other at A.
4. Join BA and CA. So, ΔABC is the required isosceles triangle.



5. From B, we draw any ray BX making an acute $\angle CBX$.
6. Locate four points B_1, B_2, B_3 and B_4 on BX such that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4$.
7. Now join B_3C and from B_4 draw a line $B_4R \parallel B_3C$ intersecting the extended line segment BC at R.
8. From point R, draw $RP \parallel CA$ meeting BA produced at P.

Then, Δ PBR is the required triangle.

Explanation,
As, we have,
$B_4R \parallel B_3C$ (by construction)

$$\frac{BC}{CR} = \frac{3}{1}$$

or,

$$\frac{CR}{BC} = \frac{1}{3}$$

Now,

$$\frac{BR}{BC} = \frac{BC + CR}{BC}$$

$$= 1 + \frac{CR}{BC}$$

$$= 1 + \frac{1}{3}$$

$$= \frac{4}{3}$$

And,

$RP \parallel CA$

$So, \Delta ABC \sim \Delta PBR$

Hence,

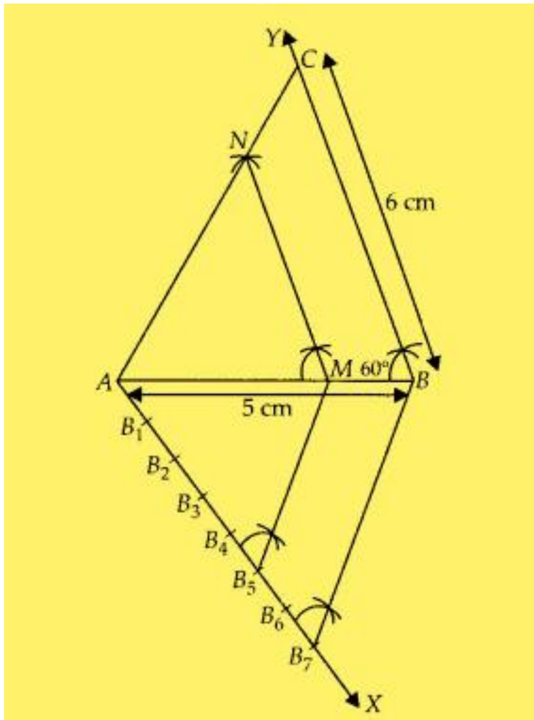
$$\frac{PB}{AB} = \frac{RP}{CA} = \frac{BR}{BC} = \frac{4}{3}$$

5. Draw a triangle ABC in which AB = 5 cm, BC = 6 cm and ABC= 60º.Construct a triangle similar to ABC with scale factor $\frac{5}{7}$. Justify the construction.

Solution:

Steps of construction:

1. Draw a line segment AB = 5 cm.
2. From point B, draw ∠ABY = 60° on which take BC = 6 cm.
3. Join AC, ΔABC is the required triangle.
4. From A, draw any ray AX downwards making an acute angle ∠BAX
5. Mark 7 points B_1, B_2, B_3, B_4, B_5, B_6 and B_7 on AX, such that $AB_1 = B_1B_2 = B_2B_3 = B_3B_4 = B_4B_5 = B_5B_6 = B_6B_7$.
6. Join B_7B and from B_5 draw $B_5M \parallel B_7B$ intersecting AB at M.
7. From point M draw MN ∥ BC intersecting AC at N. Then, ΔAMN is the required triangle whose sides are equal to $\frac{5}{7}$ of the corresponding sides of the ΔABC.



Explanation:

Here,

$B_5M \parallel B_7B$ (by construction)

$$\frac{AM}{BM} = \frac{5}{2}$$

$$or,$$

$$\frac{BM}{AM} = \frac{2}{5}$$

$$\frac{AB}{AM} = \frac{AM + BM}{AM}$$

$$= 1 + \frac{BM}{AM}$$

$$= 1 + \frac{2}{5}$$

$$= \frac{7}{5}$$

$$Also, MN \parallel BC$$

$$So, \Delta AMN \sim \Delta ABC$$

$$\frac{AM}{AB} = \frac{AN}{AC} = \frac{NM}{BC} = \frac{5}{7}$$

6. Draw a circle of radius 4 cm. Construct a pair of tangents to it, the angle between which is 60°. Also justify the construction. Measure the distance between the centre of the circle and the point of intersection of tangents.

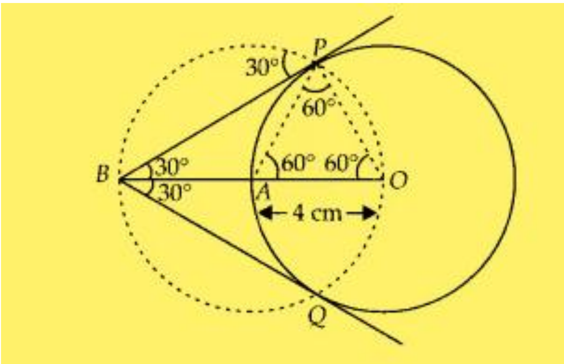
Solution:

Steps of construction:

1. Take a point O on the plane of the paper and draw a circle of radius OA = 4 cm.
2. Produce OA to B such that OA = AB = 4 cm.
3. Taking A as the centre draw a circle of radius AO = AB = 4 cm.
   Suppose it cuts the circle drawn in step 1 at P and Q.
4. Join BP and BQ to get desired tangents.

Explanation:
In ΔOAP, we have
OA = OP = 4 cm (Radius)
Also,
AP = 4 cm
(As, Radius of circle with centre A)

ΔOAP is equilateral triangle.
∠PAO = 60°
∠BAP = 120°



Therefore in ΔBAP,
BA = AP
and ∠BAP = 120°

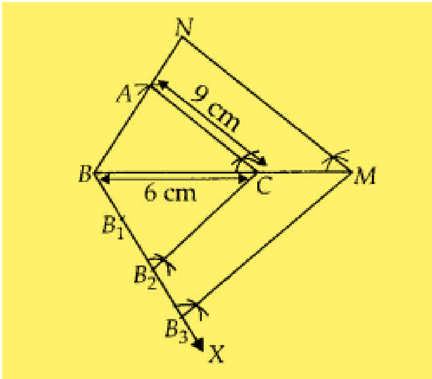
So,
∠ABP = ∠APB
= 30°
∠PBQ = 60°

7. Draw a triangle ABC in which AB = 4 cm, BC = 6 cm and AC = 9 cm. Construct a triangle similar to ΔABC with scale factor $\frac{3}{2}$. Justify the construction. Are the two triangles congruent? Note that all the three angles and two sides of the two triangles are equal.

Solution:

Steps of construction:

1. Firstly draw a line segment BC = 6 cm.
2. Taking B and C as centre, draw two arcs of radii 4 cm and 9 cm intersecting each other at A.
3. Join BA and CA. ΔABC is the required triangle.
4. From B, draw any ray BX downwards making an acute angle ∠CBX
5. Mark three points B_1, B_2, B_3, on BX, such that $BB_1 = B_1B_2 = B_2B_3$.



6. Join B_2C and from B_3 draw $B_3M \parallel B_2C$ intersecting BC at M.
7. From point M, draw MN ∥ CA intersecting the extended line segment BA to N.

Then ΔNBM is the required triangle whose sides are equal to $\frac{3}{2}$ of the ΔABC.

Explanation:

$B_3M \parallel B_2C$

$\frac{BC}{CM} = \frac{2}{1}$

Now,

$\frac{BM}{BC} = \frac{BC+CM}{BC}$

$= 1 + \frac{CM}{BC}$

$$=1+\frac{1}{2}$$

$$=\frac{3}{2}$$

Also,

$MN \parallel CA$

$\Delta ABC : \Delta NBM$

So,

$$\frac{NB}{AB}=\frac{NM}{AC}=\frac{BM}{BC}=\frac{3}{2}$$

The two triangles are not congruent because, if two triangles are congruent, then they have same shape and same size.

Therefore, all the three angles are same but three sides are not same that is one side is different.