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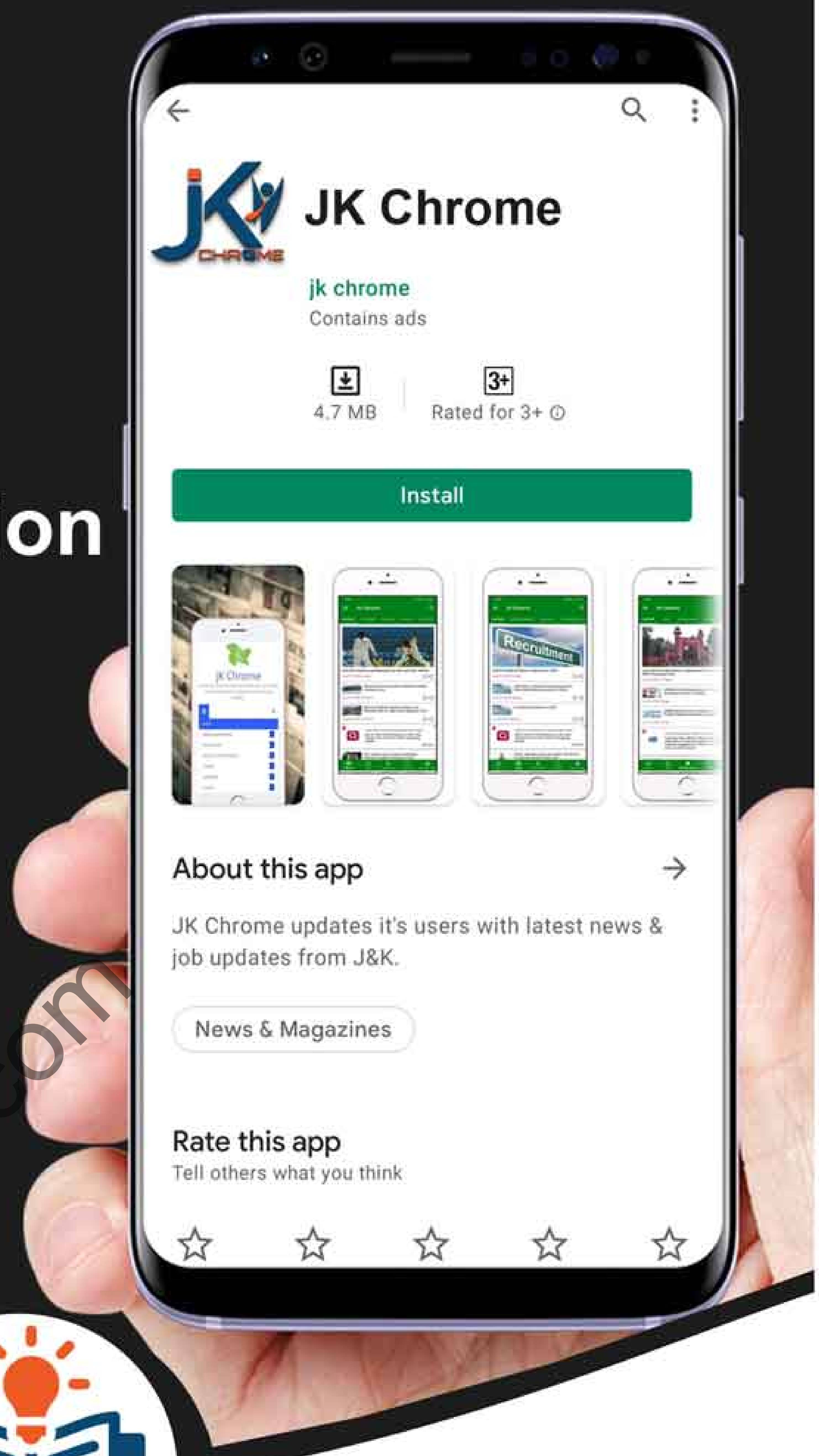

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Structure Analysis

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Topics **Page**

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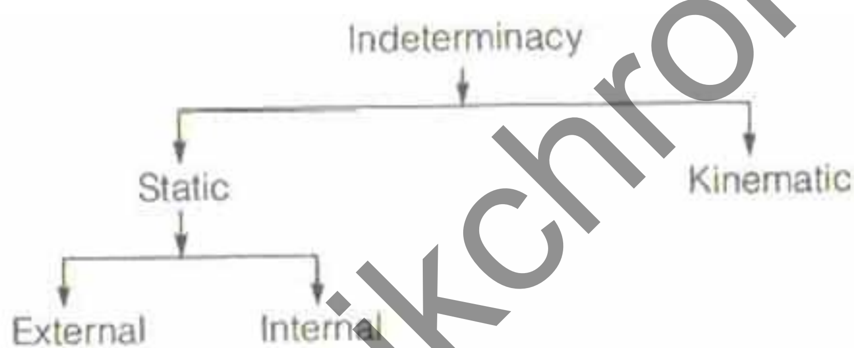
Determinacy and Indeterminacy

Statically Determinate Structures

Conditions of equilibrium are sufficient to analyse the structure. Bending moment and shear force is independent of the cross-sectional area of the components and flexural rigidity of the members. No stresses are caused due to temperature change. No stresses are caused due to lack of fit or differential settlement.

Statically Indeterminate Structures

Additional compatibility conditions are required. Bending moment and shear force depends upon the cross-sectional area and flexural rigidity of the members. Stresses are caused due to temperature variation. Stresses are caused due to lack of fit or differential settlement.



Important Terms

1. **Stable/Unstable:** A stable structure is one that will not collapse when disturbed. Stability may also be defined as "The power to recover equilibrium". In general, there are may ways that a structure may become **unstable**, including the buckling of compression members, yielding/rupture of members, however, for linear structural analysis, the main concern is instability caused by insufficient reaction points or poor layout of structural members.
2. **Internally Stable:** In internally stable structure is one that would maintain its shape if all the reactions supports were removed. A structure that is internally unstable may still be stable if it has sufficient external support reactions. An example is shown below in Figure.

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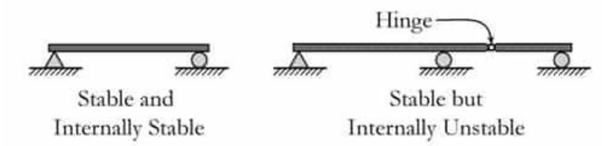

3. **External Determinacy:** The ability to calculate all of the external reaction component forces using only static equilibrium. A structure that satisfies this requirement is *externally statically determinate*. A structure for which the external reactions component forces cannot be calculated using only equilibrium is *externally statically indeterminate*.

4. **Internal Determinacy:** The ability to calculate all of the external reaction component forces *and* internal forces using only static equilibrium. A structure that satisfies this requirement is *internally statically determinate*. A structure for which the internal forces cannot be calculated using only equilibrium is *internally statically indeterminate*. Typically if one talks about 'determinacy', it is an internal determinacy that is meant.

5. **Redundant:** Indeterminate structures effectively have more unknowns than can be solved using the three equilibrium equations (or six equilibrium equations in 3D). The extra unknowns are called *redundant*.

6. **Degree of Indeterminacy:** The degree of indeterminacy is equal to the number of redundant. An indeterminate structure with 2 redundant may be said to be statically indeterminate to the second degree or "2º S. I."

Static Indeterminacy

If a structure cannot be analyzed for external and internal reactions using static equilibrium conditions alone then such a structure is called indeterminate structure.

(i) $D_S = D_{Se} + D_{Si}$

Where,

D_S = Degree of static-indeterminacy

D_{Se} = External static-indeterminacy

D_{Si} = Internal static-indeterminacy

External static indeterminacy:

It is related with the support system of the structure and it is equal to number of external reaction components in addition to number of static equilibrium equations.

(ii) $D_{Se} = r_e - 3$ For 2D

$D_{Se} = r_e - 6$ For 3D

Where, r_e = total external reactions

Internal static indeterminacy:

It refers to the geometric stability of the structure. If after knowing the external reactions it is not possible to determine all internal forces/internal reactions using static equilibrium equations alone then the structure is said to be internally indeterminate.

For geometric stability sufficient number of members are required to preserve the shape of rigid body without excessive deformation.

(iii) $D_{Si} = 3C - r_r$ …… For 2D

$D_{Si} = 6C - r_r$ …… For 3D

where, C = number of closed loops.

and

r_r = released reaction

(iv) $r_r = \sum(m_j - 1)$ …… For 2D

$r_r = 3\sum(m_j - 1)$ ……. For 3D

where m_j = number of member connecting with J number of joints.

and J = number of hybrid joint.

(v) $D_s = m + r_e - 2j$ ….. For 2D truss

$D_{Se} = r_e - 3$ & $D_{Si} = m - (2j - 3)$

(vi) $D_S = m + r_e - 3j$ ….. For 3D truss

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$D_{Se} = r_e - 6$ & $D_{Si} = m - (3j - 6)$

(vii) $D_S = 3m + r_e - 3j - r_r$ ….. 2D Rigid frame

(viii) $D_s = 6m + r_e - 6j - r_r$ ….. 3D rigid frame

(ix) $D_S = (r_e - 6) + (6C - r_r)$ ….. 3D rigid frame

Kinematic Indeterminacy

It the number of unknown displacement components are greater than the number of compatibility equations, for these structures additional equations based on equilibrium must be written in order to obtain sufficient number of equations for the determination of all the unknown displacement components. The number of these additional equations necessary is known as degree of kinematic indeterminacy or degree of freedom of the structure.

A fixed beam is kinematically determinate and a simply supported beam is kinematically indeterminate.

(i) Each joint of plane pin jointed frame has 2 degree of freedom.

(ii) Each joint of space pin jointed frame has 3 degree of freedom.

(iii) Each joint of plane rigid jointed frame has 3 degree of freedom.

(iv) Each joint of space rigid jointed frame has 6 degree of freedom.

Degree of kinematic indeterminacy is given by:

1. $D_k = 3j - r_e$ ………. For 2D Rigid frame when all members are axially extensible.
2. $D_k = 3j - r_e - m$ ………. For 2D Rigid frame if 'm' members are axially rigid / inextensible.
3. $D_k = 3(j + j') - r_e - m + r_r$ …… For 2D Rigid frame when J' = Number of Hybrid joints is available.
4. $D_k = 6(j + j') - r_e - m + r_r$ ….. For 3D Rigid frame
5. $D_k = 2(j + j') - r_e - m + r_r$ ….. For 2D Pin jointed truss.
6. $D_k = 3(j + j') - r_e - m + r_r$ …… For 3D Pin jointed truss.

Examples

Notations used in examples

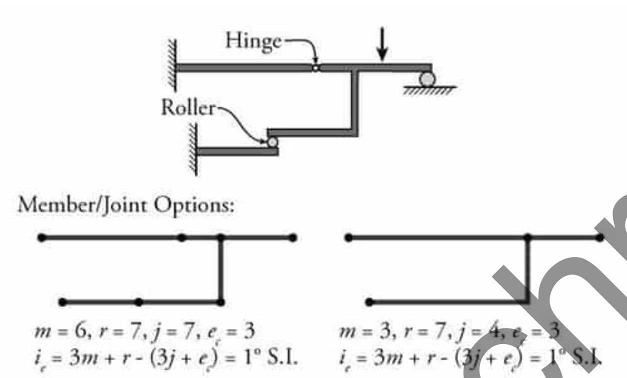
i_e is degree of Indeterminacy

e_c is the number of equations of condition,

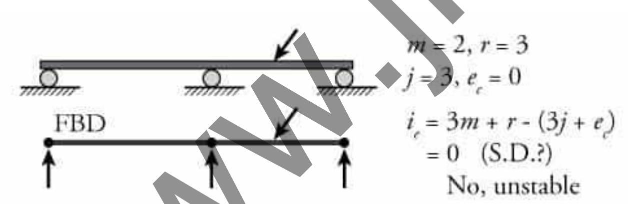
Internal Support Type	Equations of Condition
Hinge	$e_c = n - 1$
Roller	$e_c = 2 * (n - 1)$

where n is the number of members connected to the hinge or roller.

1. Determination of the Number of Members and Joints



2. Instability due to Parallel Reactions



3. Instability due to Concurrent Reactions

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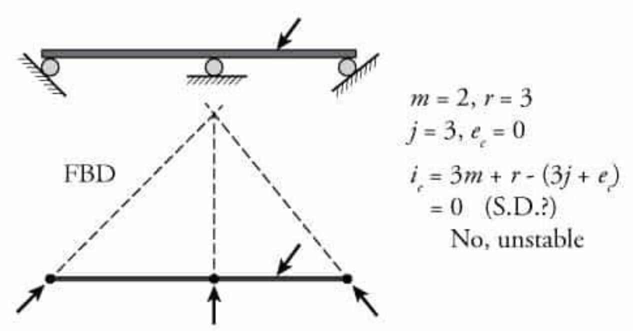
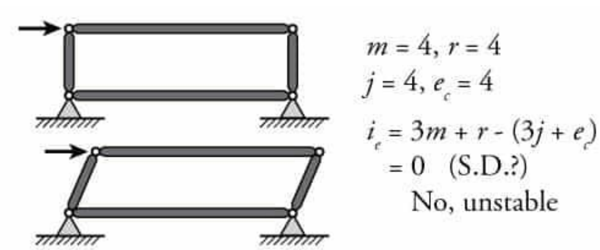
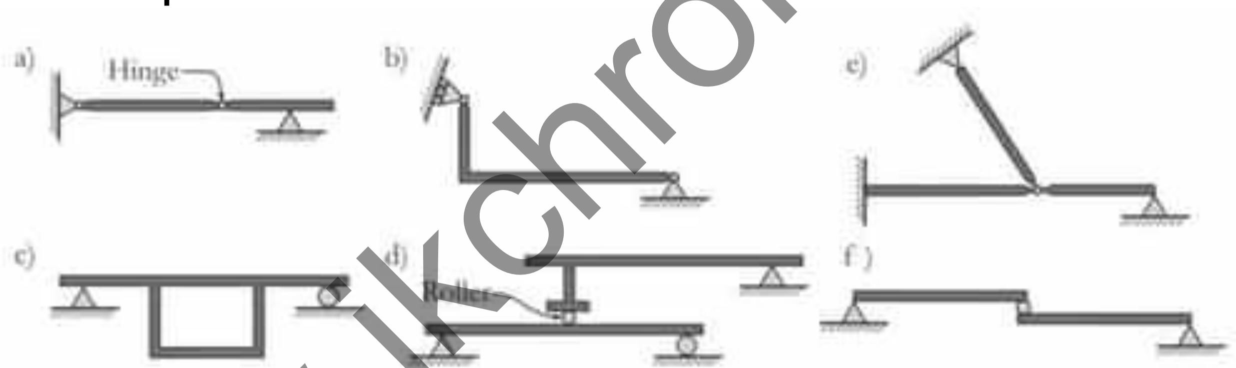


4. Instability due to an Internal Collapse Mechanism



5. Mixed up



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a) External Determinacy:

$i_e = r - (3 + e_c)$
r=4,ec=1 (The hinge on the left at the pin does not provide any additional equations of condition).
Therefore,
$i_e = 0$.
Then, is this structure statically determinate? No, it is unstable because if we take a free-body diagram of the left side of the beam, and take a sum of moments about the center hinge, the sum of moments will be non-zero due to the vertical reaction at the left pin (but we know that it has to be zero due to the existence of the pin).

Internal Determinacy:

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$i_e=(3m+r)-(3j+e_c)$
$m=2, r=4, j=3, e_c=1$ (Again, the hinge on the left at the pin does not provide any additional equations of condition).

Therefore,

$3m+r=10$, $3j+e_c=10$, and ie=0.
Then, is this structure statically determinate? No, it is unstable due to the same reason above.

b) External Determinacy:

$r=3, e_c=0$.
Therefore,
$i_e=0$.

Then is this structure statically determinate? No, because the reactions are concurrent through the pin on the right.

Internal Determinacy:

$m=2, r=3, j=3, e_c=0$.

Therefore,

$3m+r=9$ and $3j+e_c=9$,
so the structure appears internally determinate, but it is still unstable due to the concurrent reactions.

c) External Determinacy:

$r=3, e_c=0$.

Therefore,

$i_e=0$.
Since there are no sources of instability, this structure is externally statically determinate.

Internal Determinacy:

$m=6, r=3, j=6, e_c=0$.
Therefore,
$3m+r=21$ and $3j+e_c=18$,
so this structure is internally statically indeterminate to three degrees (or "3º S.I.").

d) External Determinacy:

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$r=5, e_c=2.$

Therefore,

$i_e=0.$

Since there are no sources of instability, this structure is externally statically determinate.

Internal Determinacy:

$m=5, r=5, j=6, e_c=2.$

Therefore,

$3m+r=20$ and $3j+e_c=20$,

so this structure is internally statically determinate (or "S.D.").

e) External Determinacy:

$r=7, e_c=2.$ (Due to the three members connected to the internal hinge)

Therefore,

$i_e=2.$

This structure can be described as 2 degrees externally statically indeterminate.

Internal Determinacy:

$m=3, r=7, j=4, e_c=2.$

Solving,

$3m+r=16$ and $3j+e_c=14$,

Again, this structure is found to be 2 degrees internally statically indeterminate.

f) External Determinacy:

$r=4, e_c=2.$

Therefore,

$i_e=-1.$

Due to the design of the structure, the internal roller cannot be supported and the structure is classified as unstable.

Internal Determinacy:

$m=2, r=4, j=3, e_c=2.$

Solving,

$3m+r=10$ and $3j+e_c=-11$,

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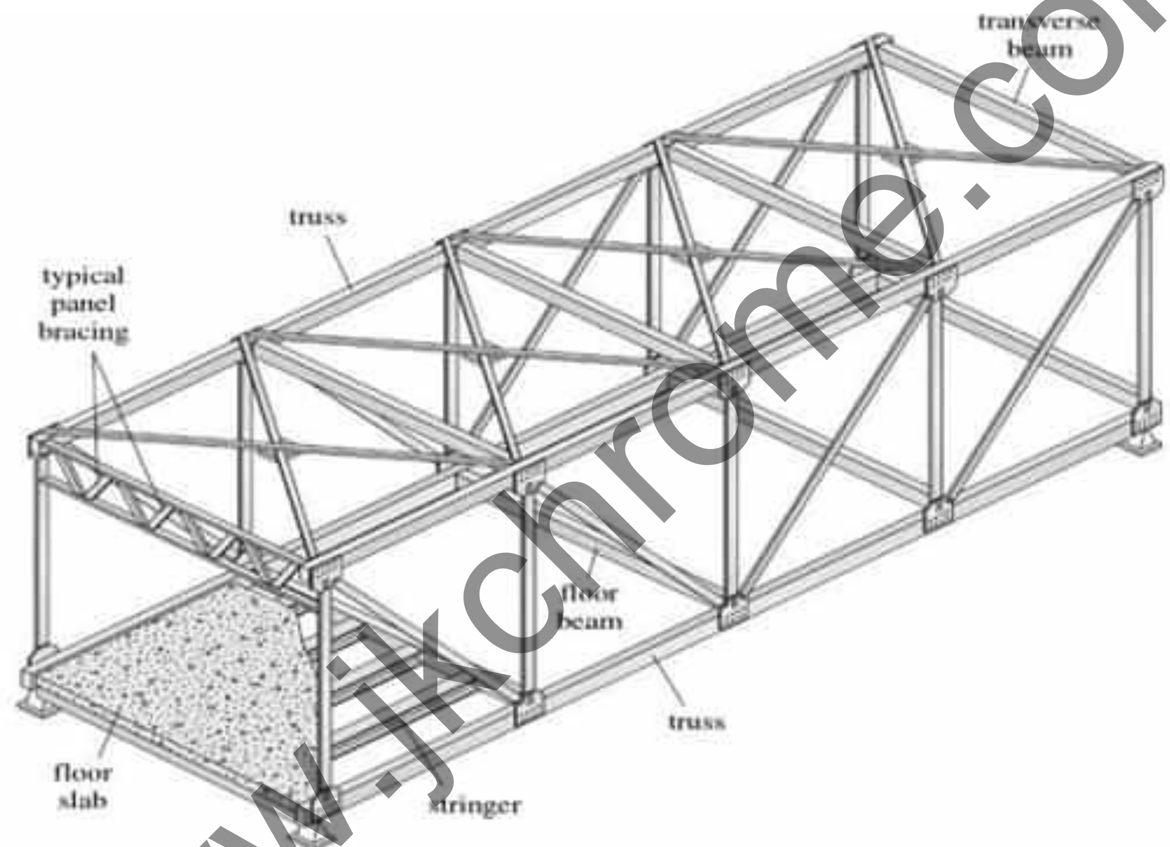
We can safely say that this structure is unstable, both by the equations of determinacy and by understanding how the structure will bend under loading.

However, if the right-hand pin were fixed-end support this case would be considered a stable, statically determinate structure.

Analysis of Trusses

Trusses are used commonly in Steel buildings and bridges.



Definition: A truss is a structure that consists of

- All straight members
- Connected together with pin joints
- Connected only at the ends of the members
- All external forces (loads & reactions) must be applied only at the joints.
- Trusses are assumed to be of negligible weight (compared to the loads they carry)

Types of Trusses

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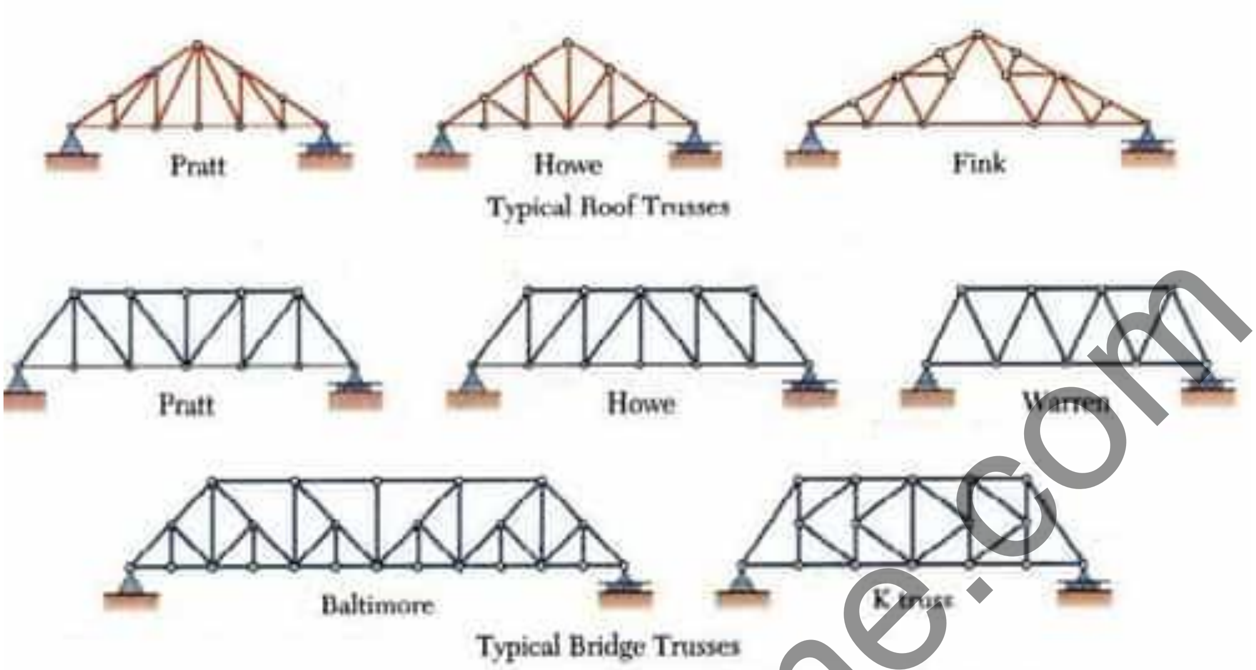


Degree of Static Indeterminacy

- $D_S = m + r_e - 2j$ where, D_S = Degree of static indeterminacy m = Number of members, r_e = Total external reactions, j = Total number of joints
- $D_S = 0 \Rightarrow$ Truss is determinate
  If $D_{se} = +1$ & $D_{si} = -1$ then $D_S = 0$ at specified point.
- $D_S > 0 \Rightarrow$ Truss is indeterminate or dedundant.

Truss Analysis: Method of Joints

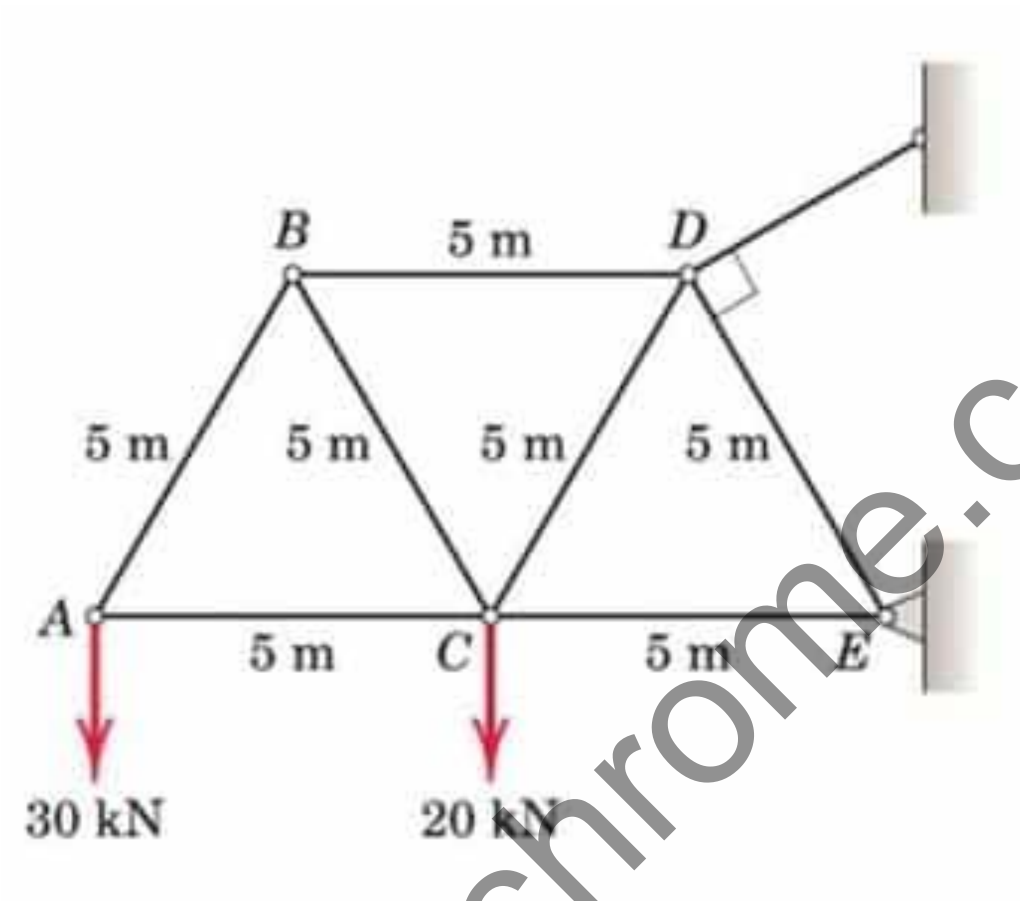
- Conditions of equilibrium are satisfied for the forces at each joint
- Equilibrium of concurrent forces at each joint
- Only two independent equilibrium equations are involved

Steps of Analysis

1. Draw Free Body Diagram of Truss
2. Determine external reactions by applying equilibrium equations to the whole truss
3. Perform the force analysis of the remainder of the truss by Method of Joints

Example 1

Determine the force in each member of the loaded truss by Method of Joints



Solution

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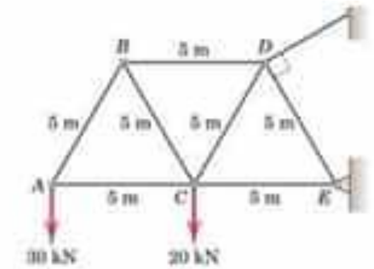
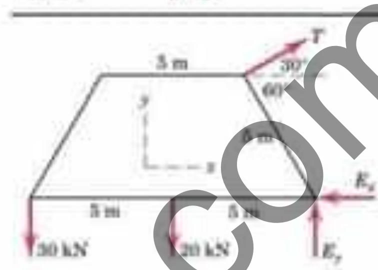
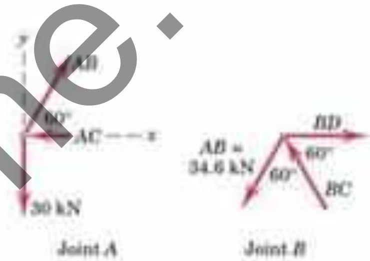


$[\Sigma M_E = 0]$ $\quad 5T - 20(5) - 30(10) = 0 \quad T = 80 \text{ kN}$

$[\Sigma F_x = 0]$ $\quad 80 \cos 30° - E_x = 0 \quad E_x = 69.3 \text{ kN}$

$[\Sigma F_y = 0]$ $\quad 80 \sin 30° + E_y - 20 - 30 = 0 \quad E_y = 10 \text{ kN}$



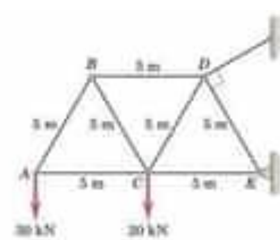


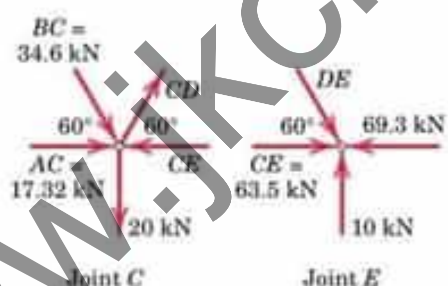
$[\Sigma F_y = 0]$ $\quad 0.866AB - 30 = 0 \quad AB = 34.6 \text{ kN } T$

$[\Sigma F_x = 0]$ $\quad AC - 0.5(34.6) = 0 \quad AC = 17.32 \text{ kN } C$

$[\Sigma F_y = 0]$ $\quad 0.866BC - 0.866(34.6) = 0 \quad BC = 34.6 \text{ kN } C$

$[\Sigma F_x = 0]$ $\quad BD - 2(0.5)(34.6) = 0 \quad BD = 34.6 \text{ kN } T$





$[\Sigma F_y = 0]$ $\quad 0.866CD - 0.866(34.6) - 20 = 0$

$CD = 57.7 \text{ kN } T$

$[\Sigma F_x = 0]$ $\quad CE - 17.32 - 0.5(34.6) - 0.5(57.7) = 0$

$CE = 63.5 \text{ kN } C$

$[\Sigma F_y = 0]$ $\quad 0.866DE = 10 \quad DE = 11.55 \text{ kN } C$

and the equation $\Sigma F_x = 0$ checks.

Truss Member Carrying Zero forces

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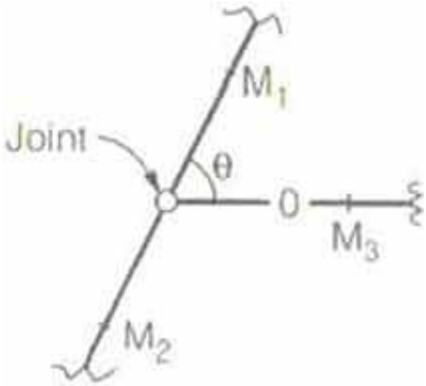
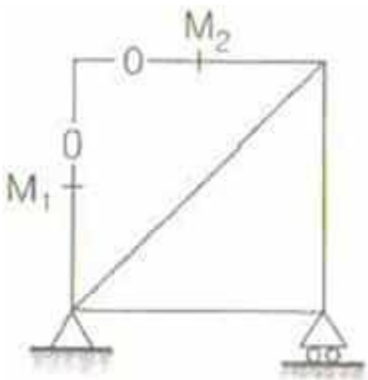
(i) M_1, M_2, M_3 meet at a joint M_1 & M_2 are collinear ⇒ M_3 carries zero force where M_1, M_2, M_3 represents member.



(ii) M_1 & M_2 are non collinear and $F_{ext} = 0$ ⇒ M_1 & M_2 carries zero force.



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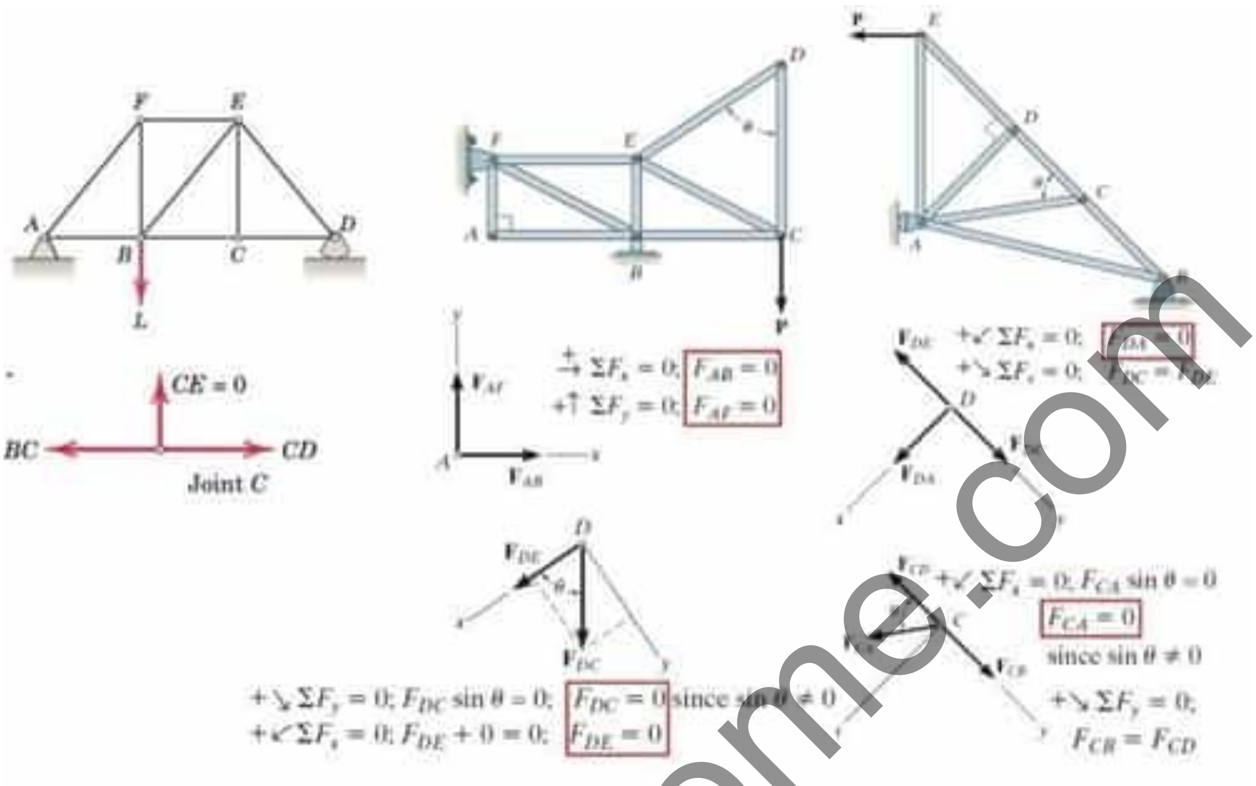

- If only two non-collinear members form a truss joint and no external load or support reaction is applied to the joint, the two members must be zero force members
- If three members form a truss joint for which two of the members are collinear, the third member is a zero-force member provided no external force or support reaction is applied to the joint.

Method of Section

- It can be used to determine three unknown member forces per FBD since all three equilibrium equations can be used
- Equilibrium under non-concurrent force system
- Not more than 3 members whose forces are unknown should be cut in a single section since we have only 3 independent equilibrium equations

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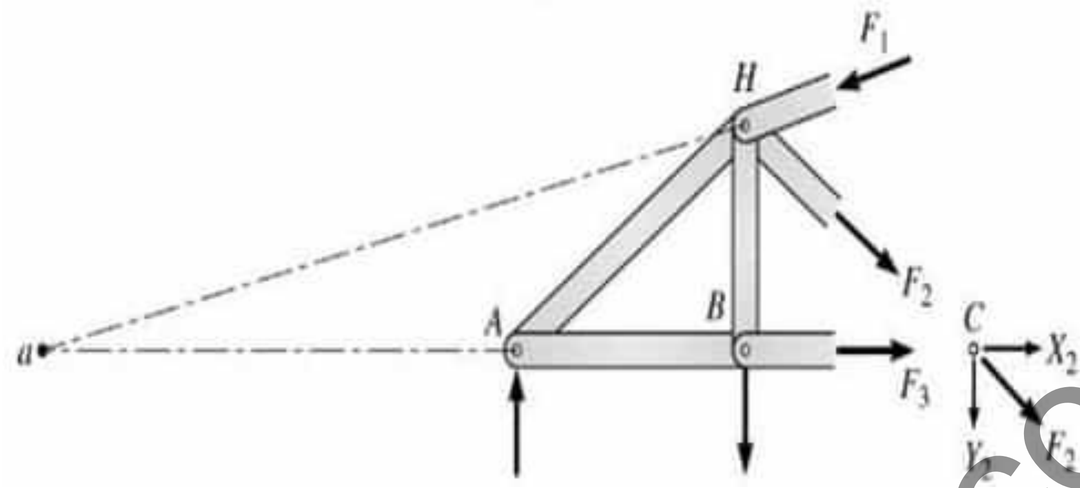
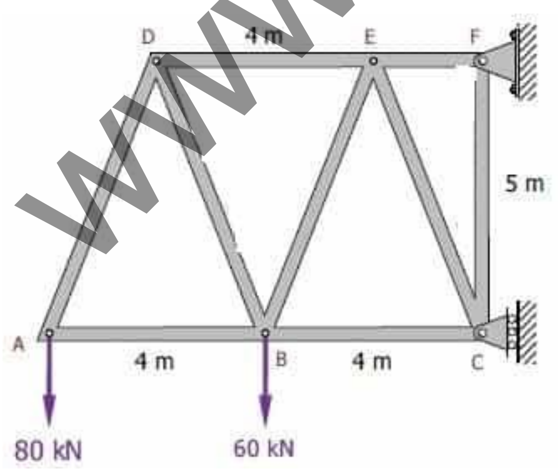


Principle:

- If a body is in equilibrium, then any part of the body is also in equilibrium.
- Forces in few particular member can be directly found out quickly without solving each joint of the truss sequentially
- Method of Sections and Method of Joints can be conveniently combined
- A section need not be straight.
- More than one section can be used to solve a given problem

Example 2

The truss in Fig given below is pinned to the wall at point F, **and supported by a roller at point C. Calculate the force (tension or compression) in members BC, BE, and DE.**



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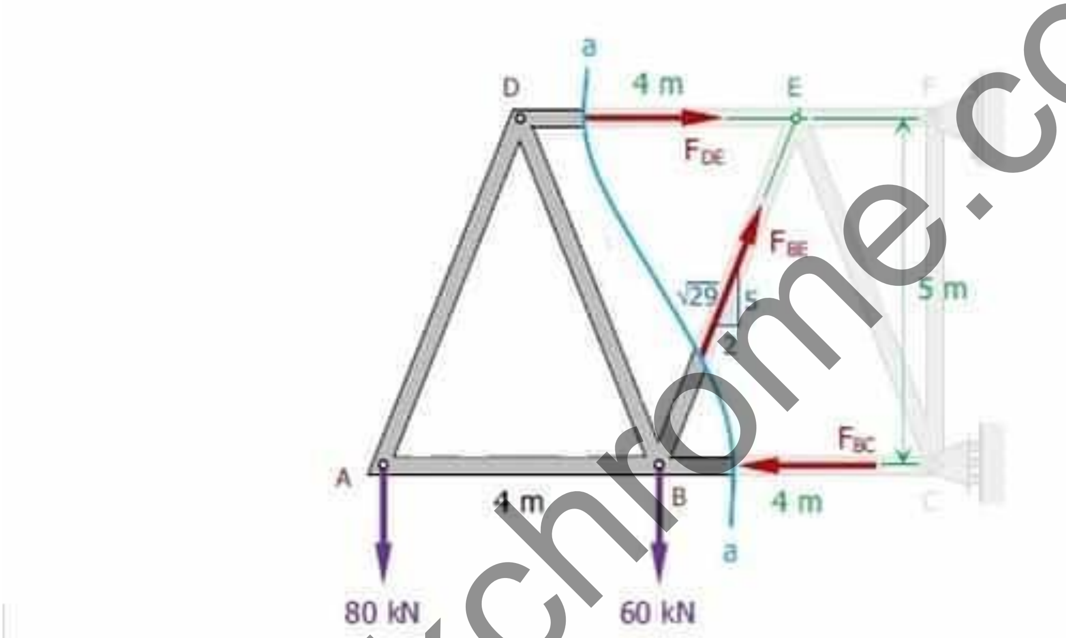
Solution

From section to the left of a-a

$\Sigma F_V = 0$

$\frac{5}{\sqrt{29}} F_{BE} = 80 + 60$

$F_{BE} = 150.78 \text{ kN tension}$



$\Sigma M_E = 0$

$5F_{BC} = 6(80) + 2(60)$

$F_{BC} = 120$ kN compression

$\Sigma M_B = 0$

$5F_{DE} = 4(80)$

$F_{DE} = 64$ kN Tension

Indeterminate Truss

(i) Final force in the truss member

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$$S = P + kX \text{ and } X = \frac{-\sum \frac{PkL}{AE}}{\sum \frac{k^2 L}{AE}}$$

sign convn → +ve for tension, −ve for compression

where,

S = Final force in the truss member

K = Force in the member when unit load is applied in the redundant member

L = Length of the member

A = Area of the member

E = Modulus of elasticity

P = Force in the member when truss become determinate after removing one of the member.

P = Zero for redundant member.

Lack of Fit in Truss

$$\frac{\partial U}{\partial X} = \Delta \text{ where, } U = \sum \frac{Q^2 L}{2AE}$$

Q = Force induce in the member due to that member which is 'Δ' too short or 'Δ' too long is pulled by force 'X'.

Deflection of Truss

$$y_C = \sum k\left[L\alpha T + \frac{PL}{AE}\right]$$

Where, y_C = Deflection of truss due to effect of loading & temp. both.

If effect of temperature is neglected then

$$y_C = \frac{\sum PkL}{AE}$$

α = Coefficient of thermal expansion

T = Change in temperature

T = +ve it temperature is increased

T = -ve it temperature is decreased

P & K have same meaning as mentioned above.

Moment Distribution Method

Introduction

- The moment distribution method is a structural analysis method for **statically indeterminate** beams and frames developed by **Hardy Cross**.
- The method only accounts for **flexural effects** and ignores axial and shear effects.
- The moment distribution method falls into the category of **displacement method** of structural analysis.
- In the slope deflection method, the end moments are computed using the slopes and deflection at the ends. Contrarily in the moment distribution method, as a first step the slopes at the ends are made zero. This is done by fixing the joints.

In the moment distribution method, every joint of the structure to be analysed is fixed so as to develop the fixed-end moments. Then each fixed joint is sequentially released and the fixed-end moments (which by the time of release are not in equilibrium) are distributed to adjacent members until equilibrium is achieved. The moment distribution method in mathematical terms can be demonstrated as the process of solving a set of simultaneous equations by means of iteration.

Important Points

1. When the member is fixed at one end and a moment is applied at the other end which is simply supported or hinged, the moment induced at

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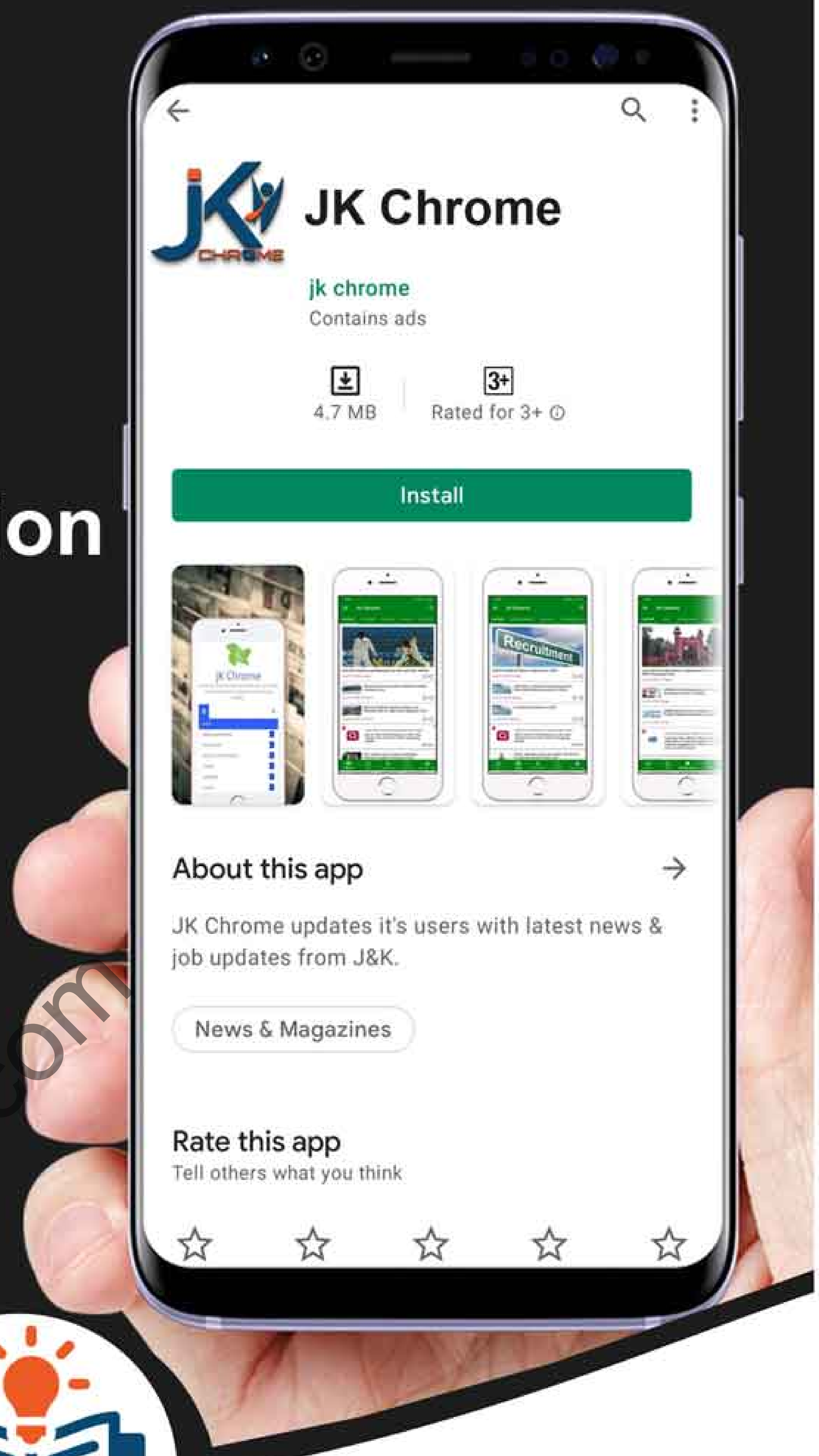

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the fixed end is one half of the applied moment. The induced moment at the fixed end is in the same direction as the applied moment.

2. If a moment is applied in a stiff joint of a structure, the moment is resisted by various members in proportion to their respective stiffnesses (i.e., moment of inertia divided by the length). If the stiffness of the member is more; then it resists more bending moment and it absorbs a greater proportion of the applied moment.
3. While distributing the moments in a rigid joint, if one end of the member is not restrained then its stiffness should be multiplied by (3/4).
4. In a fixed beam, if the support settles/subsides/sinks by an amount Δ, the moment required to make the ends horizontal is $6EI\Delta/l^2$

Basic Definition

1. **Stiffness:** Rotational stiffness can be defined as the moment required to rotate through a unit angle (radian) without the translation of either end.

$$k = \frac{F}{\Delta} or \frac{M}{\theta}$$

Where, K = Stiffness

F = Force required to produce deflection Δ

M = Moment required to produce rotation θ.

2. **Stiffness Factor:**

(i) It is the moment that must be applied at one end of a constant section member (which is unyielding supports at both ends) to produce a unit rotation of that end when the other end is **fixed**

, i.e. k = 4EI/l.

Where K = Stiffness of BA at joint B. When the farther end is fixed.

EI = Flexural rigidity

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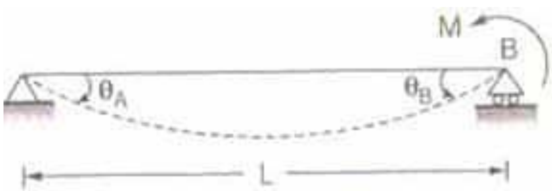
L = Length of the beam

M = Moment at B.

(ii) It is the moment required to rotate the near end of a prismatic member through a unit angle without translation, the far end being hinged is k = 3EI/l.



1. **Carry Over Factor:**
   It is the ratio of the induced moment to the applied moment. The carry-over factor is always (1/2) for members of the constant moment of inertia (prismatic section). If the end is hinged/pin-connected, the carry-over factor is zero. It should be mentioned here that carry over factors values differ for non-prismatic members. For non-prismatic beams (beams with variable moment of inertia); the carryover factor is not half and is different for both ends.

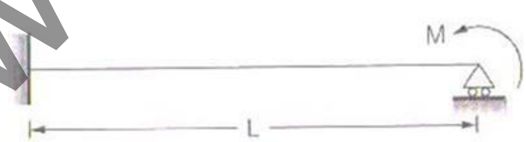
   Carryover factor = $\frac{\text{Carry over moment}}{\text{Applied moment}}$

   COF may greater than, equal to or less than 1.

   Standard Cases:

   $(i)\ COF = \frac{1}{2}$

   (ii) COF = 0



   $(iii)\ COF = \frac{a}{b}$

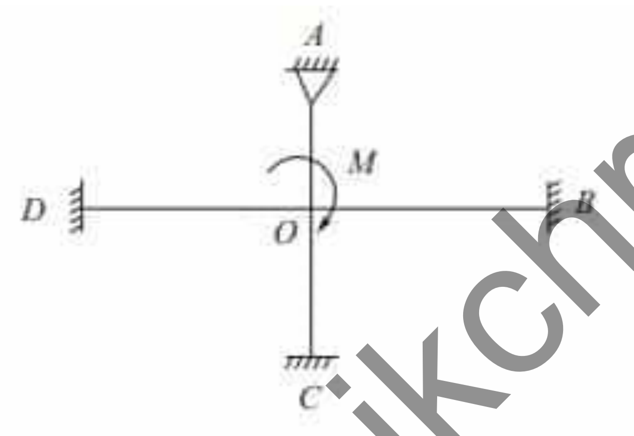
2. **Distribution Factors:**

$$DF = \frac{\text{Stifffness of a member}}{\text{Sum of stiffness of all members at that joint}}$$

or

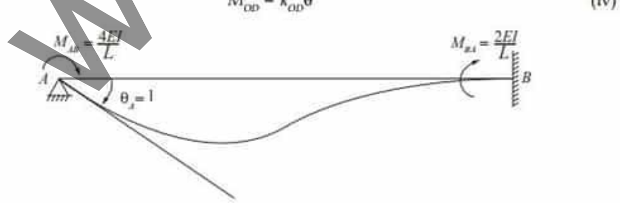
$$DF = \frac{\text{Relative stiffness of a member}}{\text{Sum of relative stiffness of all member at that joint}}$$

Consider a frame with members OA, OB, OC and OD rigidly connected at O as shown below. Let M be the applied moment at joint O in the clockwise direction. Let the joint rotate through an angle θ. The members OA, OB, OC and OD also rotate by the same angle θ



Let k_{OA} , k_{OB}, k_{OC} and k_{OD} be the stiffness values of the members OA, OB, OC and OD respectively; then

$$M_{OA} = k_{OA}\theta \quad \text{(i)}$$
$$M_{OB} = k_{OB}\theta \quad \text{(ii)}$$
$$M_{OC} = k_{OC}\theta \quad \text{(iii)}$$
$$M_{OD} = k_{OD}\theta \quad \text{(iv)}$$



(a) Beam with far end fixed

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$d_{OB} = \frac{k_{OB}}{\Sigma k}$ = distribution factor for OB

$d_{OC} = \frac{k_{OC}}{\Sigma k}$ = distribution factor for OC

$d_{OD} = \frac{k_{OD}}{\Sigma k}$ = distribution factor for OD

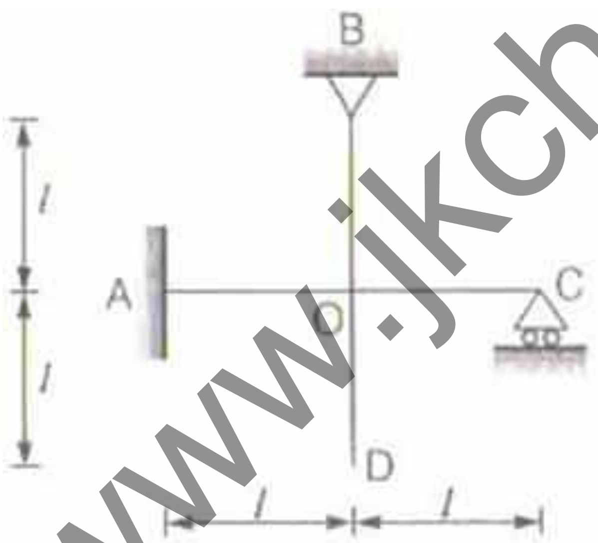
5. Relative Stiffness

(i) When farther end is fixed

Relative stiffness for member $= \frac{I}{L}$

(ii) When the farther end is hinged

Relative stiffness for member $= \frac{3I}{4L}$



Stiffness of OA

$= \frac{4EI}{l}$

Stiffness of OB

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Stiffness of OC $= \frac{3EI}{l}$

Stiffness of OD = 0

SIGN CONVENTION

Clockwise moments are considered positive and anticlockwise moments negative

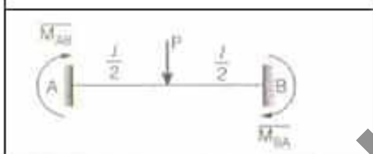
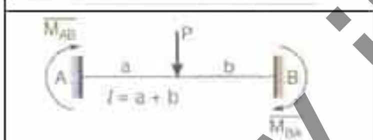
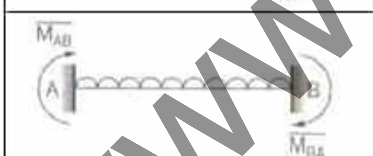
+ve → Sagging

−ve → Hogging

and All clockwise moment → +ve

and All Anti clockwise moment → −ve

Span length is l

	$\overline{M_{AB}}$	$\overline{M_{BA}}$
	$\frac{-Pl}{8}$	$\frac{Pl}{8}$
	$\frac{-Pab^2}{l^2}$	$\frac{Pa^2b}{l^2}$
	$\frac{-wl^2}{12}$	$\frac{wl^2}{12}$
	$\frac{-wl^2}{30}$	$\frac{wl^2}{20}$

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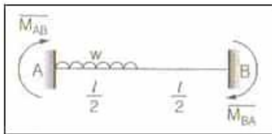
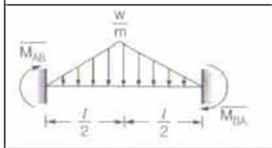
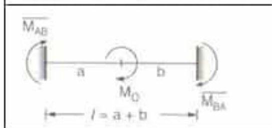
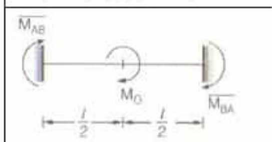
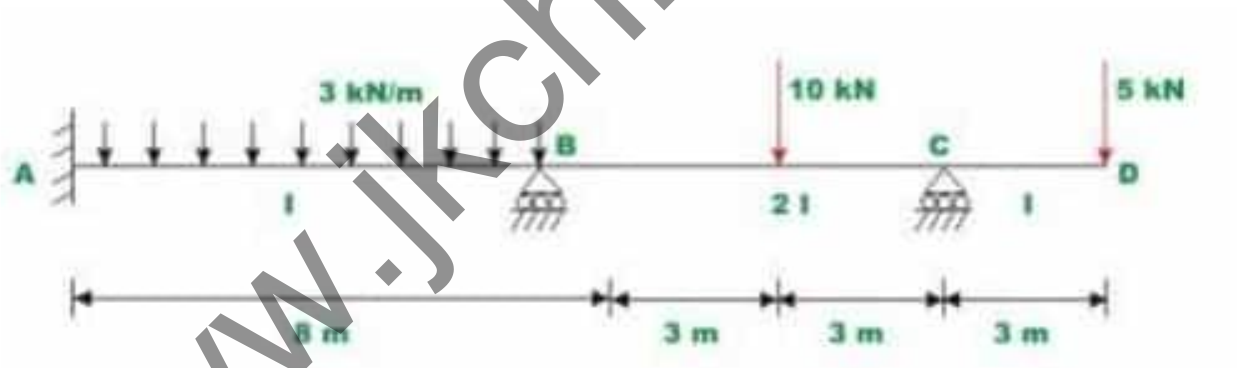
	$\frac{-11}{192}wl^2$	$\frac{5}{192}wl^2$
	$\frac{-5}{96}wl^2$	$\frac{5}{96}wl^2$
	$\frac{M_0 b(3a-l)}{L^2}$	$\frac{M_0 a(3b-l)}{L^2}$
	$\frac{M_O}{4}$	$\frac{M_O}{4}$

Example

Draw the bending moment diagram for the continuous beam ABCD loaded as shown below. The relative moment of inertia of each span of the beam is also shown in the figure.



Solution

Note that joint C is hinged and hence stiffness factor BC gets modified. Assuming that the supports are locked, calculate fixed end moments. They are

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$M_{AB}^{F} = 16$ kN.m

$M_{BA}^{F} = -16$ kN.m

$M_{BC}^{F} = 7.5$ kN.m

$M_{CB}^{F} = -7.5$ kN.m , and

$M_{CD}^{F} = 15$ kN.m

In the next step calculate stiffness and distribution factors

$$K_{BA} = \frac{4EI}{8}$$

$$K_{BC} = \frac{3}{4}\frac{8EI}{6}$$

$$K_{CB} = \frac{8EI}{6}$$

At joint B:

$$\sum K = 0.5EI + 1.0EI = 1.5EI$$

$$D_{BA}^{F} = \frac{0.5EI}{1.5EI} = 0.333$$

$$D_{BC}^{F} = \frac{1.0EI}{1.5EI} = 0.667$$

At C:

$$\sum K = EI, D_{CB}^{F} = 1.0$$

Now all the calculations are shown below

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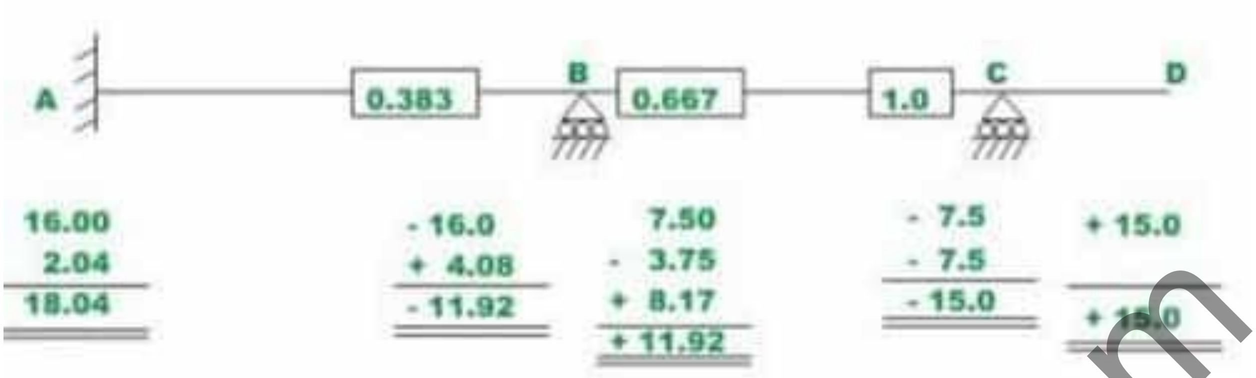
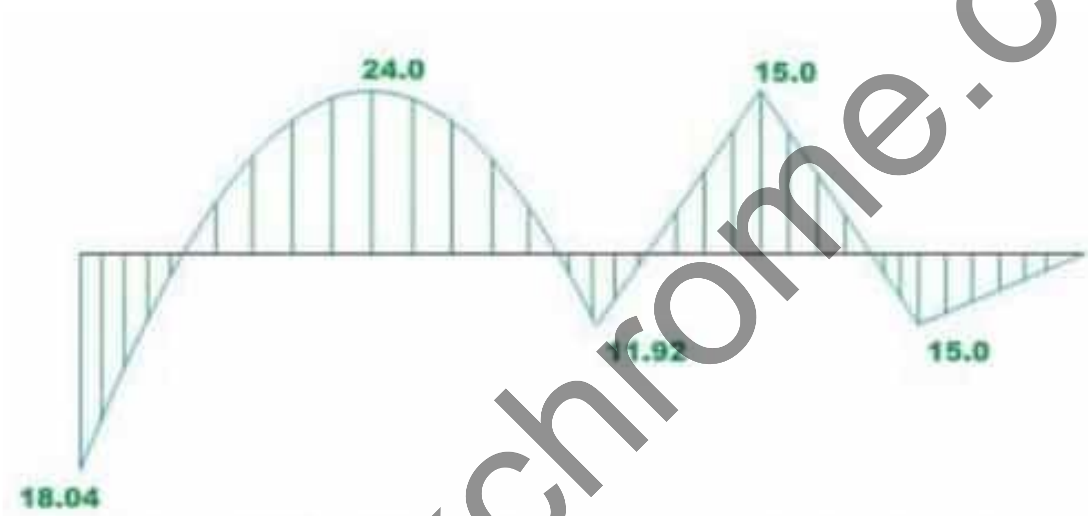


Computation

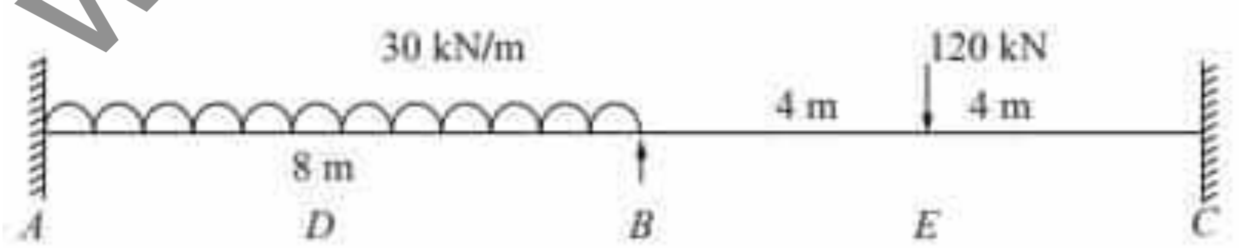


Bending Moment Diagram

Note: This problem has also been solved by the slope-deflection method

Example 2

Analyse the continuous beam by the moment distribution method. Draw the shear force diagram and bending moment diagram.



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Solution

Distribution factors

The distribution factors at joint *B* are evaluated as follows.

Table Distribution factors

Joint	*Members*	*Relative Stiffness (k)*	*Sum* Σk	*Distribution Factors* ($k/\Sigma k$)
	BA	$I/8 = 0.125I$		0.5
B			$0.25I$	
	BC	$I/8 = 0.125I$		0.5

Fixed end moments

$$M_{AB} = -30 \times \frac{8^2}{12} = -160 \text{ kNm}; \qquad M_{BC} = -\frac{120 \times 8^2}{8} = -120 \text{ kNm}$$

$$M_{BA} = +160 \text{ kNm}; \qquad M_{CB} = +120 \text{ kNm}$$

Table Moment distribution table

Joint	*A*	*B*		*C*
Members	*AB*	*BA*	*BC*	*CB*
DF	0	0.5	0.5	0
FEMS Bal	−160 –	+160 −20	−120 −20	+120 –
Co	−10	–	–	−10
Final	−170	+140	−140	+110

Using the above end moments,

$$M_D = \frac{30 \times 8^2}{8} - \left(\frac{170 + 140}{2}\right) = 85 \text{ kNm}$$

$$M_E = 120 \times \frac{8}{4} - \left(\frac{140 + 110}{2}\right) = 115 \text{ kNm}$$

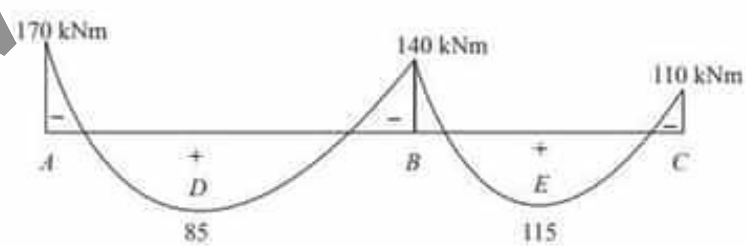

FIG. Bending moment diagram

Shear force diagrams
Equilibrium of span AB

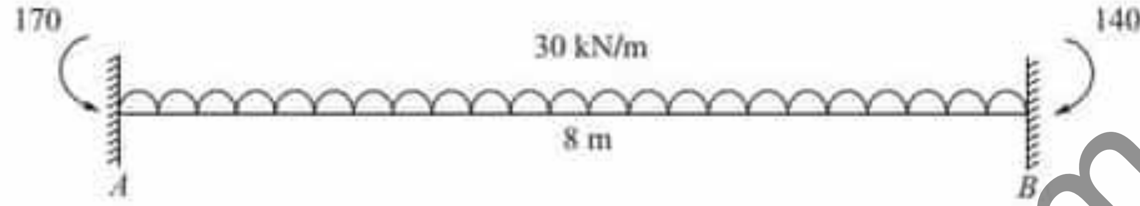

FIG. 2.29

$$\sum V = 0;\ V_{AB} + V_{BA} = 240$$

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$$\Sigma M_A = 0;\ -170 + 140 + 30 \times \frac{8^2}{2} - 8V_{BA} = 0 \qquad (2)$$

$$\boxed{\begin{array}{l} V_{BA} = 116.3 \text{ kN} \\ V_{AB} = 123.7 \text{ kN} \end{array}}$$

Equilibrium of span BC

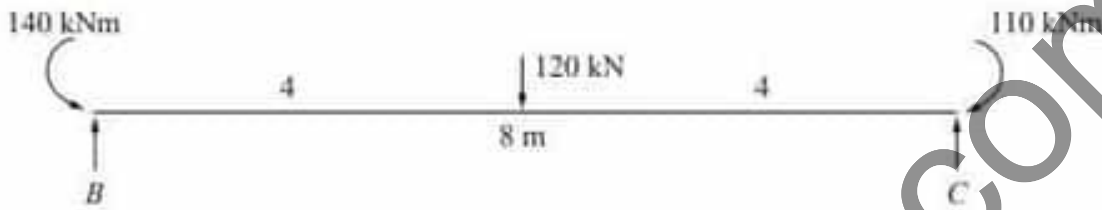

FIG. 2.30

$$\Sigma V = 0;\ V_{BC} + V_{CB} = 120 \qquad (3)$$

$$\Sigma M_B = 0;\ -140 + 110 + 120(4) - 8V_{CB} = 0$$

$$\boxed{\begin{array}{l} V_{CB} = 56.3 \text{ kN} \\ V_{BC} = 63.7 \text{ kN} \end{array}}$$

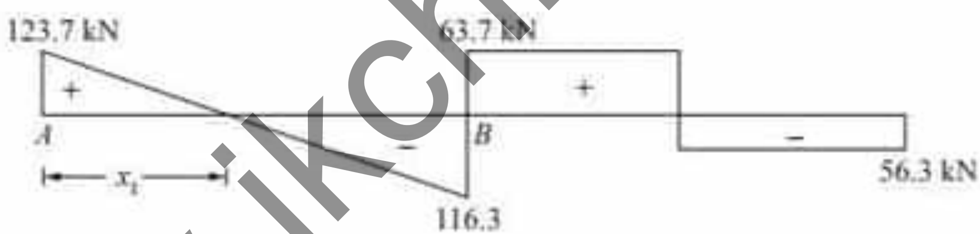

FIG. Shear force diagram

From similar Δ's

$$\frac{x_1}{(8 - x_1)} = \frac{123.7}{116.3}$$

$$116.3x_1 = 989.6 - 123.7x_1$$

$$x_1 = 4.12 \text{ m}$$

Arches and Cables

INTRODUCTION

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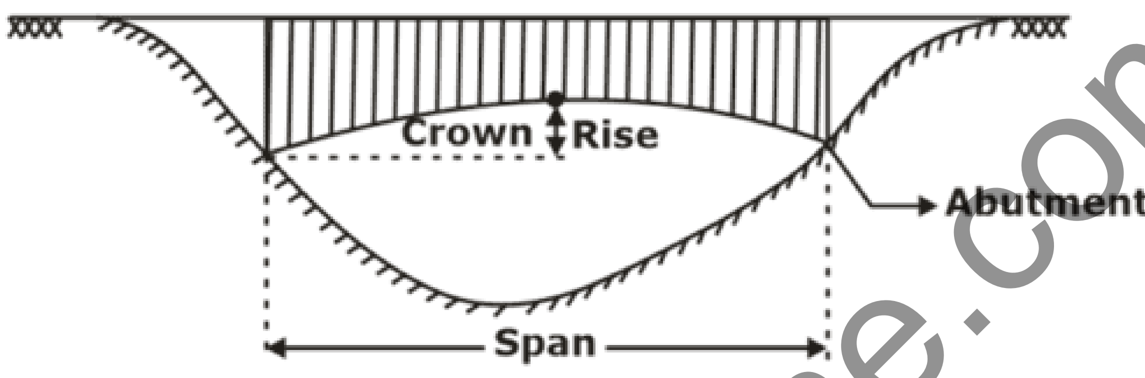
Beams generally transfer the applied load to end supports by bending and shear action but arches transfers load to abutments at spring points. The topmost point is called the crown which sometimes has a hinge. The height of the crown above the support level is known as rise. An arch is generally economical for a larger span compared to a simply supported beam.



Typical Arch Bridge

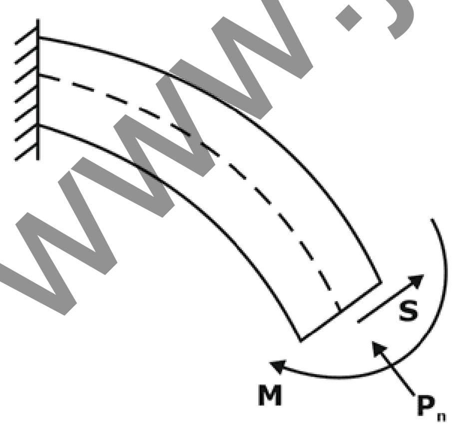
Design Forces in Arches:

An arch will be subjected to three forces.

(a) Bending Moment

(b) Normal Thrust

(c) Radial shear force



1. **TYPES OF ARCHES**

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There are three types of arches depending upon the number of hinges provided.

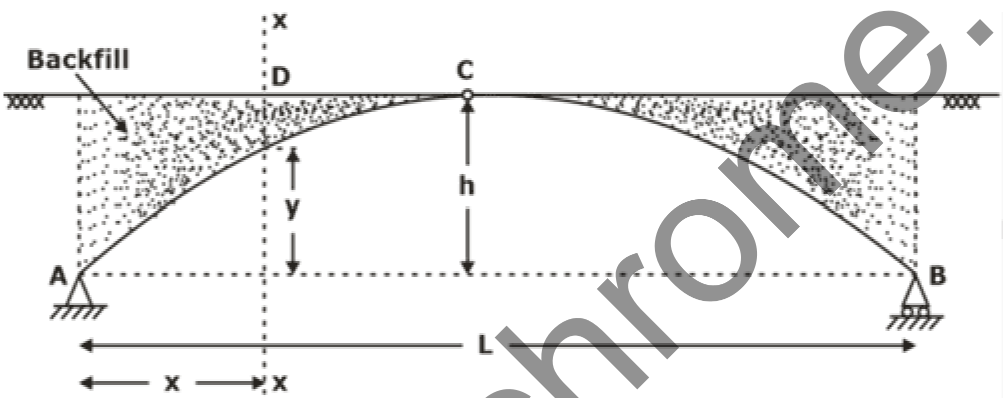
(i) Three hinged arch (Determinate)

(ii) Two hinged arch (Indeterminate to 1 degree)

(iii) Fixed arch (Indeterminate to 3 degree)

2.1. Three hinged Arch

The three hinged arches are statically determinate structure as equations of equilibrium alone are sufficient to find all the unknown quantities.



Circular Arch:

From the properly of a circle the radius r of the circular arch of span L and rise h may be found as

$$\frac{L}{2} \times \frac{L}{2} = h(2R - h)$$

$$\Rightarrow \boxed{R = \frac{L^2}{8h} + \frac{h}{2}}$$

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Taking origin at A, the coordinates of any pont d on the arch may be defined as

$$x = \left[\frac{L}{2} - R\sin\theta\right]$$

$$y = R\cos\theta - (R - h)$$

$$\Rightarrow \boxed{y = h - R(1 - \cos\theta)}$$

Parabolic Arch:

Taking spring point as the origin, its equation is given by

$$\boxed{y = \frac{4h_x}{L^2}(L - x)}$$

Bending moment at the section X-X

$$BM_{X-X} = +V_A \times x - H_A \times y$$

$$\Rightarrow BM_{X-X} = \text{Beam moment} - \text{H-moment}$$

When compared with a beam of similar span, bending moment at any section in a three hinged arch is less by an amount of '$H \times y$' or moment dur to horizontal force.

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(i) ILD for horizontal thrust (H):

Taking moment about A,

$$V_B = \frac{x}{l}$$

$$V_A = \frac{l - x}{l}$$

When the load is in portion AC taking moment about hinge C, We get

$$H.h = V_B \times \frac{l}{2} = \frac{x}{2} \times \frac{l}{2}$$

$$\boxed{H = \frac{x}{2h}}$$

When,

x = 0 → H = 0

$x = \frac{l}{2} \rightarrow H = \frac{l}{4h}$

When unit load is in portion CB, considering the left half portion and taking moment about B. We get

$$H.h = V_A \times \frac{l}{2} = \frac{l - x}{l} \times \frac{l}{2} = \frac{l - x}{2}$$

$$\boxed{H = \frac{l - x}{2h}}$$

When,

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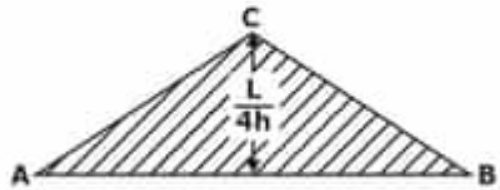
$x = \frac{l}{2}$, $H = \frac{l}{4h}$

$x = l$, $H = 0$

$\therefore$ ILD for H is a triangle with its maximum ordinate equals to $\frac{'l'}{4h}$ at hinge 'C'.



ILD for horizontal thrust

(ii) ILD for Bending Moment:

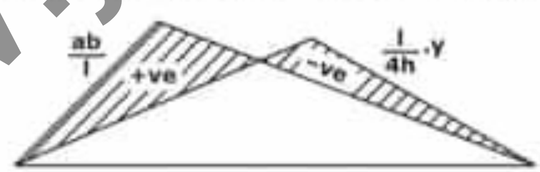
Bending moment at any given section in the arch is given as

$$BM = \text{Beam moment} - Hy$$

We know that ILD for beam moment at D is a triangle with maximum ordinate $\frac{'ab'}{l}$ at D.

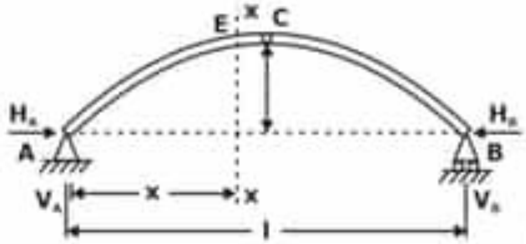
Since for H, ILD is a triangle with maximum ordinate of $\frac{'l'}{4h}$ at hinge c, H.y diagram is a triangle with $'\frac{l}{4h} \cdot y'$ as the maximum ordinate at C. This is to be subtracted from beam moment diagram.

From the above ILD, we find that to get maximum positive bending moment at D, keep the unit load D. To get maximum negative bending moment at D, keep the unit load at C. To get zero bending moment at D, keep the unit load somewhere between D and C.



ILD for BM at D

(iii) Normal thrust and Radial shear:



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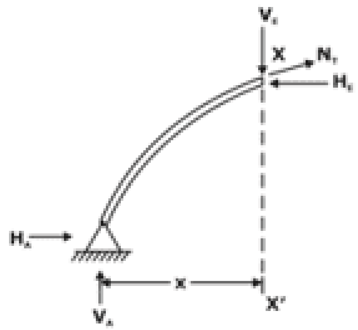
FBD for the section X – X



Normal thrust at E = Sum of components of all forces normal to the cross section at E

$$N_T = H_E \cos\theta + V_E \sin\theta$$

Radial shear at E = Sum of component of all the forces parallel to cross section at E

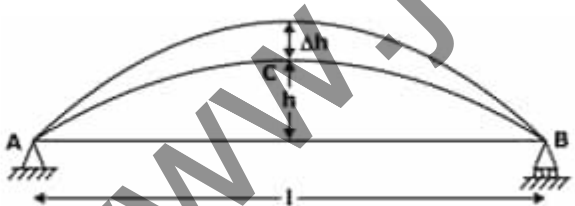
$$R_s = V_E \cos\theta - H_E \sin\theta$$

Note: If a two hinged or three hinged arch is subjected to UDL throughout its length, then BM and R_s are zero everywhere. The cross section is subjected to N_T (Normal thrust) only.

(iv) Temperature effects in 3-hinged arches

Case 1: When the arch is subjected to temperature rise alone

Since, three hinged arch is a statically determinate structure due to temperature variation or due to settlement of support, no stress are developed anywhere in the whole structure.



Although, there is no stress developed in the structure, the crown will rise by the value of Δh due to increase in temperature to accommodate the free expansion of the arch.

Here,

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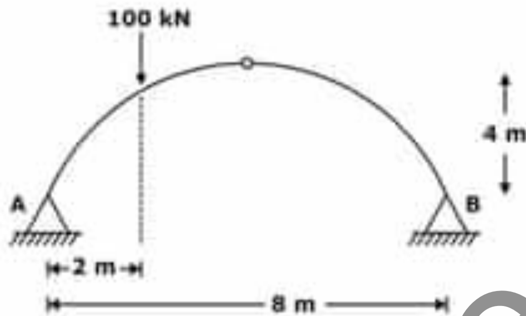
$$\Delta h = \left[\frac{l^2 + 4h^2}{4h}\right](\alpha t)$$

Where,

α = coefficient of thermal expansion.

t = change or rise in temperature

Example:



Sol.

Taking Moment about B,

$$R_A \times 8 = 100 \times 6$$

$$\Rightarrow R_A = 75\ kN$$

And,

$$R_B = 25\ kN$$

Bending moment about C would be zero.

75 x 4 - 100 x 2 - H x 4 =0

H= 25 kN

Normal thrust and radial shear at a distance 1m from A.

$$N_T = 25 \cos 45 + 75 \sin 45$$

$$\Rightarrow N_T = 70.71\ kN$$

$$R_s = V_E \cos\theta - H_E \sin\theta$$

$$\Rightarrow R_S = 35.35\ kN$$

Eddy's Theorem: This theorem states that," The Bending moment at any section of an arch is directly proportional to the vertical intercept between the actual arch and linear arch (Theoretical arch)". Where linear arch is the shape

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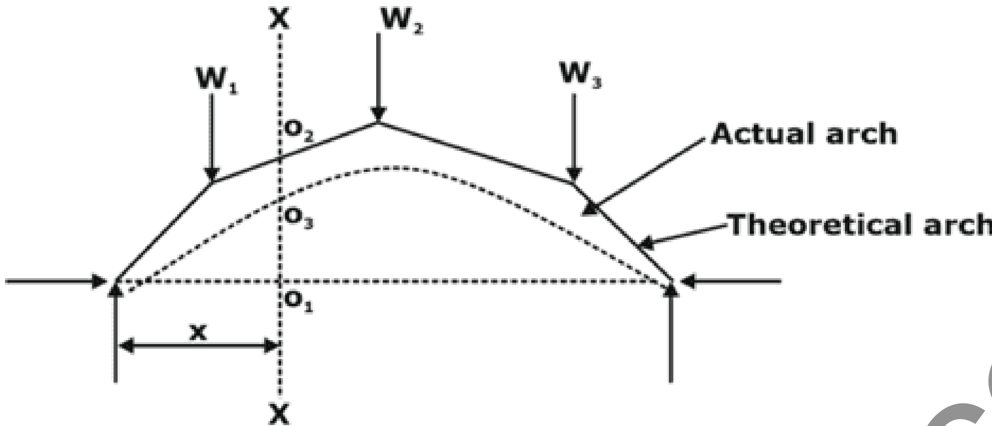
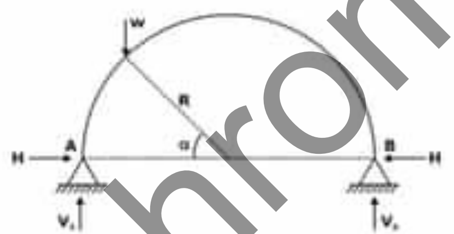
of funicular polygon due to the loading.



2.2. Two hinged arches

A two hinged arch is an indeterminate arch. The horizontal thrust is determined using Castigliano's theorem of least energy.



Two hinged circular arch

Assuming the redundant to be H, As per Castigliano's theorem

$$\frac{\partial U}{\partial H} = 0$$

Which gives the following condition

$$H = \frac{\int_0^l \frac{M_x y dx}{EI_C}}{\int_0^l \frac{y^2 dx}{EI_C}}$$

Where,

M_x = beam moment at any section x − x

I_C = Moment of inertia of the cross section of the arch at the crown.

(i) Horizontal Thrust in case of circular arch subjected to point load

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$$H = \frac{W}{\pi} \sin^2 \alpha$$

(ii) Horizontal Thrust in case of circular arch subjected to UDL

$$H = \frac{4}{3} \frac{wR}{\pi}$$

(iii) Horizontal Thrust in case of parabolic arch subjected to a point load at centre

$$H = \frac{25}{128} \frac{wL}{H}$$

(iv) Horizontal Thrust in case of parabolic arch subjected to a UDL

$$H = \frac{wl^2}{8h}$$

If there is rib shortening, temperature rise by t°C and yielding of supports then horizontal thrust is given by

$$H = \frac{\int \frac{M_x y dx}{EI_C} + \alpha tl}{\int \frac{y^2 dx}{EI_C} + \frac{l}{AE} + k}$$

Where,

αtl = due to increase in temperature

l/AE due to rib shortening

K = yielding of support/unit horizontal thrust.

In a two-hinged parabolic arch as the temperature increase, horizontal thrust increases. If the effect of rib shortening and yielding of support are considered then horizontal thrust decreases.

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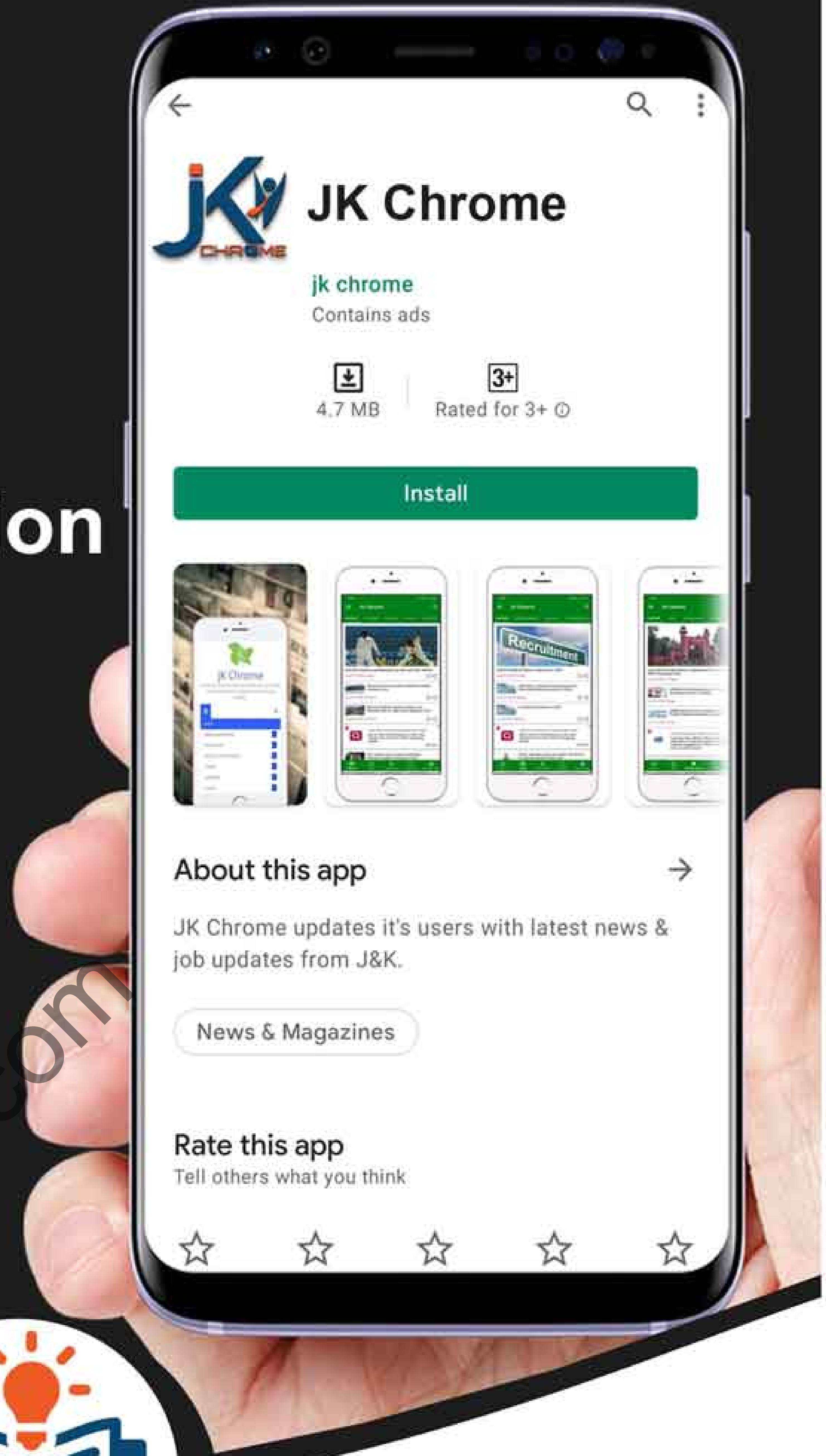

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