Strength of Materials

Index

Stress and Strain

Stress: The force of resistance per unit area, offered by a body against deformation is known as **stress**.

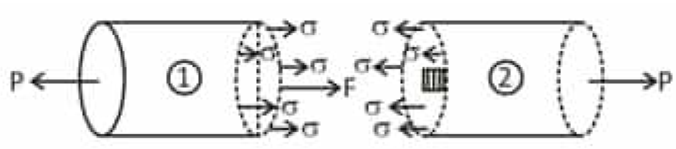


Fig.1: Stress

It is denoted by a symbol 'σ'. And mathematically expressed as

$$\sigma = \frac{P}{A}$$

TYPES OF STRESSES:

only two basic stresses exist.

(1) Normal stress and

(2) Shear stress.

(i) Normal stresses:

If the force applied are perpendicular or normal to areas concerned, then these are termed as normal stresses.

The normal stresses are generally denoted by a Greek letter (σ).

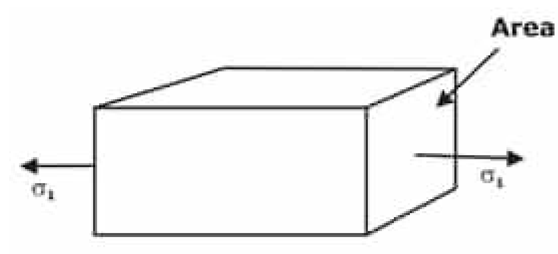


Fig.2 Uniaxial Normal Stress

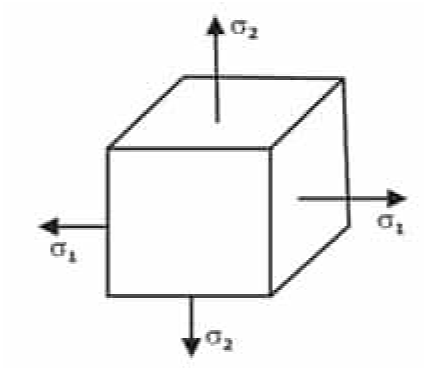


Fig.3: Biaxial Normal Stress

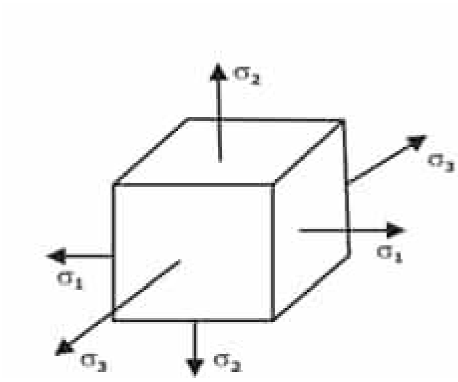


Fig.4: Triaxial Normal Stress

Tensile or compressive stresses:

The normal stresses can be either tensile or compressive depending upon the direction of the load.

Sign convention: The tensile forces are termed as (+ve) while the compressive forces are termed as negative (-ve).

(ii)Shear Stress:

when cross-sectional area of a block of material is subject to a distribution of forces which are parallel to the area concerned. Such forces are associated with a shearing of the material, are known as shear forces. The stress produced by these forces are known as shear stresses.

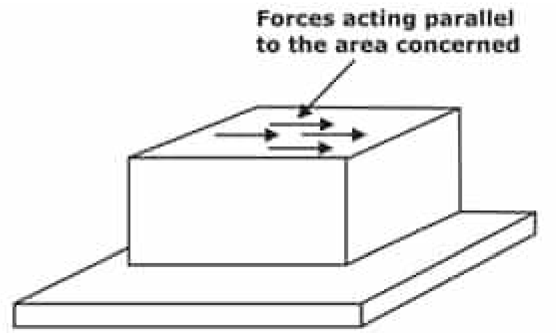


Fig.5: Shear stress

It is generally denoted by 'τ' and expressed as

$$\tau = \frac{\text{Shear resistance}}{\text{Shear area}} = \frac{P_s}{A}$$

The complementary shear stresses are equal in magnitude.

The same form of relationship can be obtained for the other two pair of shear stress components to arrive at the relations,

$\tau_{xy}=\tau_{yx}$

$\tau_{yz}=\tau_{zy}$

Sign convections for shear stresses:

Shear stress tending to turn the element Clockwise is taken as Positive.

Shear stress tending to turn the element Anticlockwise is taken as Negative.

STRAIN

When a prismatic bar is subjected to axial load, it undergoes a change in length, as indicated in Figure. This change in length is usually called deformation.

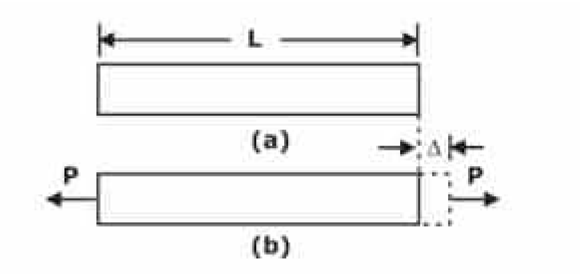


Fig.6: Deformation of bar under axial load

If the axial force is tensile, the length of the bar is increased, while if the axial force is compressive, there is shortening of the length of the bar.

The deformation (i.e. elongation or shortening) per unit length of the bar is termed as strain and denoted by ε or e.

$$\text{strain} = \frac{\text{change in length}}{\text{Original lenth}} = \frac{\Delta L}{L}$$

Classification of strain

(i) Longitudinal strain:

The ratio of axial deformation along the length of the applied load to the original length of the body is known as longitudinal (or linear) strain.

L = Length of the body,

P = Tensile force acting on the body,

δL = Increase in the length of the body in the direction of P.

Then,

$$\text{longitudinal strain} = \frac{\delta L}{L}$$

(ii) Lateral strain:

The strain at right angles to the direction of applied load is known as lateral strain. Let a rectangular bar of length L, breadth b and depth d is subjected to an axial tensile load P. The length of the bar with increase while the breadth and depth will decrease.

δL = Increase in length,

δb = Decrease in breadth, and

δd = Decrease in depth.

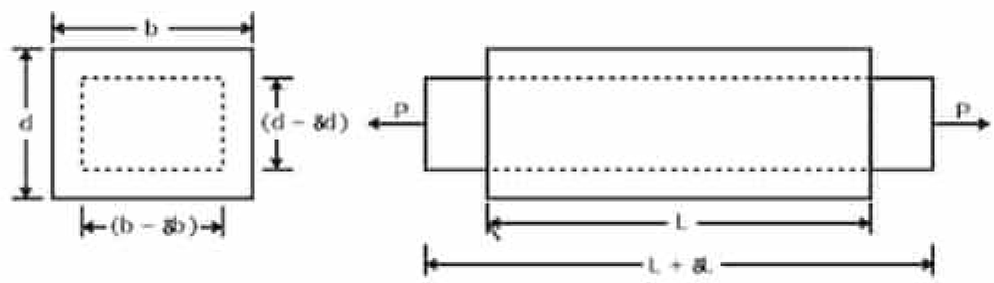


Fig.8: strain

$$\text{Then longitudinal strain} = \frac{\delta L}{L}$$

$$\text{lateral strain} = \frac{\text{Change in lateral dimension}}{\text{original lateral dimension}} = \frac{d_o - d_i}{d_i}$$

$$\text{Lateral strain} = \frac{\delta b}{b} \text{ or } \frac{\delta d}{d}$$

(iii)Shear Strain

Change in initial right angle between two-line elements which are parallel to x and y-axis respectively.

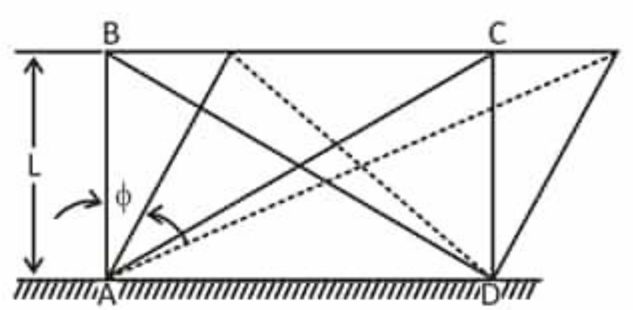


Fig.9: Shear strain

$$G = \frac{\tau}{\gamma} \Rightarrow \gamma = \frac{\tau}{G}$$

STRESS - STRAIN DIAGRAM

The mechanical properties of a material are determined in the laboratory by performing tests on small specimens of the material, in the materials testing laboratory. The most common materials test is the tension test performed on a cylindrical specimen of the material.

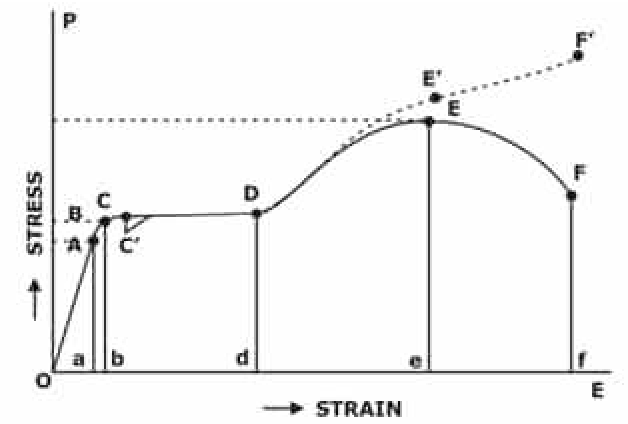


Fig.16: Stress strain Diagram

A = Proportional Limit　　Oa = Linear Deformation

B = Elastic Limit　　Ob = Elastic Deformation

C = Yield Point　　bd = Perfect Plastic Yielding

C’ = Lower Yield Point　　de = Strain hardening

E = Ultimate Strength　　ef = Necking

F = Rupture Strength/ Fracture strength

It is customary to base all the stress calculations on the original cross-sectional area of the specimen, and since the latter is not constant, the stresses so calculated are known as Nominal stresses.

LINEAR ELASTICITY: HOOKE’S LAW

The slope of stress-strain curve is called the young’s modulus of elasticity (E):

Slope of stress-strain curve,

$$E = \frac{\sigma}{\varepsilon}$$

$$\sigma = \varepsilon . E$$

This equation is known as Hooke’s law.

Type of Metal Behaviour

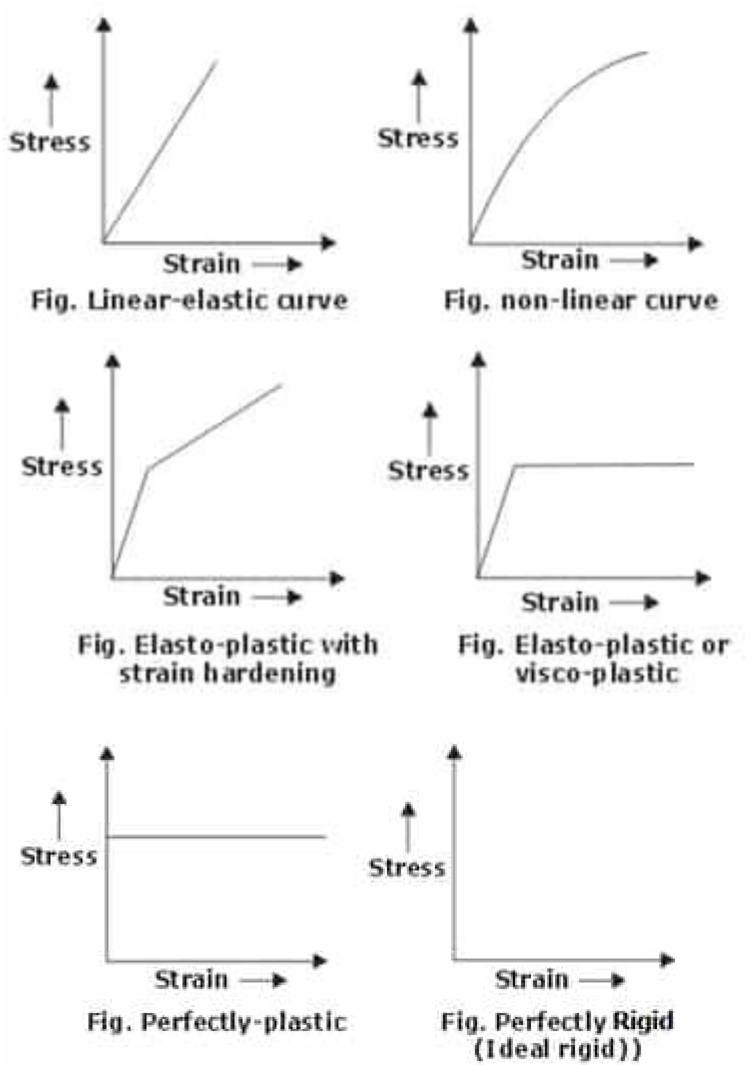


Fig.17: Stress strain Diagram

ELONGATION OF BAR UNDER DIFFERENT CONDITION

UNIFORMLY TAPERING CIRCULAR BAR

Let us now consider a uniformly tapering circular bar, subjected to an axial force P, as shown in Figure. The bar of length L has a diameter d_1 at one end and d_2 at the other end ($d_2 > d_1$).

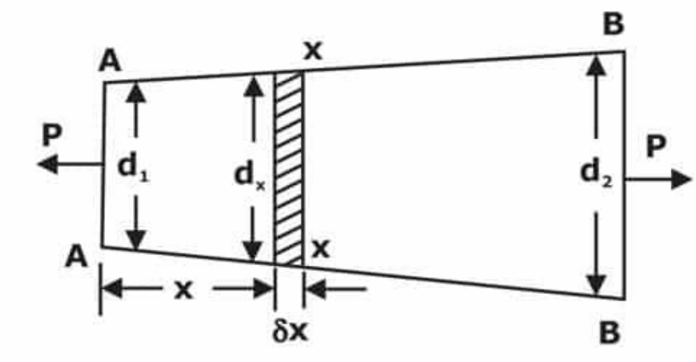


Fig.18: Uniformly tapering circular bar

$$\Delta L = \frac{4PL}{\pi E\, d_1\, d_2}$$

PRINCIPLE OF SUPERPOSITION

Bars In Series

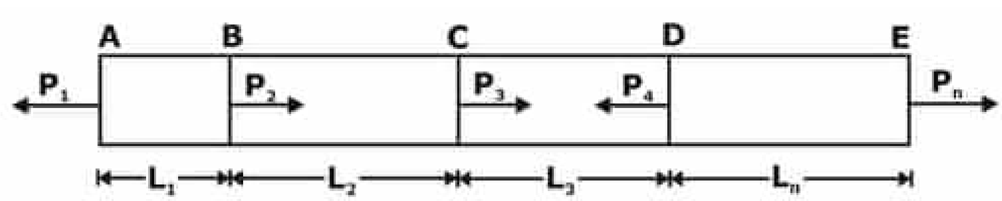


Fig.19: A prismatic bar subjected to multiple loads

It stated that

"the resultant strain in bar will be equal to the algebraic sum of the strains caused by the individual forces acting along the length of the member.

Thus, if a member of uniform section is subjected to a number of forces, the resulting deformation (ΔL) is given by

$$\Delta L = \Sigma \frac{P\,L}{A\,E} = \frac{1}{A\,E}\ [P_1L_1 + P_2L_2 + \ldots + P_n\,L_n]$$

Elastic constants

Elastic constants are those factors which determine the deformations produced by a given stress system acting on a material.

Various elastic constants are :

(i) Modulus of elasticity (E)

(ii) Poisson's ratio (μ or 1/m)

(iii) Modulus of rigidity (G or N)

(iv) Bulk modulus (K)

Materials on the basis of elastic properties

(i)Homogeneous Material

When a material exhibits same elastic properties at any point in a given directions than the material is known as homogenous material.

(ii)Isotropic Material

When a material exhibits Same elastic properties at any direction at a given point than the material is known as Isotropic Material.

(iii)Anisotropic Material

When a material exhibits different elastic properties at every direction at a every point than the material is known as Isotropic Material.

(v) Orthotropic Material

When a material exhibits Same elastic properties at only orthogonal direction at a given point than the material is known as Orthotropic Material.

For a homogeneous and isotropic material, the number of independent elastic constants are two.

Material	**No. of independent elastic constants**
Isotropic	2
Orthotropic	9
Anisotropic	21

MODULUS OF ELASTICITY

When an axial load, P is applied along the longitudinal axis of a bar due to which length of the bar will be increases in the direction of applied load and a stress, σ is induced in the bar.

The ratio of stress to longitudinal strain, within elastic limits, is called the modulus of elasticity (E):

Thus, modulus of elasticity $E = \frac{\sigma}{\varepsilon}$

POISSON'S RATIO

It is the ratio of lateral strain to the longitudinal strain.

It is an unitless quantity which is generally denoted as μ or 1/m.

$$\text{poisson's ratio} = -\frac{\text{Lateral strain}}{\text{longitudinal strain}}$$

$$\mu \text{ or } \frac{1}{m} = -\frac{\frac{d_o - d_i}{d_i}}{\frac{l_o - l_i}{l_i}}$$

Material	μ
Cork	Zero
Concrete	0.1 to 0.2
Metals	$\frac{1}{4}$ to $\frac{1}{3}$
Rubber, Clay, Paraffin	0.5 → Behaves like perfect plastic material

VOLUMETRIC STRAIN

Volumetric Strain Due to Three Mutually Perpendicular Stress

Figure shows a parallelepiped subjected to three tensile load P_1, P_2 and P_3 in the three mutually perpendicular direction.

Then,

$$\frac{\delta V}{V} = \varepsilon_1 + \varepsilon_2 + \varepsilon_3$$

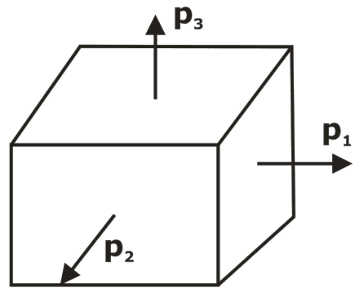


FIG. 3.3

Fig. Parallelepiped subjected to Three Mutually Perpendicular Stress

Since any axial load produces a strain in its own direction and an opposite kind of strain in every direction at right angles to this direction.

we have,

$$\text{Longitudinal strain} \quad \varepsilon_1 = \frac{\sigma_1}{E} - \mu\frac{\sigma_2}{E} - \mu\frac{\sigma_3}{E} = \frac{\sigma_1}{E} - \mu\frac{(\sigma_2 + \sigma_3)}{E}$$

$$\varepsilon_2 = \frac{\sigma_2}{E} - \mu\frac{\sigma_1}{E} - \mu\frac{\sigma_3}{E} = \frac{\sigma_2}{E} - \mu\frac{(\sigma_1 + \sigma_3)}{E}$$

$$\varepsilon_3 = \frac{\sigma_3}{E} - \mu\frac{\sigma_2}{E} - \mu\frac{\sigma_1}{E} = \frac{\sigma_3}{E} - \mu\frac{(\sigma_2 + \sigma_3)}{E}$$

Adding the three expressions of Equations we get.

$$\varepsilon_v = \frac{\delta V}{V} = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 = (1 - 2\mu)\left(\frac{\sigma_1 + \sigma_2 + \sigma_3}{E}\right)$$

Hydrostatic static state of stress-

In case of hydrostatic state of stress, the applied stress in all direction are equal and tensile in nature.

i.e. $\sigma_1 = \sigma_2 = \sigma_3 = \sigma$

$$\varepsilon_v = \frac{\delta V}{V} = (1 - 2\mu)\left(\frac{\sigma_1 + \sigma_2 + \sigma_3}{E}\right)$$

$$\varepsilon_v = \frac{\delta V}{V} = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 = (1 - 2\mu)\left(\frac{\sigma + \sigma + \sigma}{E}\right)$$

$$E\varepsilon = \sigma(1 - 2\mu)$$

since E∊ and σ in the above expression are positive numbers, must also be positive.

$$1 - 2\mu \geq 0$$

$$2\mu \leq 1$$

$$\mu \leq 0.5$$

Thus, maximum value poison's ratio is 0.5

Volumetric Strain Due to Single Direct Stress

Figure shows a rectangular bar of length L, width b and thickness t subjected to single direct load (P) acting along its longitudinal axis. Let this stress σ generated to be tensile in nature.

Then longitudinal strain, $\varepsilon_1 = \frac{\sigma}{E}$ (tensile)

Lateral strain $\varepsilon_2 = -\mu\,\varepsilon_1 = -\mu\frac{\sigma}{E}$ (compressive)

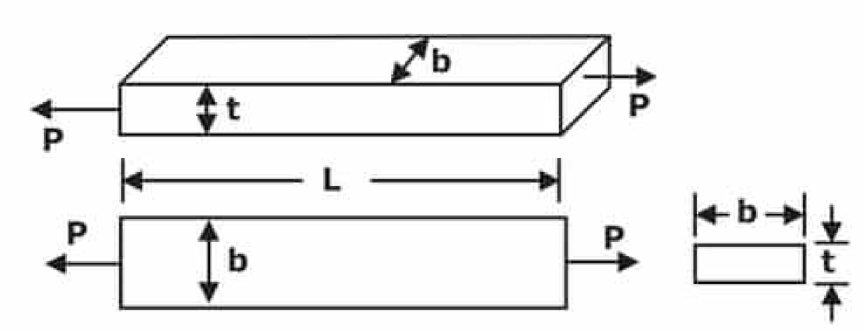


Fig. 3.2

Fig. Volumetric strain

$$\frac{\delta V}{V} = \varepsilon_v = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 = \varepsilon_x + \varepsilon_y + \varepsilon_z$$

$$\therefore \text{ Volumetric strain, } \varepsilon_v = \frac{\delta V}{V} = \frac{\sigma}{E} - 2\frac{\sigma}{E} = \frac{\sigma}{E}(1 - 2\mu)$$

SHEAR MODULUS OR MODULUS OF RIGIDITY

The shear modules or modulus of rigidity expresses the relation between shear stress and shear strain.

$$\tau = G\phi \quad \text{or} \quad \frac{\tau}{\phi} = G$$

where G = modulus of rigidity

ϕ = Shear strain (in radians) (also sometimes denoted by the symbol γ)

BULK MODULUS

When a body is subjected to three mutually perpendicular like stresses of equal intensity (σ).

Then the ratio of direct stress (σ) to the corresponding volumetric strain (ϵ_v) is defined as the bulk modulus K for the material of the body.

Which is generally denoted as 'K'

$$\text{Thus } K = \frac{\text{Direct stress}}{\text{Volumetric strain}} = \frac{\sigma}{\varepsilon_v}$$

RELATION BETWEEN DIFFERENT ELASTIC PROPERTIES

$$E = 2G(1 + \mu)$$

$$E = 3K\,(1 - 2\mu)$$

$$E = \frac{9KG}{3K + G}$$

Value of any Elastic constant should be ≥ 0

E, K, G > 0

$\mu \geq 0$ [$\mu_{cork} = 0$]

If K should be positive,

Then $1 - 2\mu \geq 0$

$$\mu \leq \frac{1}{2}$$

For any Engineering Material

$$0 \leq \mu \leq \frac{1}{2}$$

	μ	G	K
Min limit	0	$\frac{E}{2}$	$\frac{E}{3}$
Max limit	$\frac{1}{2}$	$\frac{E}{3}$	∞

$$\frac{E}{3} \le G \le \frac{E}{3} A$$

$$\frac{E}{3} \le K \le \infty$$

$$\mu \uparrow \rightarrow G \downarrow$$

$$K \uparrow$$

Always

$G \le E$

For metals

μ	G	K
$\frac{1}{4}$	0.4E	0.67E
$\frac{1}{3}$	0.375E	E

Shear Force and Bending Moment Diagrams

A **Beam** is defined as a structural member subjected to transverse shear loads during its functionality. Due to those transverse shear loads, beams are subjected to variable shear force and variable bending moment.

Shear force at a cross section of beam is the sum of all the vertical forces either at the left side or at the right side of that cross section.

Bending moment at a cross section of beam is the sum of all the moments either at the left side or at the right side of that cross section.

Types of Rigid Supports

Simple Supports

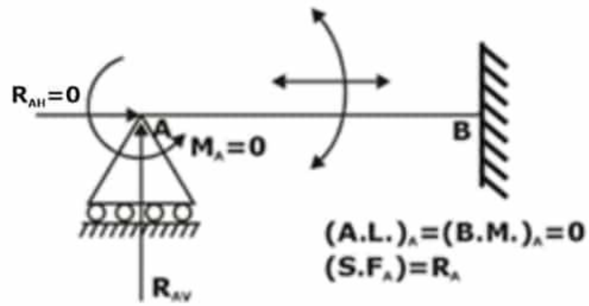
Roller Support

Hinge Support (or) Pin Support

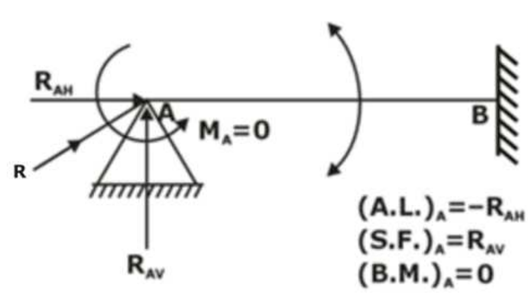
Fixed Supports

Clamped Supports (or) Built-in Supports

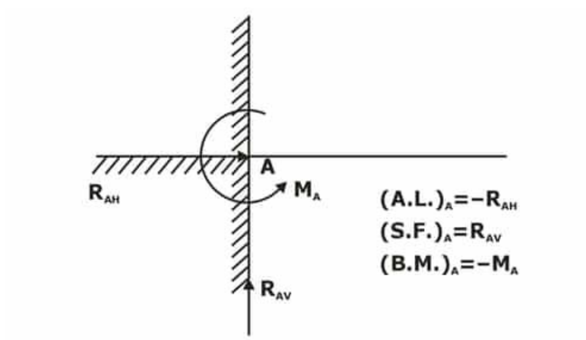
(a) Roller Support – resists vertical forces only



(b) Hinge support or pin connection – resists horizontal and vertical forces

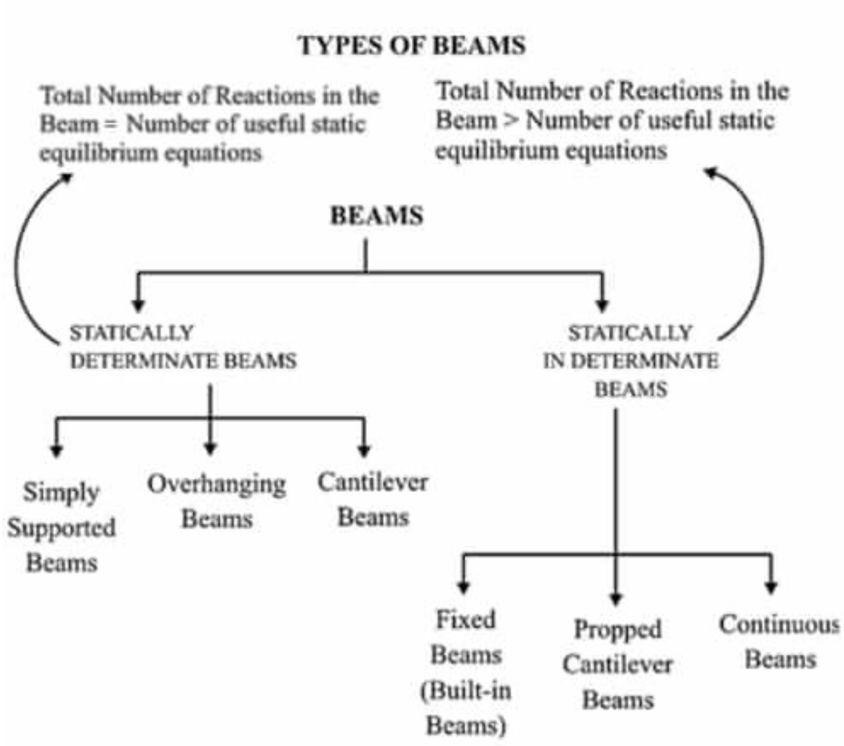


(c) Fixed support or built-in end



Note: The distance between two supports is known as “span”.

Types of Beams



Statically Determinate Beam

A beam is said to be statically determinate if all its reaction components can be calculated by applying three conditions of static equilibrium.

Statically Indeterminate Beam

When the number of unknown reaction components exceeds the static conditions of equilibrium, the beam is said to be statically indeterminate.

TYPES OF LOAD

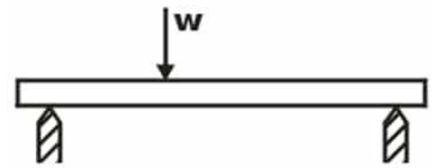
The following are the important types of load acting on a beam,

Concentrated or point load,

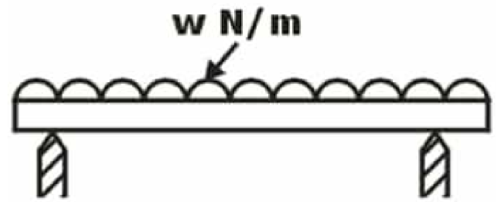
Uniformly distributed load, and

Uniformly varying load.

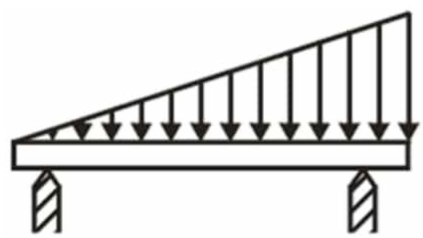
(i) Concentrated or Point Load: load act at a point.



(ii) Uniformly Distributed Load: load spread over a beam, rate of loading w is uniform along the length.



(iii) Uniformly Varying Load: load spread over a beam, rate of loading varies from point to point along the beam.

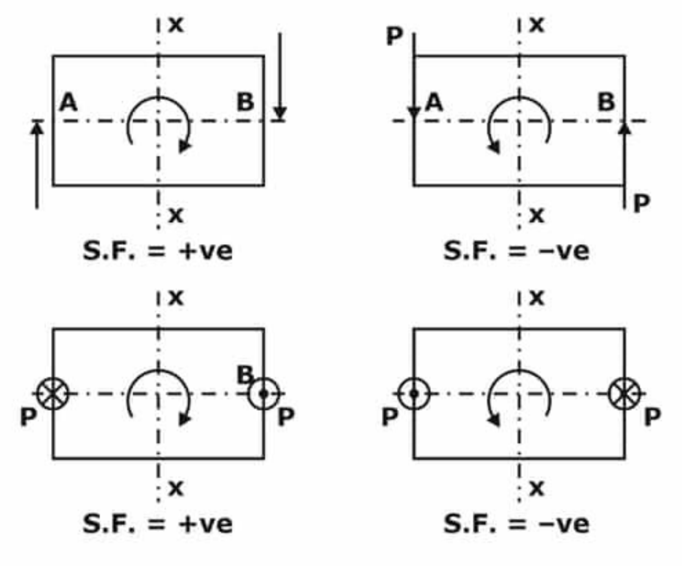


SIGN CONVENTIONS FOR SHEAR FORCE AND BENDING MOMENT

Shear force: If moving from left to right, then take all upward forces as positive and downward as negative.

Or if the shear force tries to rotate the element clockwise then it is takes as positive & if the shear force tries to rotate the element anticlockwise then it is takes as negative.

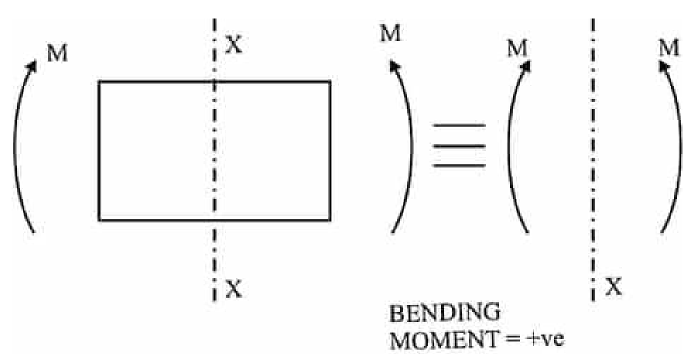
Left to right	Right to left
↑ (+ve)	↓ (+ve)
↓ (−ve)	↑ (−ve)

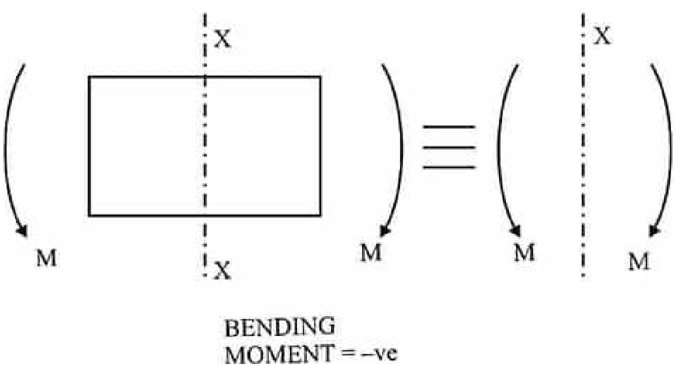


Bending moment: If moving from left to right, take clockwise moment as positive and anticlockwise as negative.

Or if forces are forming sagging moment then it is taken as positive and if forces are forming hogging moment then it is taken as negative.

Left to right	Right to left
↻(+ve)	↺(+ve)
↺(−ve)	↻(−ve)





RELATIONS BETWEEN LOAD, SHEAR FORCE AND BENDING MOMENT

Loading	Shape of SFD	Shape of BMD
No load	Straight line	Inclined straight line
UDL	Inclined straight line	2° Curve (parabola)
UVL	2° curve (parabola)	3° curve (cubic)

RELATIONS BETWEEN LOAD, SHEAR FORCE AND BENDING MOMENT

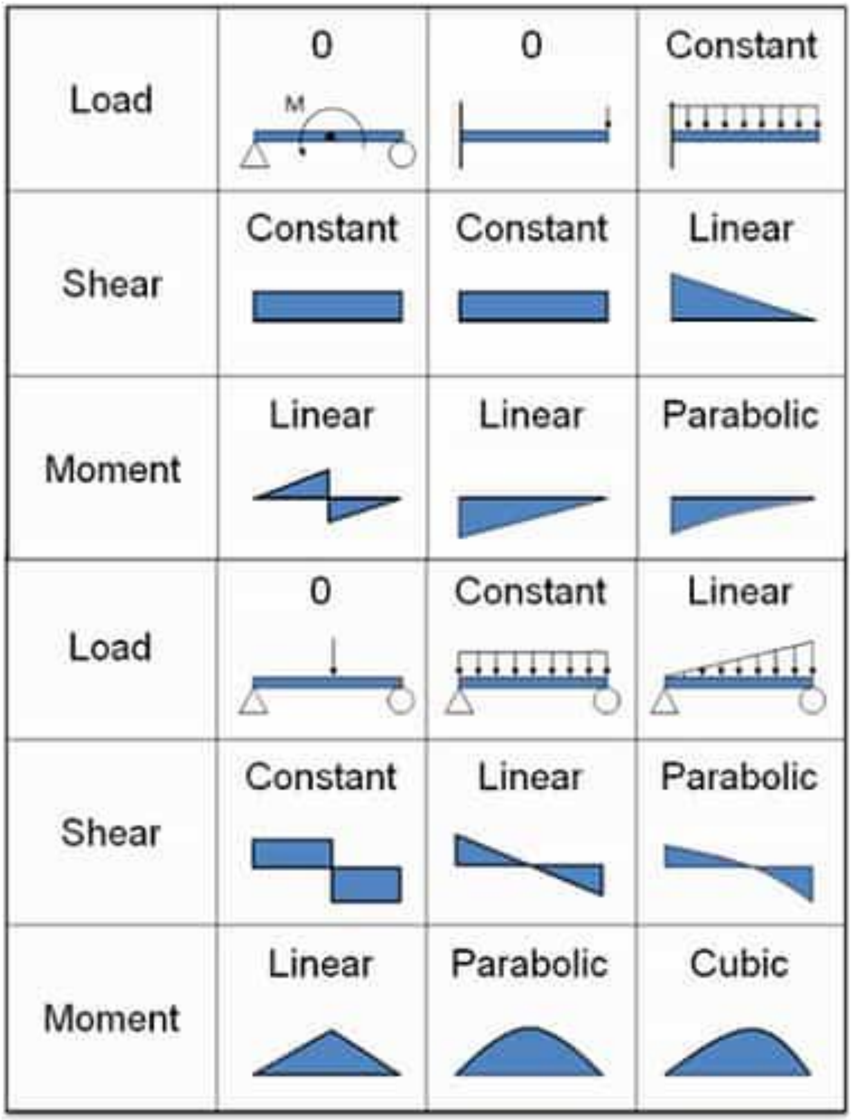
The rate of change of shear force is equal to the rate of loading.

$$\frac{dF}{dx} = -w$$

The rate of change of bending moment is equal to the shear force at the section.

$$F = \frac{dM}{dx}$$

Some Examples:-



Bending and Shear Stresses

Bending stresses on Beams:

Beam:

The term beam refers to a component that is designed to support transverse loads, that is, loads that act perpendicular to the longitudinal axis of the beam as shown in figure.

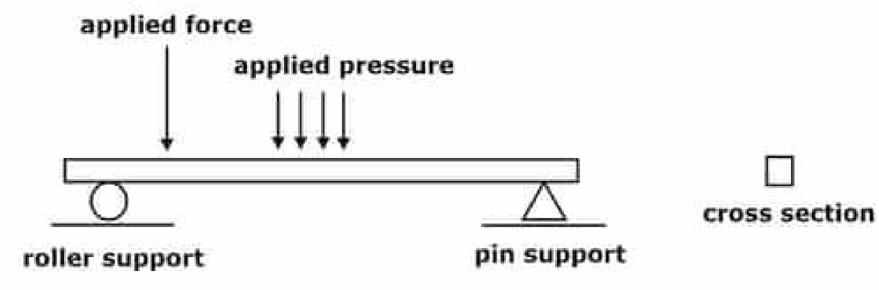


Fig: A supported beam loaded by a force and a distribution of pressure

Bending:

•When the beam is bent by the action of downward transverse loads, the fibres near the top of the beam contract in length whereas the fibres near the bottom of the beam extend.

•Somewhere in between, there will be a plane where the fibres do not change length. This is called the neutral surface. Such a deformation of beam is called the bending.

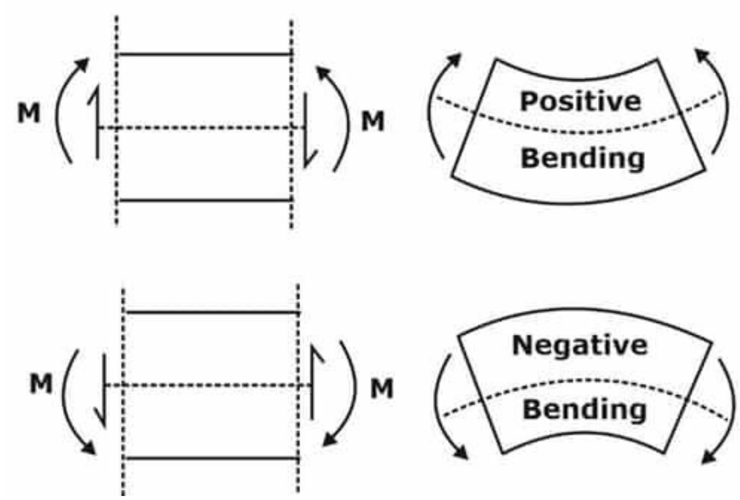


Fig: Positive and Negative bending and draw for negative bending

Bending Moment Equation:

Following assumptions are made while deriving the bending moment equation:

Assumptions:

The constraints put on the geometry would form the **assumptions:**

Beam is initially **straight** and has a **constant cross-section.**

Beam is made of **homogeneous material** and the beam has a **longitudinal plane of symmetry.**

Resultant of the applied loads lies in the plane of symmetry.

The geometry of the overall member is such that bending not buckling is the primary cause of failure.

Elastic limit is nowhere exceeded, and **'E'** is same in tension and compression.

Plane cross - sections remains plane before and after bending.

The bending equation is given as:

$$\frac{M}{I} = \frac{E}{R} = \frac{\sigma}{y}$$

where M is the Moment of resistance,
y is the distance of the considered strip from the Neutral Axis,
I is the moment of inertial of the section about the N.A,
E is the Young's modulus
σ is the bending stress
R is the radius of curvature of the beam,

From the equation, we also get,

$$\frac{1}{R} = \frac{M}{EI}$$

The term 1/R (= ρ) is known as the curvature of the section and is inversely proportional to the flexural rigidity (EI) of the section.

SECTION MODULUS

From simple bending theory equation, the maximum stress obtained in any cross-section is given as:

$$\sigma_{max} = \frac{M}{I} y_{max}$$

For any given allowable stress, the maximum moment which can be accepted by a particular shape of cross-section is therefore

$$M = \frac{I}{y_{max}} \sigma_{max}$$

For ready comparison of the strength of various beam cross-section this relationship is sometimes written in the form

$$M = Z\sigma_{max}$$

where

$$Z = \frac{I}{y_{max}}$$

is termed as section modulus.

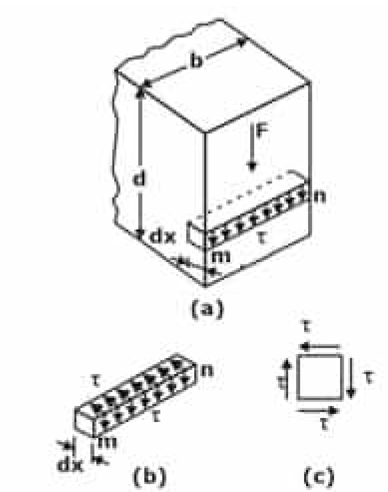
STRAIN ENERGY DUE TO BENDING:

As we know that strain energy per unit volume =

$$\frac{\sigma^2}{2E}$$

$$\text{Total strain energy (U)} = \frac{1}{2EI}\int_0^L M_x^2 \, dx$$

SHEAR STRESSES IN BEAMS:



·Consider a beam of rectangular cross section of width (b) and depth (d), subjected to a vertical force (F).

· The shear stress (τ) act parallel to the S.F. and the distribution of the shear stress is uniform across the width b of the beam.

Assumptions in finding out the expression for transverse shear stress:

For all values of y, τ is uniform across the width of the cross-section, irrespective of its shape.

is derived from the assumption that bending stress varies linearly across the section and is zero at the centroid.

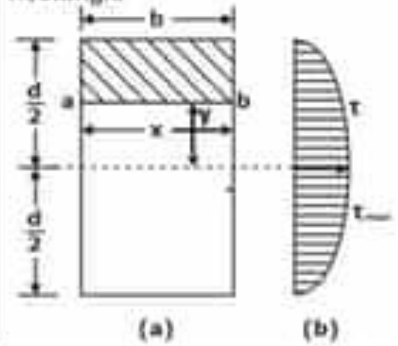
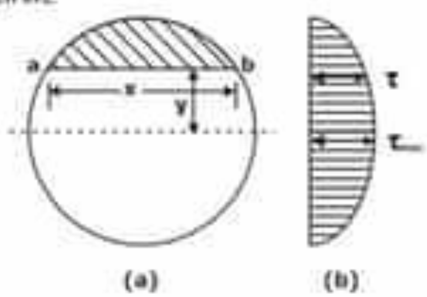
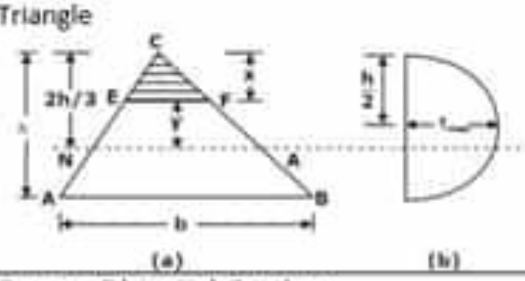
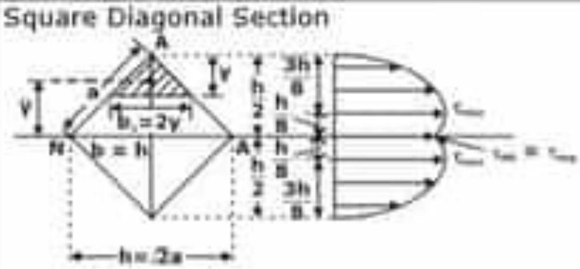
The material is homogeneous and isotropic, and the value of E is the same for tension as well as compression.

Expression for transverse shear stress:

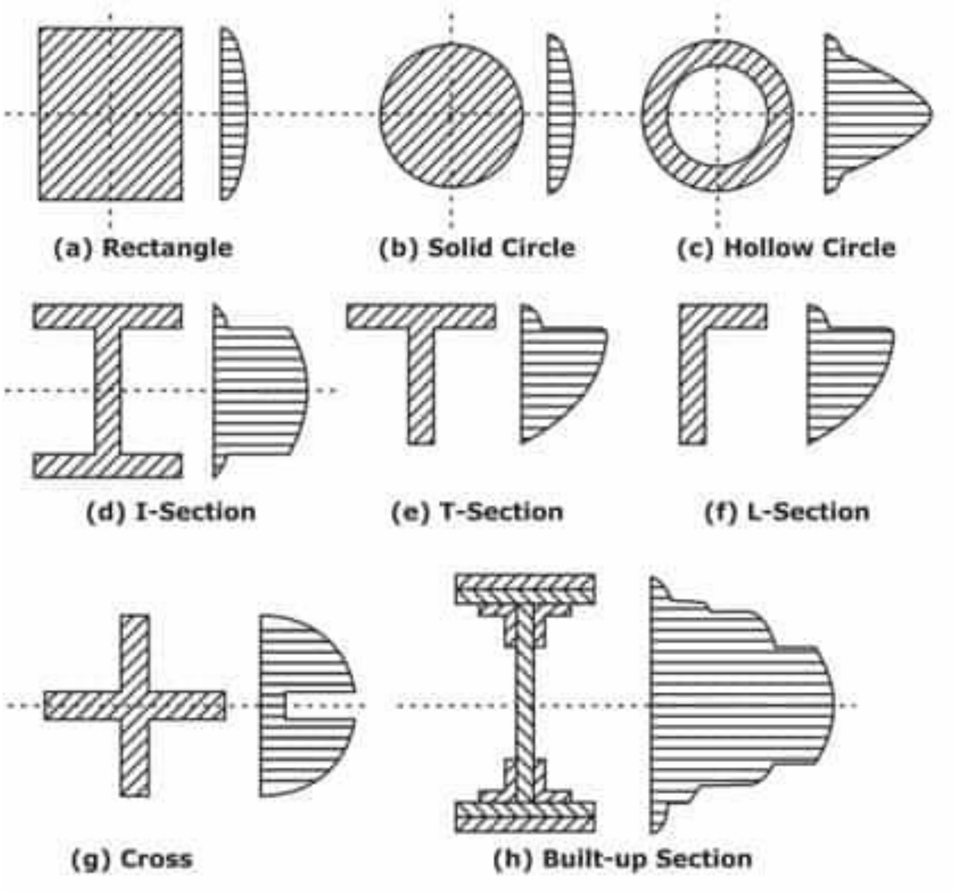
$$\boxed{\tau = \frac{F}{Iz}.A\bar{y}}$$

where τ is the shear stress, F is the shear Force, I is the moment of Inertia. Z is the section modulus of the beam, A is the area of cross section and $\bar{y}$ is the centroidal distance.

SHEAR STRESS FORMULA FOR DIFFRENT SECTIONS:

Shape	Formula	Ratio of τ_{max} to τ_{avg}
Rectangle 	$\tau_{max} = \frac{Fd^2}{8I} = \frac{Fd^2}{8\times\frac{1}{12}bd^3} = \frac{3}{2}\frac{F}{bd}$	1.5
Circle 	$\tau = \tau_{max} = \frac{F}{3I}r^2 = \frac{Fd^2}{12I}$	1.33
Triangle 	$\tau_{max} = \frac{3F}{bh} = \frac{3}{2}\frac{F}{\left(\frac{1}{2}bh\right)} = \frac{3}{2}\tau_{avg}$	1.5
Square Diagonal Section 	$\tau_{max} = \frac{9F}{2\times(\sqrt{2}a)^2} = \frac{9F}{8a^2} = \frac{9}{8}\tau_{avg}$	1.125

SHEAR STRESS DISTRIBUTION OVER OTHER SECTIONS:



Principal Stress and Strain

Stress tensor:

A tensor is a multi-dimensional array of numerical values that can be used to describe the physical state or properties of a material. A simple example of a geo-physically relevant tensor is stress.

Sign conventions:

While analysing a stress system, the general conventions have been taken as follows:

A tensile stress is positive and compressive stress, negative.

A pair of shear stresses on parallel planes forming a clockwise couple is positive and a pair with counterclockwise couple, negative.

Clockwise angle is taken as positive and counterclockwise as negative.

Methods to find stresses at a point:

The stresses on a point are determined by the following methods:

Analytical method, and

Graphical method

Sum of Direct stresses of two mutually perpendicular stresses:

Direct stress on an inclined plane at angle θ is given by:

$$\sigma_\theta = \frac{1}{2}(\sigma_x + \sigma_y) + \frac{1}{2}(\sigma_x - \sigma_y)\cos 2\theta + \tau \sin 2\theta \quad \text{..................} (1)$$

Direct stress on an inclined plane at angle (θ + 90°) will be:

$$\sigma_{(\theta+90^\circ)} = \frac{1}{2}(\sigma_x + \sigma_y) + \frac{1}{2}(\sigma_x - \sigma_y)\cos 2(\theta + 90^\circ) + \tau \sin 2(\theta + 90^\circ)$$

$$\sigma_{(\theta+90^\circ)} = \frac{1}{2}(\sigma_x + \sigma_y) - \frac{1}{2}(\sigma_x - \sigma_y)\cos 2\theta - \tau \sin 2\theta \quad \text{..................} (2)$$

On adding (1) and (2):

$$\boxed{\sigma_\theta + \sigma_{(\theta+90^\circ)} = \sigma_x + \sigma_y}$$

Since the sum of direct stresses σ_x and σ_y is constant, the sum of direct stresses on two mutually perpendicular planes at a point at angle θ and (θ + 90°) remains constant and equal to $\sigma_x + \sigma_y$.

PRINCIPAL STRESSES

In general, a body may be acted upon by direct stresses and shear stresses.

However, it will be seen that even in such complex systems of loading, there exist three mutually perpendicular planes, on each of which the resultant stress is wholly normal. These are known as principal planes and the normal stress across these planes, as principal stresses.

The larger of the two stresses σ_1 is called the major principal stress, and the smaller one σ_2 as the minor principal stress. The corresponding planes are known as major and minor principal planes.

In two-dimensional problems, the third principal stress is taken to be zero.

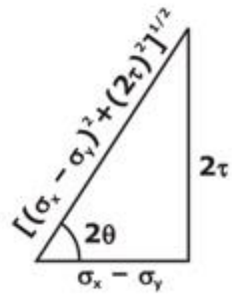
As shear stress is zero in principal planes:

$$\tau_\theta = -\frac{1}{2}(\sigma_x - \sigma_y)\sin 2\theta + \tau\cos 2\theta = 0$$

$$\frac{1}{2}(\sigma_x - \sigma_y)\sin 2\theta = \tau\cos 2\theta$$

$$\boxed{\tan 2\theta = \frac{2\tau}{\sigma_x - \sigma_y}} \qquad \text{......................... (1)}$$

which provides two values of 2θ differing by 180° or two values of θ differing by 90°. Thus, the two principal planes are perpendicular to each other.



$$\sigma_{1,2} = \frac{1}{2}(\sigma_x + \sigma_y) \pm \frac{1}{2}\frac{(\sigma_x - \sigma_y)^2}{\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2}} \pm \tau\frac{2\tau}{\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2}}$$

$$\sigma_{1,2} = \frac{1}{2}(\sigma_x + \sigma_y) \pm \frac{1}{2}\frac{(\sigma_x - \sigma_y)^2 + 4\tau^2}{\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2}}$$

$$\boxed{\sigma_{1,2} = \frac{1}{2}(\sigma_x + \sigma_y) \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2}}$$

$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_x - \sigma_y) + 4\tau^2}$$

As maximum principal stress:

$$\sigma_1 = \frac{1}{2}(\sigma_x + \sigma_y) + \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2}$$

And minimum principal stress:

$$\sigma_2 = \frac{1}{2}(\sigma_x + \sigma_y) - \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2}$$

$$\sigma_1 - \sigma_2 = \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2}$$

$$\therefore \tau_{max} = \frac{1}{2}(\sigma_1 - \sigma_2)$$

$$\boxed{\tau_{max} = \frac{1}{2}(\sigma_1 - \sigma_2) = \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2}}$$

MOHR'S STRESS CIRCLE

The stress components on any inclined plane can easily be found with the help of a geometrical construction known as Mohr’s stress circle.

Some points about Mohr's Circle:

Diameter of the Mohr’s circle = $\sigma_1 - \sigma_2$

Centre of the Mohr's circle = $\frac{\sigma_x - \sigma_y}{2}$

Radius of Mohr's circle = $\frac{\sigma_1 - \sigma_2}{2} = \tau_{max}$

Value of shear stress at the Principal planes is equal to zero.

Maximum shear stress occurs at the σ_n reaches the centre of the circle.

When the normal stresses are equal in all the perpendicular planes with no shear stress, the Mohr’s circle reduces to a point. Located at the point equal to the value of the normal stress.

Deflection of Beams

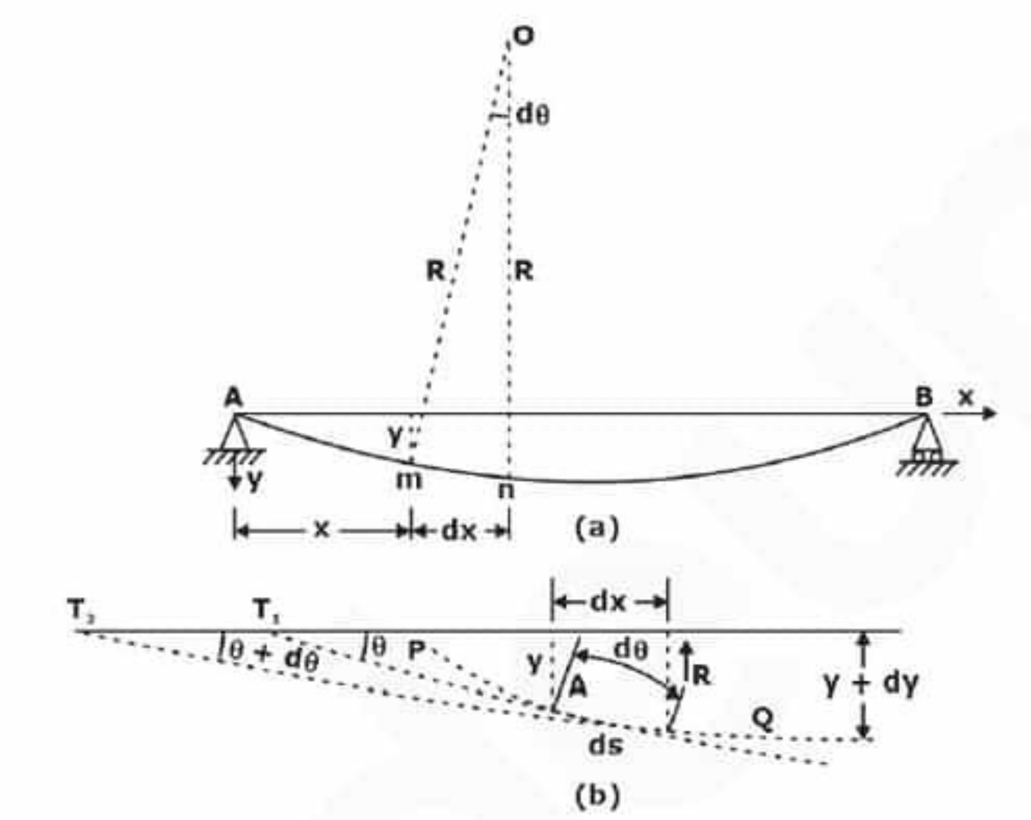
The deformation of a beam is usually expressed in terms of its deflection from its original unloaded position. The deflection is measured from the original neutral surface of the beam to the neutral surface of the deformed beam. The configuration assumed by the deformed neutral surface is known as the elastic curve of the beam.

Slope of a Beam: Slope of a beam is the angle between deflected beam to the actual beam at the same point.

Deflection of Beam: Deflection is defined as the vertical displacement of a point on a loaded beam. There are many methods to find out the slope and deflection at a section in a loaded beam.

The maximum deflection occurs where the slope is zero. The position of the maximum deflection is found out by equating the slope equation zero. Then the value of x is substituted in the deflection equation to calculate the maximum deflection

DIFFERENTIAL EQUATION OF THE DEFLECTION CURVE OF BEAM



$$\frac{1}{R} = -\frac{\frac{d^2y}{dx^2}}{\left[1+\left(\frac{dy}{dx}\right)\right]^{3/2}}$$

Methods of Determining Beam Deflections

Numerous methods are available for the determination of beam deflections. These methods include:

Double Integration Method:

This is most suitable when concentrated or udl over entire length is acting on the beam. A double integration method is a powerful tool in solving deflection and slope of a beam at any point because we will be able to get the equation of the elastic curve.

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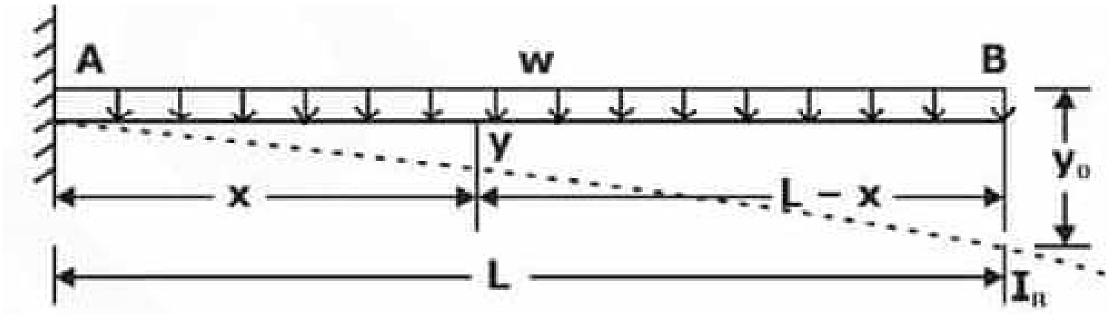


In calculus, the radius of curvature of a curve y = f(x) is given by:

$$N^2 - \left[N + \frac{1}{2}N(N-1)\right] = \frac{1}{2}N(N-1)$$

In the derivation of flexure formula, the radius of curvature of a beam is
ρ=EI/M

Deflection of beams is so small, such that the slope of the elastic curve dy/dx is very small, and squaring this expression the value becomes practically negligible, hence:

$$\rho = \frac{1}{\frac{d^2y}{dx^2}} = \frac{1}{y''}$$

$$\text{Thus, } EI / M = 1 / y''$$

$$y'' = \frac{M}{EI}$$

If EI is constant, the equation may be written as:

EIy"=M

where x and y are the coordinates shown in the figure of the elastic curve of the beam under load.

y is the deflection of the beam at any distance x.

E is the modulus of elasticity of the beam,

I represent the moment of inertia about the neutral axis, and

M represents the bending moment at a distance x from the end of the beam.

The product **EI is called the flexural rigidity** of the beam.

$$EI \frac{d^2y}{dx^2} = -M$$

Integrating one time:

$$EI \frac{dy}{dx} = -\int M$$

The first integration y'(dy/dx) yields the Slope of the Elastic Curve.

Second Integration:

$$EI\, y = -\int \int M$$

The second integration y gives the **Deflection of the Beam at any distance x**.

The resulting solution must contain two constants of integration since EI y" = M is of second order.

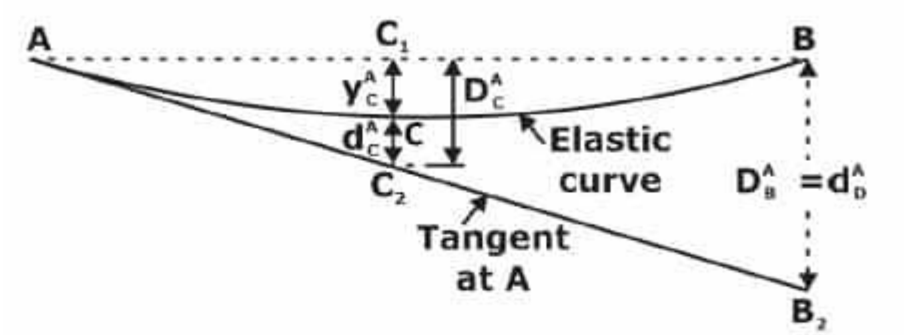
These two constants must be evaluated from known conditions concerning the slope deflection at certain points of the beam.

For instance, in the case of a simply supported beam with rigid supports, at x = 0 and x = L, the deflection y = 0, and in locating the point of maximum deflection, we simply set the slope of the elastic curve y' to zero

Area Moment Method (Mohr's Method):

Another method of determining the slopes and deflections in beams is the area-moment method, which involves the area of the moment diagram. The moment-area method is a

The moment-area method is a semi graphical procedure that utilizes the properties of the area under the bending moment diagram. It is the quickest way to compute the deflection at a specific location if the bending moment diagram has a simple shape.



$$D_B^A = d_B^A$$

Theorems of Area-Moment Method:

Theorem 1

The angle between the tangent of the deflection curve of two points A and B is equal to the negative area of M/EI diagram between the points.

$$\theta_B^A = \frac{1}{EI}(\text{Area AB})$$

Theorem 2

The deviation of B from tangent at A is equal to the negative of the statical moment (or the first moment) with respect to B, of the M/EI diagram area between A and B.

$$EId_B^A = -\sum_B^A A_m \overline{X}$$

Method of Superposition: The method of superposition, in which the applied loading is represented as a series of simple loads for which deflection formulas are available. Then the desired deflection is computed by adding the contributions of the component loads(principle of superposition).

Mostly direct formula is used in questions, hence it is advised to look for the beam deflection formula which are directly asked from this topic rather than going for long derivations.

Deflection for Common Loadings:

Concentrated load at the free end of cantilever beam (origin at A):

Maximum Moment, **$M = -PL$**

Slope at end: **$\theta = PL^2/2EI$**

Maximum deflection: **$\delta = PL^3/3EI$**

Deflection Equation (y is positive downward): **$EIy=(Px^2)(3L-x)/6$**

2 .Concentrated load at any point on the span of cantilever beam

Maximum Moment: **$M = -wa$**

Slope at end: **$\theta = wa^2/2EI$**

Maximum deflection: **$\delta = wa^3(3L-a)/6EI$**

Deflection Equation (y is positive downward),

$EIy=Px^2(3a-x)/6$ for $0 < x < a$

$EIy=Pa^2(3x-a)/6$ for $a < x < L$

Uniformly distributed load over the entire length of cantilever beam

Maximum Moment: **$M=-wL^2/2$**

Slope at end: **$\theta = wL^3/6EI$**

Maximum deflection: **$\delta = wL^4/8EI$**

Deflection Equation (y is positive downward): **$EIy=wx^2(6L^2-4Lx+x^2)/120L$**

Triangular load, full at the fixed end and zero at the free end

Maximum Moment: $M=-wL^2/6$

Slope at end: **$\theta = wL^3/24EI$**

Maximum deflection, **$\delta = wL^4/30EI$**

Deflection Equation (y is positive downward): **$EIy=wx^2(10L^3-10L^2x+5Lx^2-x^3)/120L$**

Moment load at the free end of cantilever beam

Maximum Moment: **$M=-M$**

Slope at end: **$\theta = ML/EI$**

Maximum deflection: $\boldsymbol{\delta=ML^2/2EI}$

Deflection Equation (y is positive downward): $\boldsymbol{EIy=Mx^2/2}$

Concentrated load at the midspan of simple beam

Maximum Moment: **M=PL/4**

Slope at end: $\boldsymbol{\theta_A=\theta_B = WL^2/16EI}$

Maximum deflection: $\boldsymbol{\delta=PL^3/48EI}$

Deflection Equation (y is positive downward): $\boldsymbol{EIy=Px\{(3/4)L^2-x^2)\}/12}$ **for** $\boldsymbol{0<x<L/2}$

Uniformly distributed load over the entire span of simple beam

Maximum Moment: $\boldsymbol{M=wL^2/8}$

Slope at end: $\boldsymbol{\theta_L=\theta_R=wL^3/24EI}$

Maximum deflection: $\boldsymbol{\delta = 5wL^4/384EI}$

Deflection Equation (y is positive downward): $\boldsymbol{EIy=wx(L^3-2Lx^2+x^3)/24}$

9.**Triangle load with zero at one support and full at the other support of simple beam**

Maximum Moment: $\boldsymbol{M=w_oL^2/9\sqrt{3}}$

Slope at end,

$\boldsymbol{\theta_L= 7wL^3/360EI}$

$\boldsymbol{\theta_R= 8wL^3/360EI}$

Maximum deflection: $\boldsymbol{\delta=2.5wL^4/384EI}$ **at x=0.519L**

Deflection Equation (y is positive downward), $\boldsymbol{EIy=wx(7L^4-10L^2x+3x)/360L}$

Triangular load with zero at each support and full at the midspan of simple beam

Maximum Moment: $\boldsymbol{M=wL^2/12}$

Slope at end, $\boldsymbol{\theta_L=\theta_R=5wL^3/192EI}$

Maximum deflection: $\boldsymbol{\delta=wL^4/120EI}$

Deflection Equation (y is positive downward): **$EIy=w_o x(25L^4-40L^2x^2+16x^4)/960L$** for $0<x<L/2$

Conjugate Beam method

Support conditions for the real and conjugate beam:

Rules	Existing support condition of actual beam	Corresponding support condition for the conjugate beam
Rule -1	Fixed end	Free end
Rule -2	Free end	Fixed end
Rule -3	Simple support at the end	Simple support at the end.
Rule -4	Simple support not at the end	Unsupported hinge
Rule -5	Unsupported hinge	Simple support

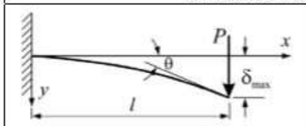
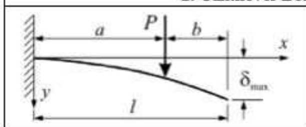
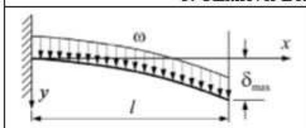
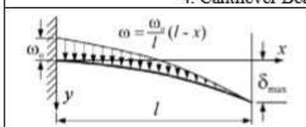
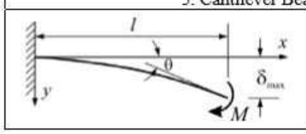
DEFLECTIONS BY CASTIGLIANO'S THEOREM:

$$\delta_i = \frac{\partial U}{\partial W_i} = \int_0^l \frac{M}{EI}\frac{M}{\partial W_i}dx$$

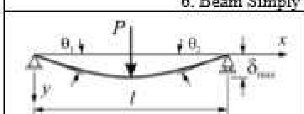
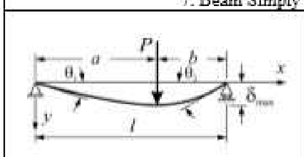
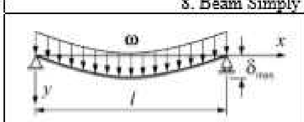
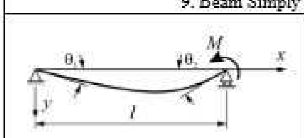
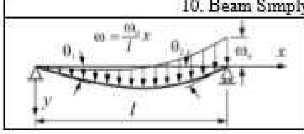
$$\theta_i = \frac{\partial U}{\partial M_i} = \int_0^l \frac{M}{EI}\frac{M}{\partial M_i}dx = \frac{1}{EI}\int_0^l M.\frac{\partial M}{\partial W_i}dx$$

Beam Deflection Formula:

Cantilever Beams:

BEAM TYPE	SLOPE AT FREE END	DEFLECTION AT ANY SECTION IN TERMS OF x	MAXIMUM DEFLECTION
1. Cantilever Beam – Concentrated load P at the free end			
	$\theta = \frac{Pl^2}{2EI}$	$y = \frac{Px^2}{6EI}(3l - x)$	$\delta_{max} = \frac{Pl^3}{3EI}$
2. Cantilever Beam – Concentrated load P at any point			
	$\theta = \frac{Pa^2}{2EI}$	$y = \frac{Px^2}{6EI}(3a - x)$ for $0 < x < a$ $y = \frac{Pa^2}{6EI}(3x - a)$ for $a < x < l$	$\delta_{max} = \frac{Pa^2}{6EI}(3l - a)$
3. Cantilever Beam – Uniformly distributed load ω (N/m)			
	$\theta = \frac{\omega l^3}{6EI}$	$y = \frac{\omega x^2}{24EI}\left(x^2 + 6l^2 - 4lx\right)$	$\delta_{max} = \frac{\omega l^4}{8EI}$
4. Cantilever Beam – Uniformly varying load: Maximum intensity ω_o (N/m)			
	$\theta = \frac{\omega_o l^3}{24EI}$	$y = \frac{\omega_o x^2}{120lEI}\left(10l^3 - 10l^2x + 5lx^2 - x^3\right)$	$\delta_{max} = \frac{\omega_o l^4}{30EI}$
5. Cantilever Beam – Couple moment M at the free end			
	$\theta = \frac{Ml}{EI}$	$y = \frac{Mx^2}{2EI}$	$\delta_{max} = \frac{Ml^2}{2EI}$

Simply supported Beams:

BEAM TYPE	SLOPE AT ENDS	DEFLECTION AT ANY SECTION IN TERMS OF x	MAXIMUM AND CENTER DEFLECTION
6. Beam Simply Supported at Ends – Concentrated load P at the center			
	$\theta_1 = \theta_2 = \frac{Pl^2}{16EI}$	$y = \frac{Px}{12EI}\left(\frac{3l^2}{4} - x^2\right)$ for $0 < x < \frac{l}{2}$	$\delta_{max} = \frac{Pl^3}{48EI}$
7. Beam Simply Supported at Ends – Concentrated load P at any point			
	$\theta_1 = \frac{Pb(l^2 - b^2)}{6lEI}$ $\theta_2 = \frac{Pab(2l - b)}{6lEI}$	$y = \frac{Pbx}{6lEI}\left(l^2 - x^2 - b^2\right)$ for $0 < x < a$ $y = \frac{Pb}{6lEI}\left[\frac{l}{b}(x - a)^3 + \left(l^2 - b^2\right)x - x^3\right]$ for $a < x < l$	$\delta_{max} = \frac{Pb\left(l^2 - b^2\right)^{3/2}}{9\sqrt{3}\,lEI}$ at $x = \sqrt{\left(l^2 - b^2\right)/3}$ $\delta = \frac{Pb}{48EI}\left(3l^2 - 4b^2\right)$ at the center, if $a > b$
8. Beam Simply Supported at Ends – Uniformly distributed load ω (N/m)			
	$\theta_1 = \theta_2 = \frac{\omega l^3}{24EI}$	$y = \frac{\omega x}{24EI}\left(l^3 - 2lx^2 + x^3\right)$	$\delta_{max} = \frac{5\omega l^4}{384EI}$
9. Beam Simply Supported at Ends – Couple moment M at the right end			
	$\theta_1 = \frac{Ml}{6EI}$ $\theta_2 = \frac{Ml}{3EI}$	$y = \frac{Mlx}{6EI}\left(1 - \frac{x^2}{l^2}\right)$	$\delta_{max} = \frac{Ml^2}{9\sqrt{3}\,EI}$ at $x = \frac{l}{\sqrt{3}}$ $\delta = \frac{Ml^2}{16EI}$ at the center
10. Beam Simply Supported at Ends – Uniformly varying load: Maximum intensity ω_o (N/m)			
	$\theta_1 = \frac{7\omega_o l^3}{360EI}$ $\theta_2 = \frac{\omega_o l^3}{45EI}$	$y = \frac{\omega_o x}{360lEI}\left(7l^4 - 10l^2x^2 + 3x^4\right)$	$\delta_{max} = 0.00652\frac{\omega_o l^4}{EI}$ at $x = 0.519l$ $\delta = 0.00651\frac{\omega_o l^4}{EI}$ at the center

Torsion

It's magnitude is given as the product of the force and the distance between the force.

Torque, $T = P \times d$

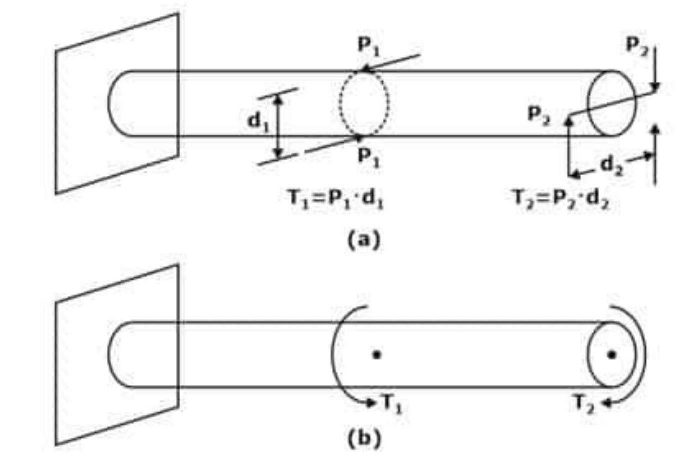


Fig.: Magnitude and representation of Torque

Figure shows a bar or shaft of circular section, subjected to torque T. Such a case is a case of pure torsion,

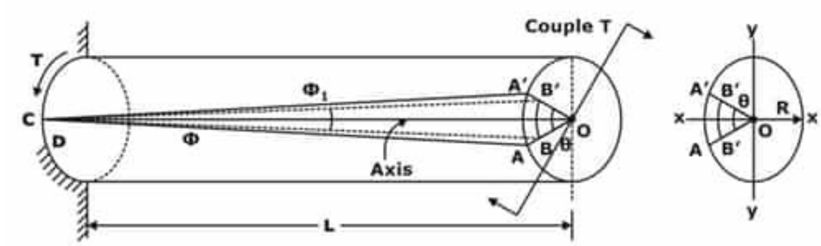


Fig.: Shaft is under pure torsion

$$\frac{\tau}{R} = \frac{T_R}{J} = \frac{G\theta}{L}$$

J/R is known as torsional section modulus.,& GJ is known as torsional rigidity of the bar or the shaft.

The above relation states that the intensity of shear stress at any point in the cross-section of a shaft subjected to pure torsion is proportional to its distance from the center and the variation of shear stress with respect to radial distance is linear.

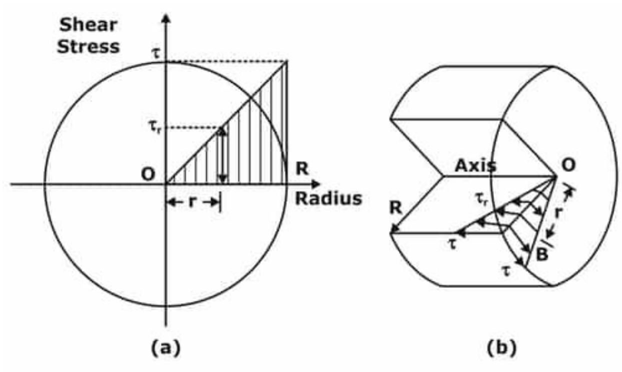
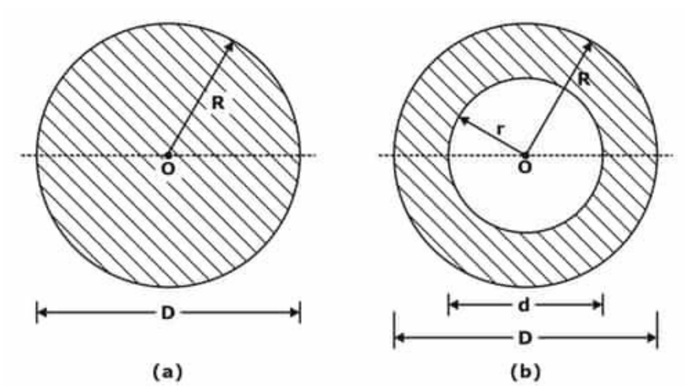


Fig.: Variation of Torsional Shear Stress

Polar moment of inertia



(a) For a solid shaft of circular section,

Torsional section modulus

$$Z_p = \frac{J}{R} = \frac{J}{D/2} = \frac{\pi}{16} D^3$$

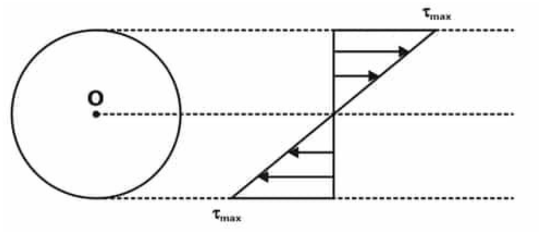
(b) For a hollow circular shaft,

$$Z_p = \frac{J}{R} = \frac{\frac{\pi}{2}\left(R^4 - r^4\right)}{R}$$

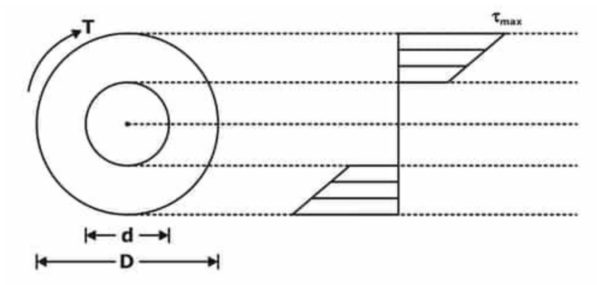
SHEAR STRESS DISTRIBUTION IN DIFFERENT SECTIONS

It is zero at the center and increases in the radially outward direction and become maximum at the outer periphery And for hollow circular shaft, it is minimum at inner radius and maximum at the outer periphery.

(a) Solid circular section:



(b) Hollow circular section



POWER TRANSMITTED

$$P = \frac{2\pi NT}{60} \text{ W}$$

DESIGN OF SHAFT

While designing a shaft, we calculate the maximum torque that can be transmitted from the shaft.

The resisting couple should be equal to the applied torque. Hence

$$\frac{T}{J} = \frac{\tau}{R}$$

$$T_R = T = \tau \frac{J}{R}$$

MAXIMUM TORQUE TRANSMITTED BY A CIRCULAR SHAFT

(a) CIRCULAR SOLID SHAFT

The maximum torque transmitted by a circular solid shaft is obtained from the maximum shear stress-induced at the outer surface of the solid shaft.

$$T = \tau \times \frac{\pi}{2} \times \frac{D^3}{8} = \tau \times \frac{\pi D^3}{16} = \frac{\pi}{16} \tau D^3$$

(b) HOLLOW CIRCULAR SHAFTS

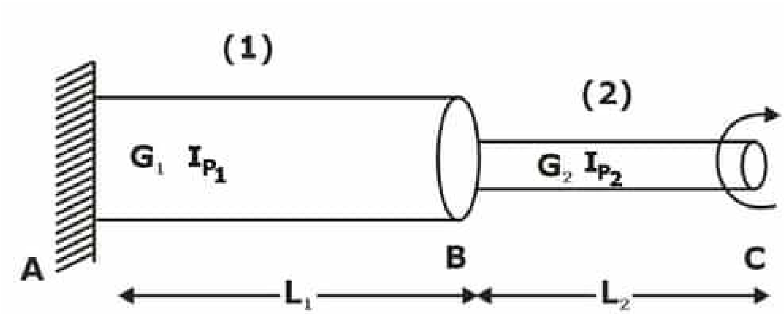
Torque transmitted by a hollow circular shaft Is obtained in the same way as for a solid shaft,

$$T = \tau \frac{\pi}{16}\left[\frac{D_0^4 - D_i^4}{D_0}\right]$$

COMPOSITE SHAFTS:

(i) Series connection:

If two or more shaft of different material, diameter or basic forms are connected together in such a way that each carries the same torque, then the shafts are said to be connected in series & the composite shaft so produced is therefore termed as series connection.



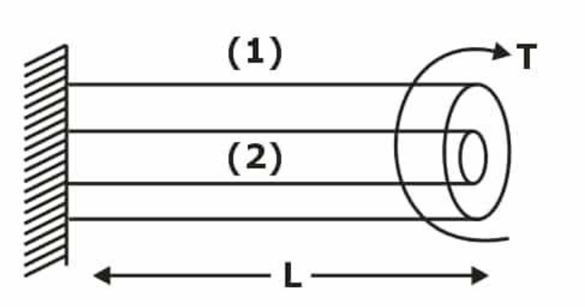
for two shafts in series $T_1 = T_2 = T$

$\theta_{AC} = \theta_{AB} + \theta_{BC}$

$$\theta_{AC} = T\left(\frac{L_1}{G_1 J_1} + \frac{L_2}{G_2 J_2}\right)$$

(ii)parallel connection:

If two shafts are loaded in such a way that angle of twist on both the shaft is the same then this type of connection is known as a parallel connection of shaft.



For parallel connection of shaft

Torque is cumulative, $T = T_1 + T_2$ and $\theta_1 = \theta_2$

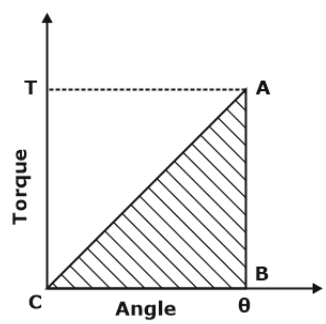
$$\frac{T_1 L_1}{G_1 J_1} = \frac{T_2 L_2}{G_2 J_2}$$

STRAIN ENERGY IN TORSION

Consider a solid shaft of length L, under the action of torque T.

The torsional strain energy of shaft is equal to the work done in twisting.

$$U = \frac{1}{2} T.\theta = \text{Area under } T - \theta \text{ diagram}$$



$$U = \frac{\tau^2}{2G} \times \text{Volume of the shaft}$$

TORSIONAL STIFFNESS (K)

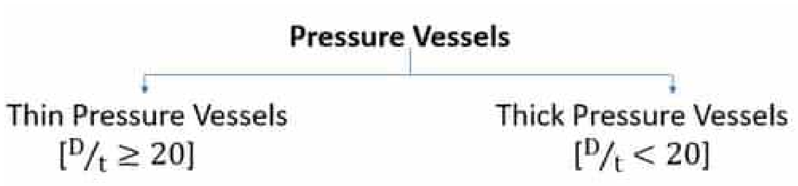
Torsional stiffness is defined as the amount of torque or twisting couple required to produce a twist of unit radian. And it represented by 'K'

$$\theta = \frac{TL}{GJ} \quad K = \frac{GJ}{L}$$

Thin Cylinders & Buckling of column

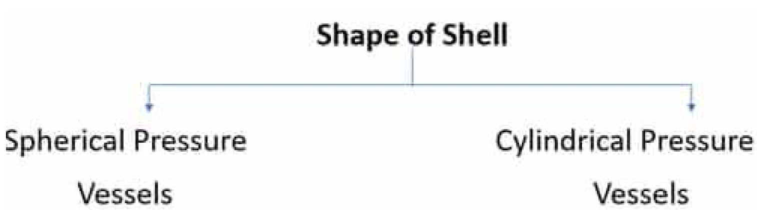
Thin pressure vessel is defined as a closed cylindrical or spherical container designed to hold or store fluids at a pressure substantially different from ambient pressure. Pressure vessels can be **classified** as

(i) on the basis of ratio of diameter to its thickness



where, D is the inner diameter of the shell & t is the thickness of the shell.

(ii) On the basis of shape of the pressure vessel



However, Spherical pressure vessels are better, but due to fabrication difficulty, cylindrical pressure vessels are most commonly used.

Common examples of pressure vessels are steam boilers, reservoirs, tanks, working chambers of engines, gas cylinders etc.

THIN CYLINDRICAL SHELL SUBJECT TO INTERNAL PRESSURE

Consider a thin cylinder of internal diameter d and wall thickness t, subject to internal gauge pressure P. The following stresses are induced in the cylinder-
(a) Circumferential tensile stress (or hoop stress) σ_H.
(b) Longitudinal (or axial) tensile stress σ_L.
(c) Radial compressive stress σ_R which varies from a value at the inner surface equal to the atmosphere pressure at the outside surface.

Assumptions followed in thin pressure vessels

Stresses are assumed to be distributed uniformly

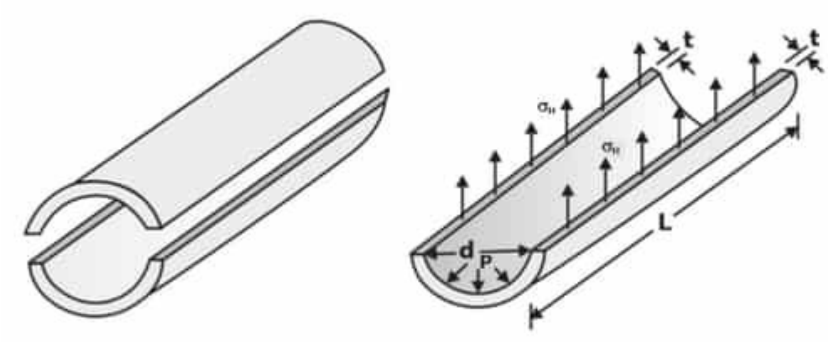
Area is calculated considering the pressure vessel as thin

Radial stresses are neglected

Biaxial state of stress is assumed to be applicable

(a) Circumferential stress or Hoop stress, σ_H

There are normal stresses which act in the direction of circumference. Due to internal fluid pressure these are tensile in nature. In thin pressure vessels, hoop stresses are assumed to be uniform across thickness.



In the figure we have shown a one half of the cylinder. This cylinder is subjected to an internal pressure P.

Pressure force by fluid ≤ Resisting force owing to hoop stresses σ_H
P x Projected Area ≤ $2.\sigma_h.L.t$

$$\frac{Pd}{2t} \le \sigma_H$$

For equilibrium, $\sigma_H = \frac{Pd}{2t}$

$P.d.L \le 2.\sigma_h.L.t$

In η_L is the efficiency of the Longitudinal riveted joint,

$$\sigma_H = \frac{pd}{2t\eta_L}$$

Similarly,

(b) Longitudinal stress (or axial stress) σ_L

Pressure force by fluid ≤ Resisting force owing to longitudinal stresses σ_L

$$P \times \frac{\pi}{4} d^2 \leq \sigma_L \pi d t$$

$$\frac{Pd}{4t} \leq \sigma_L$$

For equilibrium, $\sigma_L = \frac{Pd}{4t}$

In η_L is the efficiency of the circumferential riveted joint,

$$\sigma_L = \frac{pd}{4t\eta_c}$$

Thus, the magnitude of the longitudinal stress is one half of the circumferential stress, both the stresses being of tensile nature.

Hoop strain or Circumferential strain -

$$\varepsilon_1 = \varepsilon_H = \frac{\sigma_1}{E} - \mu \frac{\sigma_2}{E} = \frac{1}{E}(\sigma_H - \mu \sigma_L)$$

$$\frac{\Delta d}{d} = \varepsilon_H = \frac{Pd}{4tE}[2 - \mu]$$

Longitudinal Strain or axial strain

$$\epsilon_L = \epsilon_2 = \frac{1}{E}(\sigma_L - \mu \sigma_H) = \frac{1}{E}\left[\frac{Pd}{4t} - \mu \frac{Pd}{2t}\right]$$

$$\frac{\Delta L}{L} = \varepsilon_L = \frac{Pd}{4tE}[1 - 2\mu]$$

Ratio of Hoop Strain to Longitudinal Strain

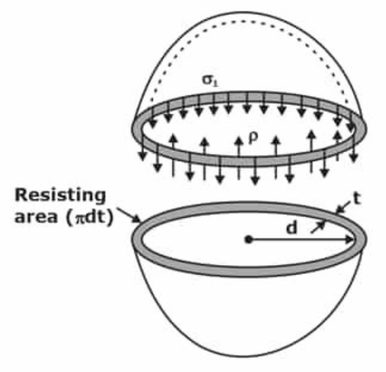
$$\frac{\epsilon_H}{\epsilon_L} = \frac{\text{circumeferentialstrain}}{\text{longitudinalstrain}} = \frac{\frac{Pd}{4tE}(2-\mu)}{\frac{Pd}{4tE}(1-2\mu)} = \frac{(2-\mu)}{(1-2\mu)}$$

Volumetric Strain or Change in the Internal Volume

$$\varepsilon_v = \frac{\Delta V}{V} = \frac{Pd}{4tE}[5 - 4\mu]$$

THIN SPHERICAL SHELLS

Figure shows a thin spherical shell of internal diameter ‘d’ and thickness ‘t’ and subjected to an internal fluid pressure ‘P’.



Hoop stress/longitudinal stress

Pressure force by fluid ≤ Resisting force owing to Hoop/Longitudinal stresses

$$P \times \frac{\pi}{4} d^2 \le \sigma \pi d t$$

$$\frac{Pd}{4t} \le \sigma$$

For equilibrium, $\sigma = \sigma_H = \sigma_L = \frac{Pd}{4t}$

Hoop stress/longitudinal strain

$$\varepsilon_L = \varepsilon_h = \frac{pd}{4tE}(1-\mu)$$

Volumetric strain of sphere

$$\varepsilon_L = \frac{3pd}{4tE}(1-\mu)$$

Columns and Struts:

A structural member subjected to an axial compressive force is called strut. As per definition strut may be horizontal, inclined or even vertical.

The vertical strut is called a column.

Euler's Column Theory

Assumptions of Euler's theory:

Euler's theory is based on the following assumptions:

(i). Axis of the column is perfectly straight when unloaded.

(ii). The line of thrust coincides exactly with the unstrained axis of the column.

(iii). Flexural rigidity EI is uniform.

(iv) Material is isotropic and homogeneous.

Limitation of Euler's Formula

There is always crookedness in the column and the load may not be exactly axial.

This formula does not take into account the axial stress and the buckling load is given by this formula may be much more than the actual buckling load.

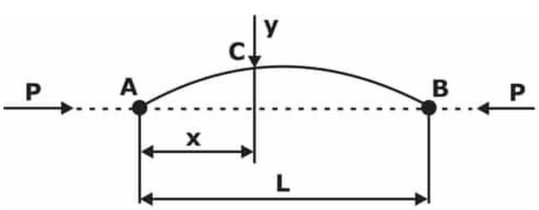
Euler's Buckling (or crippling load)

The maximum load at which the column tends to have lateral displacement or tends to buckle is known as buckling or crippling load. Load columns can be analysed with the Euler's column formulas can be given as

$$P_e = \frac{\pi^2 EI}{L_e^2}$$

where, E = Modulus of elasticity, L_e= Effective Length of column, and I = Moment of inertia of column section.

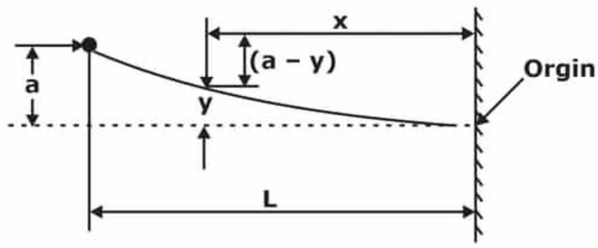
For both end hinged:



in case of Column hinged at both end L_e = L

$$P_e = \frac{\pi^2 EI}{L^2}$$

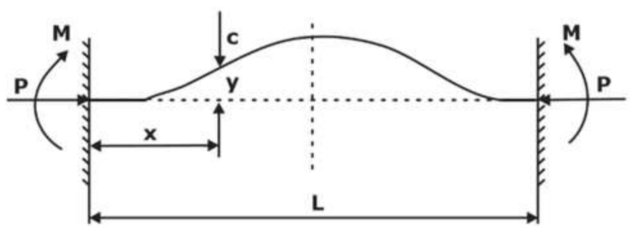
For one end fixed and other free:



in case of column one end fixed and other free**:** L_e = 2L

$$P_e = \frac{\pi^2 EI}{4L^2}$$

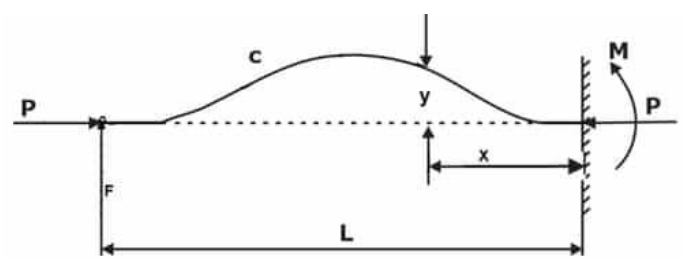
For both end fixed:



in case of Column with both end Fixed L_e = L/2

$$P_e = \frac{4\pi^2 EI}{L^2}$$

For one end fixed and other hinged:



in case of Column with one end fixed and other hinged L_e = L/√2

$$P_e = \frac{2\pi^2 EI}{L^2}$$

Effective Length for different End conditions

End Condition	Both ends hinged	One and fixed and other free	Both ends fixed	One and fixed and other hinged
Le	L	2L	$\frac{L}{2}$	$\frac{L}{\sqrt{2}}$

Slenderness Ratio (S)

The slenderness ratio of a compression member is defined as the ratio of its effective length to least radius of gyration.

$$\text{Slenderness ratio (S)} = \frac{L_e}{K} = \frac{\text{Effective length of member}}{\text{Least radius of gyration}}$$

S.No.	Type of column	Slenderness ratio
1	Short Columns	0-40
2	Intermediate columns	40-125
3	Long Columns	>125

Modes of failure of Columns

Type of Column	Mode of Failure
Short column	Crushing
Long column	Buckling
Intermediate column	Combined Crushing and Buckling

Rankine's Formula:

Rankine proposed an empirical formula for columns which coven all Lasts ranging from very short to very long struts. He proposed the relation

$$\frac{1}{P_R} = \frac{1}{P_c} + \frac{1}{P_E}$$

$P_c = \sigma_C . A$ = ultimate load for a strut

Eulerian crippling load for the standard case

$$P_E = \frac{\pi^2 EI}{L^2}$$

$$P_R = \frac{\sigma_c .A}{1+\left(\frac{\sigma_c}{\pi^2 E}\right)\left(\frac{L}{k}\right)^2} = \frac{\sigma_c .A}{1+a\left(\frac{L}{k}\right)^2}$$

$$\text{Where a} = \text{Rankine's constant} = \left(\frac{\sigma_c}{\pi^2 E}\right)$$

Testing of Materials

HARDNESS TESTING

Hardness represents the resistance of a material to indentation, penetration and scratching. In hardness testing, a loaded ball or diamond is pressed against the surface of a material which causes the plastic deformation of the same. This deformation is measured by one of the following methods:

(i) Brinell Hardness test-

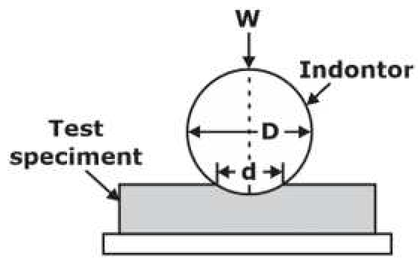
In this method, a steel hardened ball is pressed into the surface of the material under a specified load. The load is held in position for a fixed period and then released. This leaves a permanent impression in the surface of the material. Then either measure the diameter or the depth of the impression.

The Brinell specimen Hardness Number (BHN) is defined as the ratio of the applied load to the spherical area of the impression.

$$BHN = \frac{2P}{\pi D(D - \sqrt{D^2 - d^2})}$$

Where, P is in Newton.

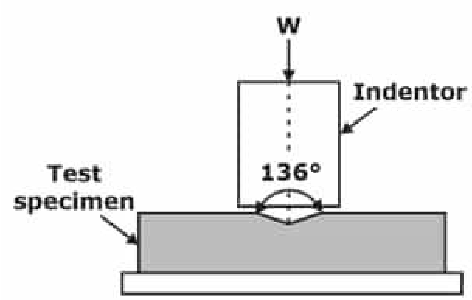
Conversion tables are also available to determine the hardness number.



(ii) Vicker Pyramid Diamond Method

This method is also similar to the Brinell method except that the indenter is a 136° pyramid diamond on a square base. As hardness of diamond is excessively high. It can be used for the whole range of materials.

The Vicker Pyramid Number (VPN) is defined as the ratio of applied load to the impressed area. The area is calculated by measuring the length of the diagonal of the square impression on the surface of the material.



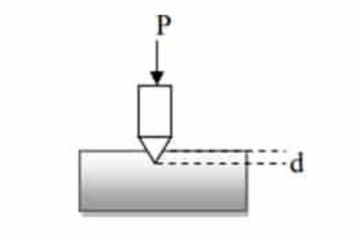
(iii) Rockwell Hardness Method –

The scale ranges between 0-100. It uses either a diamond 120° cone indenter or ball indenter made of hardened steel.

Depending on the combination of indenter and load there are several Rockwell hardness scales. Three most commonly used Rockwell hardness scales are given in table.

Rockwell Scale	Minor load, kg	Major load, kg	Total load, kg	indenter
R_A	10	50	60	120° Diamond Cone
R_B	10	90	100	1/16" steel ball
R_C	10	140	150	120° Diamond Cone

The applied load depends on the hardness of material. As a thumb rule the load used for measuring the hardness of steel = $30D^2$ kg; where D is the diameter of the ball. If D = 10mm the load to be used = 3000kg.



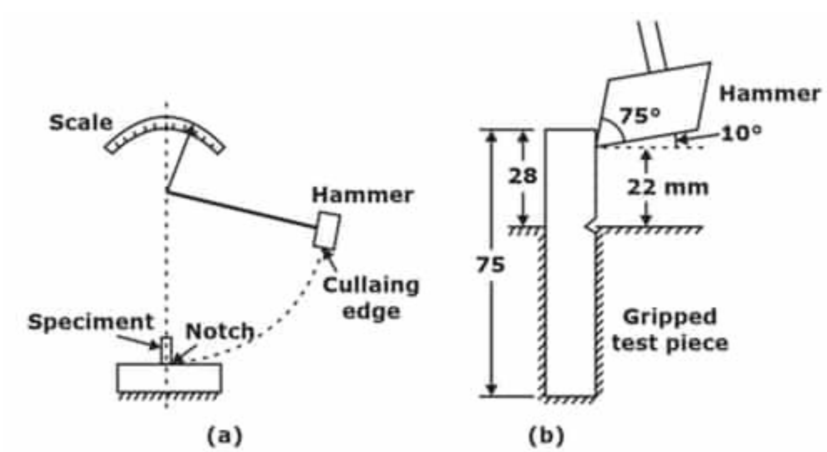
IMPACT TESTING

Static tests are useful only when the loads are static in nature. These tests do not indicate the resistance of a material against shock or impact loads to which usually the automobile parts are subjected to. In such cases, an impact test has to be undertaken. An impact test indicates the toughness of a material which is defined as the energy absorbed by the specimen without fracture.

The following are the main types of impact tests undertaken:

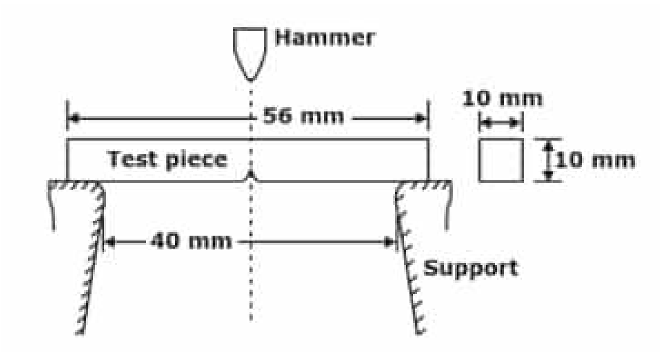
(i) Izod Impact Test

Figure shows an Izod impact testing machine. It consists of an anvil in which a notched specimen can be fixed. The specimen is taken of some standard dimensions. While fixing, care is to be taken to have the notch on the side of the falling hammer and the level with the level of top face of the hammer.



(ii) Charpy Impact Test

This test is similar to the Izod impact test except that instead of fixing the notched specimen in the anvil, it is supported at each end as a beam as shown in Figure. The hammer strikes at notch in the centre. Impact tests are important as they can reveal the temper brittleness in heat treated materials such as nickel chrome steels.



Testing of materials:

Testing of materials is very important part of the from the point of view of design and manufacturing. It provides the information of material properties, help in ensuring the quality, help in preventing the failures and also helps to make choices among different available materials.

There are mainly two types of testing performed on materials named Mechanical testing and Non-destructive testing. Here, only mechanical testing will be discussed.

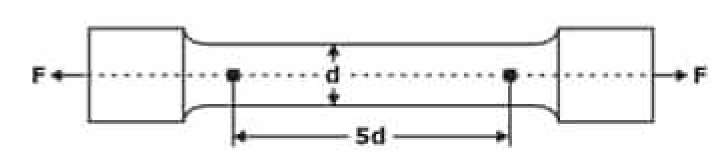
Mechanical Testing

Mechanical testing is a destructive type of testing that utilizes static or dynamic forces to reveal the properties of the material. Mechanical testing includes the different types of testing such as Tensile test, hardness testing, impact test, fatigue test, creep test, bend test, etc.

1. Tensile testing:

This test is performed on the universal testing machines (UTM). In this test, the specimen is subjected to uniaxial tensile force in a controlled way until its failure. This test helps us in accessing the following properties ductility, yield strength, tensile strength, Young's modulus (E), and Poisson's ratio (μ).

Select the standard specimen and grip it in the crossheads with proper adjustment. While setting up the job, use the adjusting knob to make zero at lower points to zero to remove the dead weight of the lower table. Now, lock the job, fix the extensometer between the gauge length (to find the extension), and apply the



Ductility: It relates the elongation during the tensile test of the material and it is defined as the percentage elongation.

$$\text{Ductility} = \frac{\text{Elongation}}{\text{Gauge length}} \times 100\%$$

Tensile strength: It is defined as the maximum load per unit cross-section area which the material can bear before breaking. It is given as:

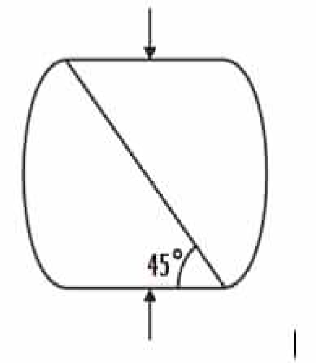
$$\text{Tensile strength } (\sigma) = \frac{\text{Maximum Force(P)}}{\text{Original cross} - \text{section area}(A_0)}$$

Yield Strength: It is the strength of the material above which permanent deformation takes place in the material under stress.

Young's Modulus (E): It is also known as the modulus of elasticity (E) of the material and it represents the stiffness of the material. It is the measure of the regain of shape and size of the material on the removal of the load.

Compression Test:

A compression test is also carried on the universal testing machine (UTM). Here, the load applied is compressive in nature and specimen is loaded till it fails.



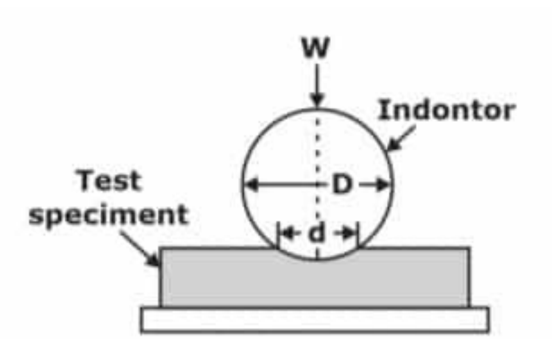
The compression test is generally carried out for the Brittle materials.

2. Hardness Test

Hardness is the surface property of the material which shows the resistance of a material against indentation, penetration and scratching.

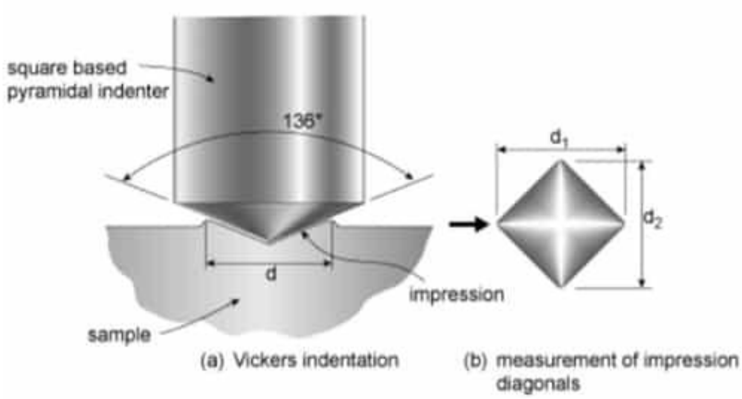
Hardness test are as follows:

1. Brinell Hardness test: In Brinell hardness test a steel or tungsten carbide ball is used to make a impression in the material under a specified load.



$$BHN = \frac{2W}{\pi D(D - \sqrt{(D^2 - d^2)})}$$

2.Vickers Hardness test: It uses the pyramid indenter of square shape and length of the diagonals of the indentation is measured to calculate the hardness number. It is suitable for very hard and tough materials.



Vickers hardness number is given by:

$$VHN = 1.854\frac{F}{D^2}$$

Where F is the applied force (in kg) and D is the average diameter of the diagonals measured.

3. Rockwell Hardness test: Rockwell test sues the diamond cone-shaped or spherical ball type of indenter for the indentation purpose. There are many scales in the Rockwell testing but C scale is the most commonly used scale and hardness on it is denoted as HRC.

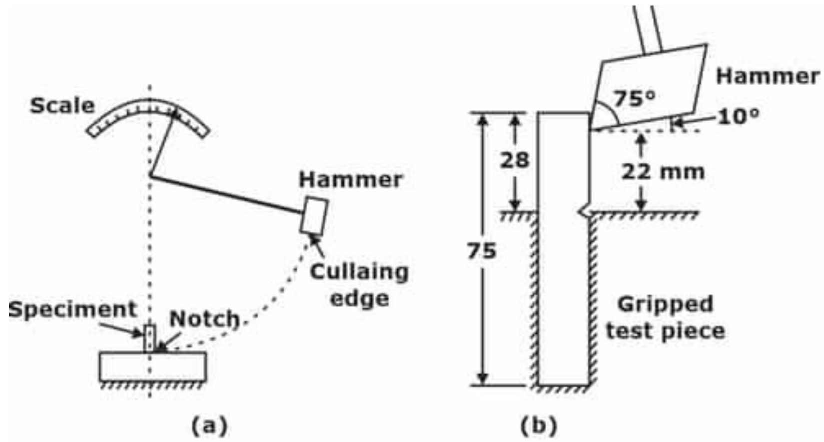
Rockwell Scale	Minor load, kg	Major load, kg	Total load, kg	Indenter
R_A	10	50	60	120° Diamond Cone
R_B	10	90	100	1/16" steel ball
R_C	10	140	150	120° Diamond Cone

3. Toughness or Impact test

Toughness tests are carried at high strain rates and the energy absorbed by the materials in breaking the specimen is considered the toughness of the material. There are two types of impact tests named Izod and Charpy tests.

Izod test:

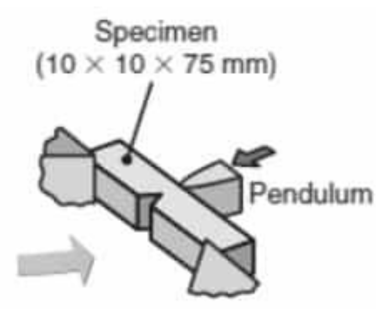
For Izod impact testing, the specimen is kept vertically as a cantilever beam. The specimen is kept in such a way that the notch side faces the striking hammer.



Charpy test:

The Charpy test shows whether a metal is either brittle or ductile and it is used for predicting ductile to brittle transition.

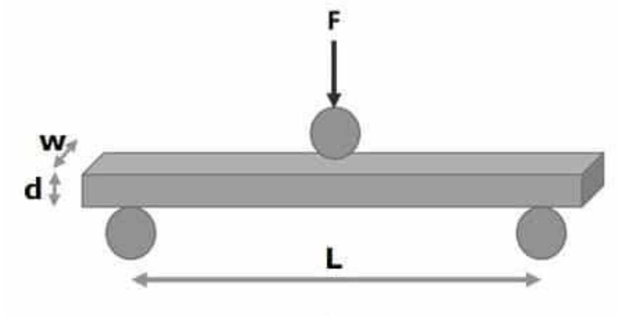
In the Charpy test, the specimen is placed horizontally and fixed at both ends i.e. it is a simply supported beam. Striking hammer strikes from the opposite side of the notch.



4. Flexural or Bending Test

The bend test is a qualitative test that can be used to access the ductility and soundness of a workpiece. Generally, it is used for the butt-welded joints to control their quality.

In bend tests, the Rectangular specimen is supported at both ends, and then the load is applied vertically at one or two points. The fracture stress in bending is called as the modulus of rupture, flexural strength.

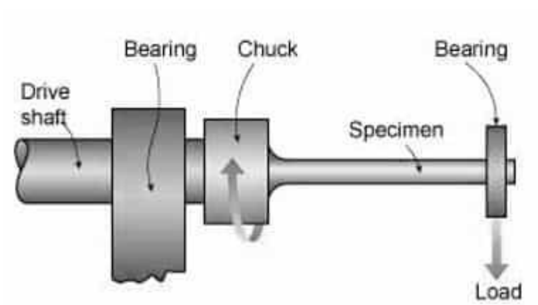


5. Shear Test:

The shear test is used to determine the shear strength of the material which is the maximum shear stress that the material can bear before the appearance of any failure. It plays a key role in the design of fasteners such as bolts and screws.

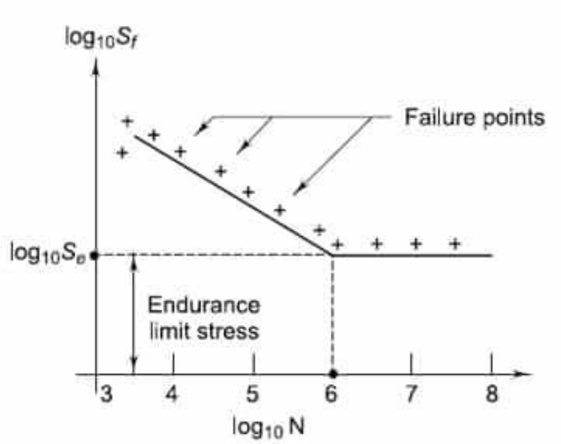
6. Fatigue Test:

Fatigue is the permanent failure of the material due to fluctuating stresses and failure takes place below the yield point of the metal. The number of cycles at which failure occurs is measured and these can vary from a couple of hundreds to millions of cycles.



The failure of the specimen under rotating loading is termed fatigue failure. Rotating loading results in completely reversed stresses.

The results of the fatigue test are plotted as an S–N curve which is the graphical representation of stress amplitude and the number of stress cycles (N) before the fatigue failure on a log-log graph paper.


$\log_{10} S_f$
Failure points
$\log_{10} S_e$
Endurance limit stress
3
4
5
6
7
8
$\log_{10} N$