QUADRATIC AND CUBIC EQUATIONS

QUADRATIC POLYNOMIALS

An expression in the form of $ax^2 + bx + c$, where a,b,c are real numbers but $a \neq 0$, is called a quadratic polynomial. For examples $2x^2 - 5x + 3$, $-x^2 + 2x$, $3x^2 - 7$, $\sqrt{2}x^2 + 7x + 2$, etc.

QUADRATIC EQUATIONS

A quadratic expression when equated to zero is called a quadratic equation. Hence an equation in the form of $ax^2 + bx + c = 0$, where a, b, c are real numbers and $a \neq 0$, is called a quadratic equation. For examples,

$$2x^2 - 5x + 3 = 0, \; -x^2 + 2x = 0,$$

$$3x^2 - 7 = 0 \text{ and } \sqrt{2}x^2 + 7x + 2 = 0, \text{etc.}$$

Illustration 1: Which of the following is not a quadratic equation?

(a) $x^2 - 2x + 2(3 - x) = 0$
(b) $x(x + 1) + 1 = (x - 2)(x - 5)$
(c) $(2x - 1)(x - 3) = (x + 5)(x - 1)$
(d) $x^3 - 4x^2 - x + 1 = (x - 2)^3$

Solution: (b) Hint: $x(x + 1) + 1 = (x - 2)(x - 5)$

$\Rightarrow$ $x^2 + x + 1 = x^2 - 7x + 10$

$\Rightarrow$ $8x - 9 = 0$, which is not a quadratic equation.

Discriminant (D)

For the quadratic equation $ax^2 + bx + c = 0$,

$$D = b^2 - 4ac$$

Here, D is the symbol of discriminant.

Roots or Solution of a Quadratic Equation

(i) If $D > 0$, then the quadratic equation $ax^2 + bx + c = 0$ has two distinct roots given by

$$\alpha = \frac{-b + \sqrt{D}}{2a} \text{ and } \beta = \frac{-b - \sqrt{D}}{2a}$$

Here α and β are symbols of roots of the quadratic equation.

(ii) If $D = 0$, then the quadratic equation $ax^2 + bx + c = 0$ has two equal roots given by

$$\alpha = \beta = -\frac{b}{2a}$$

Illustration 2: If $ax^2 + bx + c = 0$ has equal roots, then $c =$

(a) $-\frac{b}{2a}$ **(b)** $\frac{b}{2a}$

(c) $-\frac{b^2}{4a}$ **(d)** $\frac{b^2}{4a}$

Solution: (d) $ax^2 + bx + c = 0$ has equal roots if disc. $b^2 - 4ac = 0$

$\Rightarrow$ $b^2 = 4ac$

$\Rightarrow$ $c = \frac{b^2}{4a}$

Illustration 3: If $x^2 + 4x + k = 0$ has real roots, then

(a) $k \geq 4$ **(b)** $k \leq 4$
(c) $k \leq 0$ **(d)** $k \geq 0$

Solution: (b) Since $x^2 + 4x + k = 0$ has real roots.

$\therefore$ Disc. $(4)^2 - 4k \geq 0$

$\Rightarrow$ $16 - 4k \geq 0$

$\Rightarrow$ $4k \leq 16$

$\Rightarrow$ $k \leq 4$

Properties of Quadratic Equations and Their Roots

(i) If D is a perfect square then roots are rational otherwise irrational.

(ii) If $p + \sqrt{q}$ is one root of a quadratic equation, then their conjugate $p - \sqrt{q}$ must be the other root and vice-versa, where p is rational and $\sqrt{q}$ is a surd.

(iii) If a quadratic equation in x has more than two roots, then it is an identity in x.

SUM AND PRODUCT OF ROOTS

If α and β are the roots of a quadratic equation $ax^2 + bx + c = 0$, Then,

$$\text{Sum of roots, } \alpha + \beta = -\frac{b}{a} = -\frac{\text{coefficient of } x}{\text{coefficient of } x^2}$$

$$\text{Product of roots, } \alpha\beta = \frac{c}{a} = \frac{\text{constant term}}{\text{coefficient of } x^2}$$

Illustration 4: Find the sum and product of roots of $-2x^2 + 3x - 5 = 0$.

Solution: Sum of roots $= -\frac{b}{a} = \frac{3}{2} = \frac{3}{2}$

Product of roots $= \frac{c}{a} = \frac{5}{2} = \frac{5}{2}$

FORMATION OF AN EQUATION WITH GIVEN ROOTS

If α and β are the roots of a quadratic equation, then the quadratic equation will be

$$x^2 - (\alpha + \beta)\, x + \alpha . \beta = 0$$

i.e., x^2 – (Sum of the roots) x + Product of the roots = 0

Illustration 5: If α and β are the roots of the equation $3x^2 - x + 4 = 0$, then find the quadratic equation whose roots are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$.

Solution: $\alpha + \beta = \frac{1}{3} = \frac{1}{3}, \; \alpha . \beta = \frac{4}{3}$

Now, $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{a + b}{ab}$

$$= \frac{\frac{1}{3}}{\frac{4}{3}} = \frac{1}{4}$$

$$\frac{1}{\alpha} \cdot \frac{1}{\beta} = \frac{1}{ab} = \frac{1}{\frac{4}{3}} = \frac{3}{4}$$

Hence required quadratic equation,

$$x^2 \quad \frac{1}{4}x + \frac{3}{4} = 0$$

$\Rightarrow$ $4x^2 - x + 3 = 0$

Practice Exercise

Level - I

1. Which of the following is a quadratic equation ?

(a) $x^{\frac{1}{2}} + 2x + 3 = 0$

(b) $(x-1)(x+4) = x^2 + 1$

(c) $x^4 - 3x + 5 = 0$

(d) $(2x+1)(3x-4) = 2x^2 + 3$

2. Solve $x - \frac{1}{x} = 1\frac{1}{2}$

(a) $-\frac{1}{2}, 2$ (b) $\frac{1}{2}, 2$

(c) $\frac{1}{2}, \frac{2}{3}$ (d) None of these

3. If $2x^2 - 7xy + 3y^2 = 0$, then the value of $x : y$ is

(a) 3 : 2 (b) 2 : 3

(c) 3 : 1 or 1 : 2 (d) 5 : 6

4. Father's age is 4 less than five times the age of his son and the product of their ages is 288. Find the father's age.

(a) 40 years (b) 36 years

(c) 26 years (d) 42 years

5. The sum of a rational number and its reciprocal is $\frac{13}{6}$, find the number.

(a) $\frac{2}{3}$ or $\frac{3}{2}$ (b) $\frac{3}{4}$ or $\frac{4}{3}$

(c) $\frac{2}{5}$ or $\frac{5}{2}$ (d) None of these

6. $\sqrt{6+\sqrt{6+\sqrt{6+.........}}} = ?$

(a) 2.3 (b) 3

(c) 6 (d) 6.3

7. If $x^2 + 2 = 2x$, then the value of $x^4 - x^3 + x^2 + 2$ is

(a) 1 (b) 0

(c) –1 (d) $\sqrt{2}$

8. Minimum value of $x^2 + \frac{1}{x^2+1} - 3$ is

(a) 0 (b) – 1

(c) – 3 (d) – 2

9. One root of $x^2 + kx - 8 = 0$ is square of the other. Then the value of k is

(a) 2 (b) 8

(c) –8 (d) –2

10. If the roots x_1 and x_2 of the quadratic equation $x^2 - 2x + c = 0$ also satisfy the equation $7x_2 - 4x_1 = 47$, then which of the following is true?

(a) $c = -15$ (b) $x_1 = -5, x_2 = 3$

(c) $x_1 = 4.5, x_2 = -2.5$ (d) None of these

11. For what value of k, are the roots of the quadratic equation $(k+1)x^2 - 2(k-1)x + 1 = 0$ real and equal?

(a) $k = 0$ only (b) $k = -3$ only

(c) $k = 0$ or $k = 3$ (d) $k = 0$ or $k = -3$

12. If the roots of the equation

$$(a^2 + b^2)x^2 - 2ab(a+c)x + (b^2 + c^2) = 0$$

are equal, then which one of the following is correct?

(a) $2b = a + c$ (b) $b^2 = ac$

(c) $b + c = 2a$ (d) $b = ac$

13. If α and β are the roots of the equation $x^2 - 2x + 4 = 0$, then what is the value of $\alpha^3 + \beta^3$?

(a) 16 (b) – 16

(c) 8 (d) – 8

14. If p and q are the roots of the equation $x^2 - px + q = 0$, then what are the values of p and q respectively?

(a) 1, 0 (b) 0, 1

(c) – 2, 0 (d) – 2, 1

15. What is the value of $\sqrt{5\sqrt{5\sqrt{5\sqrt{5...\infty}}}}$?

(a) 5 (b) $\sqrt{5}$

(c) 1 (d) $(5)^{1/4}$

16. If r and s are roots of $x^2 + px + q = 0$, then what is the value of $\frac{1}{r^2} + \frac{1}{s^2}$?

(a) $p^2 - 4q$ (b) $\frac{p^2-4q}{2}$

(c) $\frac{p^2-4q}{q^2}$ (d) $\frac{p^2-2q}{q^2}$

17. Find the solution of $\frac{2x}{x-3} + \frac{1}{2x+3} + \frac{3x+9}{(x-3)(2x+3)} = 0$

(a) 0 (b) – 1

(c) 3 (d) – 3

18. If the roots of $x^2 - kx + 1 = 0$ are non-real, then

(a) $-3 < k < 3$ (b) $-2 < k < 2$

(c) $k > 2$ (d) $k < -2$

19. If $ax^2 + bx + c = 0$ has real and different roots, then

(a) $b^2 - 4ac = 0$ (b) $b^2 - 4ac > 0$

(c) $b^2 - 4ac < 0$ (d) $b^2 - 4ac \le 0$

20. If $\sqrt{3x^2+x+5} = x-3$, then the given equator has ...... solution/solutions.

(a) $x=-4$ (b) $x=\frac{1}{2}$

(c) $x=-4, \frac{1}{2}$ (both) (d) No solution

21. The sum of two numbers p and q is 18 and the sum of their reciprocals is $\frac{1}{4}$. Then the numbers are

(a) 10, 8 (b) 12, 6

(c) 9, 9 (d) 14, 4

22. If the roots of the equation $x^2-bx+c=0$ differ by 2, then which of the following is true?

(a) $c^2=4(c+1)$ (b) $b^2=4c+4$

(c) $c^2=b+4$ (d) $b^2=4(c+2)$

23. The sum of a number and its reciprocal is one-fifth of 26. What is the sum of that number and its square?

(a) 3 (b) 4

(c) 5 (d) 6

24. Two numbers are such that the square of greater number is 504 less than 8 times the square of the other. If the numbers are in the ratio 3 : 4. Find the number.

(a) 15 and 20 (b) 6 and 8

(c) 12 and 16 (d) 9 and 12

25. The equation $x+\sqrt{x-2}=4$ has

(a) two real roots and one imaginary root

(b) one real and one imaginary root

(c) two imaginary roots

(d) one real root

26. The equation $\sqrt{x+10}-\frac{6}{\sqrt{x+10}}=5$ has

(a) an extraneous root between –5 and –1

(b) an extraneous root between –10 and –6

(c) two extraneous roots

(d) a real root between 20 and 25

[An extraneous root means a root which does not satisfy the equation.]

27. If $\log_{10}(x^2-3x+6)=1$, then the value of x is

(a) 10 or 2 (b) 4 or –2

(c) 4 only (d) 4 or –1

28. The roots of the equation $2\sqrt{x}+2x^{-\frac{1}{2}}=5$ can be found by solving

(a) $4x^2-25x+4=0$ (b) $4x^2+25x-4=0$

(c) $4x^2-17x+4=0$ (d) None of these

29. Two numbers whose sum is 6 and the absolute value of whose difference is 8 are the roots of the equation

(a) $x^2-6x+7=0$ (b) $x^2-6x-7=0$

(c) $x^2+6x-8=0$ (d) $x^2-6x+8=0$

30. The roots of the equation $x^2+2\sqrt{3}x+3=0$ are

(a) real and equal (b) rational and equal

(c) rational and unequal (d) imaginary

31. The roots of the equation $ax^2+bx+c=0$ will be reciprocal if

(a) $a=b$ (b) $a=bc$

(c) $c=a$ (d) $b=c$

32. If $\frac{b}{x-a}=\frac{x+a}{b}$ then the value of x in terms of a and b is

(a) $\pm\sqrt{a^2+b^2}$ (b) $+\sqrt{a^2+b^2}$

(c) $-\sqrt{a^2+b^2}$ (d) None of these

33. For what value of b and c would the equation $x^2+bx+c=0$ have roots equal to b and c.

(a) (0, 0) (b) (1, –2)

(c) (1, 2) (d) Both (a) and (b)

34. One of the factors of the expression $4\sqrt{3}x^2+5x-2\sqrt{3}$ is: [*SSC-Sub. Ins.-2013*]

(a) $4x+\sqrt{3}$ (b) $4x+3$

(c) $4x-3$ (d) $4x-\sqrt{3}$

35. If $x+\frac{1}{x}=3$, then the value of $\frac{3x^2-4x+3}{x^2-x+1}$ is [*SSC 10+2-2014*]

(a) $\frac{4}{3}$ (b) $\frac{3}{2}$

(c) $\frac{5}{2}$ (d) $\frac{5}{3}$

36. If $x=p+\frac{1}{p}$ and $y=p-\frac{1}{p}$, then value of $x^4-2x^2y^2+y^4$ is [*SSC 10+2-2014*]

(a) 24 (b) 4

(c) 16 (d) 8

37. If $x=3+2\sqrt{2}$, then $\frac{x^6+x^4+x^2+1}{x^3}$ is equal to [*SSC 10+2-2014*]

(a) 216 (b) 192

(c) 198 (d) 204

38. A certain number of capsules were purchased for ₹ 216. 15 more capsules could have been purchased in the same amount if each capsule was cheaper by ₹ 10. What was the number of capsules purchased? [*IBPS Clerk-2013*]

(a) 6 (b) 14

(c) 8 (d) 12

(e) 9

Level - II

1. The discriminant of $ax^2 - 2\sqrt{2}x + c = 0$ with *a*, *c* are real constants is zero. The roots must be

(a) equal and integral (b) rational and equal
(c) real and equal (d) imaginary

2. If one root of the equation $ax^2 + bx + c = 0$ is three times the other, then ______

(a) $b^2 = 16\,ac$ (b) $b^2 = ac$
(c) $3b^2 = 16\,ac$ (d) None of these

3. If the product of roots of the equation $x^2 - 3\,(2a + 4)\,x + a^2 + 18a + 81 = 0$ is unity, then a can take the values as

(a) $3, -6$ (b) $10, -8$
(c) $-10, -8$ (d) $-10, -6$

4. If the roots of the equation $(a^2 + b^2)\,x^2 - 2(ac + bd)x + (c^2 + d^2) = 0$ are equal, then which of the following is true?

(a) $ab = cd$ (b) $ad = bc$
(c) $ad = \sqrt{bc}$ (d) $ab = \sqrt{cd}$

5. For what values of *c* in the equation $2x^2 - (c^3 + 8c - 1)x + c^2 - 4c = 0$ the roots of the equation would be opposite to signs?

(a) $c \in (0,4)$ (b) $c \in (-4,0)$
(c) $c \in (0,3)$ (d) $c \in (-4,4)$

6. If $x^2 - 3x + 2$ is a factor of $x^4 - ax^2 + b = 0$, then the values of *a* and *b* are

(a) $-5, -4$ (b) $5, 4$
(c) $-5, 4$ (d) $5, -4$

7. If α and β are the roots of the quadratic equation $ax^2 + bx + c = 0$, then the value of $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}$ is

(a) $\frac{3bc - a^3}{b^2 c}$ (b) $\frac{3abc - b^3}{a^2 c}$
(c) $\frac{3abc - b^2}{a^3 c}$ (d) $\frac{ab - b^2 c}{2b^2 c}$

8. If *a*, *b* are the two roots of a quadratic equation such that $a + b = 24$ and $a - b = 8$, then the quadratic equation having *a* and *b* as its roots is

(a) $x^2 + 2x + 8 = 0$ (b) $x^2 - 4x + 8 = 0$
(c) $x^2 - 24x + 128 = 0$ (d) $2x^2 + 8x + 9 = 0$

9. If $m + \frac{1}{m-2} = 4$ then, what is value of

$$(m-2)^2 + \frac{1}{(m-2)^2} = ?$$

(a) -2 (b) 0
(c) 2 (d) 4

10. If $x^2 + y^2 + \frac{1}{x^2} + \frac{1}{y^2} = 4$, then the value of $x^2 + y^2$ is

(a) 2 (b) 4
(c) 8 (d) 16

11. Let *x*, *y* be two positive numbers such that $x + y = 1$. Then, the minimum value of $\left(x + \frac{1}{x}\right)^2 + \left(y + \frac{1}{y}\right)^2$ is

(a) 12 (b) 20
(c) 12.5 (d) 13.3

12. Solve the simultaneous equations

$$\sqrt{\frac{x}{y}} + \sqrt{\frac{y}{x}} = \frac{5}{2};\, x + y = 10$$

(a) 8, 6 (b) 8, 2
(c) 4, 6 (d) 5, 5

13. If roots of an equation $ax^2 + bx + c = 0$ are positive, then which one of the following is correct?

(a) Signs of *a* and *c* should be like
(b) Signs of *b* and *c* should be like
(c) Signs of *a* and *b* should be like
(d) None of the above

14. If the sum of the squares of the roots of $x^2 - (p-2)\,x - (p+1) = 0$ $(p \in R)$ is 5, then what is the value of *p*?

(a) 0 (b) –1
(c) 1 (d) $\frac{3}{2}$

15. If α and β are the roots of the equation $x^2 + 6x + 1 = 0$, then what is $|\alpha - \beta|$ equal to?

(a) 6 (b) $3\sqrt{2}$
(c) $4\sqrt{2}$ (d) 12

16. If $\frac{1}{2 - \sqrt{-2}}$ is one of the roots of $ax^2 + bx + c = 0$, where *a*, *b*, *c* are real, then what are the values of *a*, *b*, *c* respectively?

(a) 6, –4, 1 (b) 4, 6, –1
(c) 3, –2, 1 (d) 6, 4, 1

17. If α and β are the roots of the equation $ax^2 + bx + c = 0$, then the equation whose roots are $\frac{1}{\alpha + \beta}, \frac{1}{\alpha} + \frac{1}{\beta}$ is equal to

(a) $acx^2 + (a^2 + bc)x + bc = 0$
(b) $bcx^2 + (b^2 + ac)x + ab = 0$
(c) $abx^2 + (c^2 + ab)x + ca = 0$
(d) None of these

18. Find the roots of the equation $a^3x^2 + abcx + c^3 = 0$

(a) $\alpha^2\beta, \beta^2\alpha$ (b) α, β^2
(c) $\alpha^2\beta, \beta\alpha$ (d) $\alpha^3\beta, \beta^3\alpha$

19. A natural number when increased by 12, equals 160 times its reciprocal. Find the number.

(a) 3 (b) 5
(c) 8 (d) 16

20. Solve: $\frac{1}{a+b+x} = \frac{1}{a}+\frac{1}{b}+\frac{1}{x};\ a \neq 0, x \neq 0$

(a) a, b (b) $-a, b$
(c) $0, a$ (d) $-a, -b$

21. Which is not true?

(a) Every quadratic polynomial can have at most two zeros.
(b) Some quadratic polynomials do not have any zero. [*i.e.* real zero]
(c) Some quadratic polynomials may have only one zero. [*i.e.* one real zero]
(d) Every quadratic polynomial which has two zeros.

22. The expression $a^2 + ab + b^2$ is _____ for $a < 0, b < 0$

(a) $\neq 0$ (b) < 0
(c) > 0 (d) $= 0$

23. For what value of c the quadratic equation $x^2 - (c + 6)\, x + 2(2c - 1) = 0$ has sum of the roots as half of their product?

(a) 5 (b) -4
(c) 7 (d) 3

24. If α and β are the roots of the equation $ax^2 + bx + c = 0$, then the equation whose roots are $\alpha + \frac{1}{\beta}$ and $\beta + \frac{1}{\alpha}$ is

(a) $abx^2 + b(c + a)x + (c + a)^2 = 0$
(b) $(c + a)x^2 + b(c + a)\, x + ac = 0$
(c) $cax^2 + b(c + a)x + (c + a)^2 = 0$
(d) $cax^2 + b(c + a)x + c(c + a)^2 = 0$

25. If $x^2 + ax + b$ leaves the same remainder 5 when divided by $x - 1$ or $x + 1$, then the values of a and b are respectively

(a) 0 and 4 (b) 3 and 0
(c) 0 and 5 (d) 4 and 0

26. The condition that both the roots of quadratic equation $ax^2 + bx + c = 0$ are positive is

(a) a and c have an opposite sign that of b
(b) b and c have an opposite sign that of a
(c) a and b have an opposite sign that of c
(d) None of these

27. If the equation $x^2 - bx + 1 = 0$ does not possess real roots, then which one of the following is correct?

(a) $-3 < b < 3$ (b) $-2 < b < 2$
(c) $b > 2$ (d) $b < -2$

28. If the roots of the quadratic equation $3x^2 - 5x + p = 0$ are real and unequal, then which one of the following is correct?

(a) $p = 25/12$ (b) $p < 25/12$
(c) $p > 25/12$ (d) $p \leq 25/12$

29. If the roots of the equation $x^3 - ax^2 + bx - c = 0$ are three consecutive integers, then what is the smallest possible value of b?

(a) $-\frac{1}{\sqrt{3}}$ (b) -1
(c) 0 (d) 1

30. If α, β are the roots of the equation $2x^2 - 3x - 6 = 0$, find the equation whose roots are $\alpha^2 + 2$ and $\beta^2 + 2$.

(a) $4x^2 + 49x + 118 = 0$ (b) $4x^2 - 49x + 118 = 0$
(c) $4x^2 - 49x - 118 = 0$ (d) $4x^2 + 49x - 118 = 0$

31. Sum of the areas of two squares is 468 m^2. If the difference of their perimeters is 24 m, find the sides of the two squares.

(a) 9 m, 6 m (b) 18 m, 12 m
(c) 18 m, 6 m (d) 9 m, 12 m

32. The sum of the ages of Puneet and his father is 45 years and the product of their ages is 126. What is the age of Puneet ? [*SSC CGL-2013*]

(a) 3 years (b) 5 years
(c) 10 years (d) 45 years

Level-I

1. **(d)** Equations in options (a) and (c) are not quadratic equations as in (a) max. power of x is fractional and in (c), it is not 2 in any of the terms.

For option (b), $(x-1)(x+4) = x^2+1$

or $x^2+4x-x-4 = x^2+1$

or $3x-5=0$

which is not a quadratic equations but a linear.

For option (d), $(2x+1)(3x-4) = 2x^2+3$

or $6x^2-8x+3x-4 = 2x^2+3$

or $4x^2-5x-7=0$

which is clearly a quadratic equation.

2. **(a)** $x-\frac{1}{x}=1\frac{1}{2} \Rightarrow \frac{x^2-1}{x}=\frac{3}{2}$

$\Rightarrow 2(x^2-1)=3x \Rightarrow 2x^2-2=3x$

$\Rightarrow 2x^2-3x-2=0$

$\Rightarrow 2x^2-4x+x-2=0$

$\Rightarrow 2x(x-2)+1(x-2)=0$

$\Rightarrow (2x+1)(x-2)=0$

Either $2x+1=0$ or $x-2=0$

$\Rightarrow 2x=-1$ or $x=2$

$\Rightarrow x=\frac{-1}{2}$ or $x=2$

$\therefore x=\frac{-1}{2}$, 2 are solutions.

3. **(c)** $2x^2-7xy+3y^2=0$

$2\left(\frac{x}{y}\right)^2-7\left(\frac{x}{y}\right)+3=0$

$\frac{x}{y}=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{7\pm\sqrt{49-24}}{2\times 2}=\frac{7\pm 5}{4}=3,\frac{1}{2}$

$\Rightarrow \frac{x}{y}=\frac{3}{1}$ or $\frac{x}{y}=\frac{1}{2}$

4. **(b)** Let the son's age be x years.

So, father's age $= 5x-4$ years.

$\therefore x(5x-4)=288$

$\Rightarrow 5x^2-4x-288=0 \Rightarrow 5x^2-40x+36x-288=0$

$\Rightarrow 5x(x-8)+36(x-8)=0$

$\Rightarrow (5x+36)(x-8)=0$

Either $x-8=0$ or $5x+36=0 \Rightarrow x=8$ or $x=\frac{-36}{5}$

x cannot be negative; therefore, $x=8$ is the solution.

$\therefore$ Son's age = 8 years and Father's age = $5x-4$ = 36 years.

5. **(a)** Let the number be x.

Then, $x+\frac{1}{x}=\frac{13}{6} \Rightarrow \frac{x^2+1}{x}=\frac{13}{6} \Rightarrow 6x^2-13x+6=0$

$\Rightarrow 6x^2-9x-4x+6=0 \Rightarrow (3x-2)(2x-3)=0$

$\Rightarrow x=\frac{2}{3}$ or $x=\frac{3}{2}$.

Hence, the required number is $\frac{2}{3}$ or $\frac{3}{2}$.

6. **(b)** $\sqrt{6+\sqrt{6+\sqrt{6+\ldots\ldots\ldots\ldots}}}$

$\sqrt{6+x}=x$

$6+x=x^2$

$x^2-x-6=0$

$x^2-3x+2x-6=0$

$(x-3)(x+2)=0$

$x=3$

7. **(b)** $x^2+2=2x \Rightarrow x^2-2x+2=0$

$x^2-2x+2\,)\,x^4-x^3+x^2+2\,(\,x^2+x+1$

$x^4-2x^3+2x^2$

$-\quad+\quad-$

x^3-x^2+2

x^3-2x^2+2x

$-\quad+\quad-$

x^2-2x+2

x^2-2x+2

0

$\therefore x^4-x^3+x^2+2$

$=(x^2-2x+2)(x^2+x+1)=0$

8. **(d)** $x^2\geq 0$

$\therefore$ Minimum value

$=0+\frac{1}{1}-3=-2$

9. **(d)** Given $x^2+kx-8=0$

Let a and b be the roots of given equation and $b=a^2$ (given)

Sum of roots $=a+b=-k=a+a^2$ .....(1)

Product of roots $=ab=-8=a^3 \Rightarrow a=-2$

Using $a=-2$ in (1), $-k=-2+4=2$ or $k=-2$

10. **(a)** $7x_2 - 4x_1 = 47$

$x_1 + x_2 = 2$

Solving $11x_2 = 55$

$x_2 = 5$ & $x_1 = -3$

$\therefore c = -15$

11. **(c)** Since, the roots of the equation $(k+1)x^2 - 2(k-1)x + 1 = 0$ are real and equal.

$\therefore \quad \{-2(k-1)\}^2 - 4(k+1) = 0 \qquad (\because b^2 - 4ac = 0)$

$\Rightarrow \quad 4(k^2 - 2k + 1) - 4(k+1) = 0$

$\Rightarrow \quad k^2 - 2k + 1 - 1 = 0$

$\Rightarrow \quad k^2 - 3k = 0$

$\Rightarrow \quad k = 0, k = 3$

12. **(b)** Since roots of the given equation are equal.

$\therefore \quad D = 0$

On solving we get $b^2 = ac$

13. **(b)** Use $\alpha^3 + \beta^3 = (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)$

14. **(a)**

15. **(a)** Let $x = \sqrt{5\sqrt{5\sqrt{5 \ldots \infty}}}$

$\Rightarrow \quad x^2 = 5x \Rightarrow x = 0, 5$

16. **(d)** $\dfrac{1}{r^2} + \dfrac{1}{s^2} = \dfrac{s^2 + r^2}{(rs)^2} = \dfrac{(s+r)^2 - 2sr}{(rs)^2} = \dfrac{p^2 - 2q}{q^2}$

17. **(b)** Clearly, the given equation is valid if $x - 3 \neq 0$ and $2x + 3 \neq 0$ i.e.,. when $x \neq \dfrac{-3}{2}, 3$

Now, $\dfrac{2x}{x-3} + \dfrac{1}{2x+3} + \dfrac{3x+9}{(x-3)(2x+3)} = 0$

$\Rightarrow \quad 2x(2x+3) + (x-3) + 3x + 9 = 0$

[Multiplying throughout by $(x-3)(2x+3)$]

$\Rightarrow \quad 4x^2 + 6x + x - 3 + 3x + 9 = 0$

$\Rightarrow \quad 4x^2 + 10x + 6 = 0$

$\Rightarrow \quad 2x^2 + 5x + 3 = 0$

$\Rightarrow \quad 2x^2 + 2x + 3x + 3 = 0$

$\Rightarrow \quad 2x(x+1) + 3(x+1) = 0$

$\Rightarrow \quad (2x+3)(x+1) = 0$

$\Rightarrow \quad x + 1 = 0 \Rightarrow x = -1 \; [\because 2x + 3 \neq 0]$

Hence, $x = -1$ is the only solution of the given equation.

18. **(b)** Since the roots of $x^2 - kx + 4 = 0$ are non-real.

$\therefore$ Disc., $(-k^2) - 4 < 0 \Rightarrow k^2 - 4 < 0$

$\Rightarrow k^2 < 4 \Rightarrow |k| < 2 \Rightarrow -2 < k < 2$

19. **(b)**

20. **(d)** Square both sides, we shall get $x = -4, \dfrac{1}{2}$. But both of them do not satisfy the given equation.

21. **(b)** $p + q = 18$ ...(1)

and $\dfrac{1}{p} + \dfrac{1}{q} = \dfrac{1}{4}$ ...(2) (Given)

i.e $\dfrac{p+q}{pq} = \dfrac{1}{4} \Rightarrow \dfrac{18}{pq} = \dfrac{1}{4}$

$\Rightarrow \quad pq = 72$ ...(3)

From (1) and (3), $p(18 - p) = 72$

$\Rightarrow \quad p^2 - 18p + 72 = 0 \Rightarrow (p-6)(p-12) = 0$

$\Rightarrow \quad p = 6, 12$ when $p = 6, q = 12$; when $p = 12, q = 6$

Hence the numbers are 12, 6.

22. **(b)** Let the roots be α and $\alpha + 2$.

Then $\alpha + \alpha + 2 = b \Rightarrow \alpha = (b-2)/2$ (1)

and $\alpha(\alpha+2) = c \quad \Rightarrow \alpha^2 + 2\alpha = c$ (2)

Putting the value of α from (1) in (2).

$((b-2)/2)^2 + 2(b-2)/2) = c$

$\Rightarrow \quad (b^2 + 4 - 4b)/4 + b - 2 = c$

$\Rightarrow \quad b^2 + 4 - 8 = 4c$

$\Rightarrow \quad b^2 = 4c + 4$

23. **(c)** Let the number be x. Then,

$x + \dfrac{1}{x} = \dfrac{26}{5}$

$\Rightarrow \quad \dfrac{x^2 + 1}{x} = \dfrac{26}{5}$

$\Rightarrow \quad 5x^2 - 26x + 5 = -0$

$\Rightarrow \quad (x-5)(5x-1) = 0$

$\Rightarrow \quad x = 5$ or $\dfrac{1}{5}$

24. **(d)**

25. **(d)** $x + \sqrt{x-2} = 4$

$\sqrt{x-2} = 4 - x$

Squaring on the both sides

$x - 2 = 16 + x^2 - 8x$

$x^2 - 9x + 18 = 0$

$(x-6)(x-3) = 0$

$x = 6$ or 3

But by checking, only $x = 3$ satisfies the equation.

26. **(b)** $\sqrt{x+10} - \dfrac{6}{\sqrt{x+10}} = 5$

$x + 10 - 6 = 5\sqrt{x+10}$

$x + 4 = 5\sqrt{x+10}$

Squaring on both sides,

$x^2 + 8x + 16 = 25x + 250$

$x^2 - 17x - 234 = 0$

$x^2 - 26x + 9x - 234 = 0$

$x(x-26) + 9(x-26) = 0$

$(x-26)(x+9) = 0$

$x = 26$ (or) -9

Here $x = -9$ is not satisfying. So it is extraneous.

27. **(d)** $\log_{10}(x^2 - 3x + 6) = 1$
$x^2 - 3x + 6 = 10^1$
$x^2 - 3x - 4 = 0$
$(x-4)(x+1) = 0$
$x = 4$ or -1

28. **(c)** $2\sqrt{x} + \dfrac{2}{\sqrt{x}} = 5$

$2x + 2 = 5\sqrt{x}$
$\Rightarrow 4x^2 + 8x + 4 = 25x$
$\Rightarrow 4x^2 - 17x + 4 = 0$

29. **(b)** Let α and β are the roots
$\alpha + \beta = 6$
$\alpha - \beta = 8$
$2\alpha = 14$
$\alpha = 7$
$\beta = -1$
$\alpha + \beta = 6, \alpha\beta = -7$
The quadratic equation is $x^2 - 6x - 7 = 0$

30. **(a)** $b^2 - 4ac = (2\sqrt{3})^2 - 4(1)(3) = 0$. So the roots are real and equal.

31. **(c)** Since roots are reciprocal,

product of the roots = 1 $\Rightarrow \dfrac{c}{a} = 1$

$\Rightarrow c = a$.

32. **(a)** $\dfrac{b}{x-a} = \dfrac{x+a}{b}$
$x^2 - a^2 = b^2$
$x^2 = b^2 + a^2$

$x = \pm\sqrt{a^2 + b^2}$

33. **(d)** Solve using options. It can be seen that $b = 0$ and $c = 0$ the condition is satisfied. It is also satisfied at $b = 1$ and $c = -2$.

34. **(d)** $4\sqrt{3}\,x^2 + 5x - 2\sqrt{3}$

$= 4\sqrt{3}\,x^2 + 8x - 3x - 2\sqrt{3}$

$= 4x(\sqrt{3}x + 2) - \sqrt{3}(\sqrt{3}x + 2)$

$= \left(4x - \sqrt{3}\right)\left(\sqrt{3}x + 2\right)$

35. **(c)** $\dfrac{3x^2 - 4x + 3}{x^2 - x + 1} = \dfrac{\frac{3x^2}{x} - \frac{4x}{x} + \frac{3}{x}}{\frac{x^2}{x} - \frac{x}{x} + \frac{1}{x}}$

$$\frac{3\left(x + \frac{1}{x}\right) - 4}{\left(x + \frac{1}{x}\right) - 1} = \frac{3 \times 3 - 4}{3 - 1} = \frac{5}{2}$$

36. **(c)** $x^4 - 2x^2y^2 + y^4 = (x^2 - y^2)^2 = [(x+y)(x-y)]^2$

$= \left(2p \times \dfrac{2}{p}\right)^2 = 16$

37. **(d)** We have, $x = 3 + 2\sqrt{2}$

$\dfrac{1}{x} = \dfrac{1}{3 + 2\sqrt{2}} \times \dfrac{3 - 2\sqrt{2}}{3 - 2\sqrt{2}} = 3 - 2\sqrt{2}$

$x + \dfrac{1}{x} = 6$

$\dfrac{x^6 + x^4 + x^2 + 1}{x^3} = x^3 + x + \dfrac{1}{x} + \dfrac{1}{x^3}$

$= \left(x^3 + \dfrac{1}{x^3}\right) + \left(x + \dfrac{1}{x}\right)$

$= \left(x + \dfrac{1}{x}\right)\left(x^2 + \dfrac{1}{x^2} - 1\right) + \left(x + \dfrac{1}{x}\right)$

$= \left(x + \dfrac{1}{x}\right)\left[\left(x + \dfrac{1}{x}\right)^2 - 3\right] + \left(x + \dfrac{1}{x}\right)$

$= 6[6^2 - 3] + 6 = 198 + 6 = 204$

38. **(d)** Let x be the price of one capsule
y be the total number of capsule.

$xy = 216$ ...(1)

$(x - 10)(y + 15) = 216$ ...(2)

From eqs (1) and (2)

$$\left(\frac{216}{y} - 10\right)(y + 15) = 216$$

$$(216 - 10y)(y + 15) = 216y$$
$$216y + 216 \times 15 - 10y^2 - 150y = 216y$$
$$216y + 3240 - 10y^2 - 150y = 216y$$
$$-10y^2 - 150y + 3240 = 0$$
$$y^2 + 15y - 324 = 0$$
$$y = 12, -27$$

Number of capsules cannot be negative.

Level-II

1. **(c)** $ax^2 - 2\sqrt{2}x + c = 0$

$(2\sqrt{2})^2 - 4ac = 0$
$4ac = 8$
$ac = 2$

$c = \dfrac{2}{a}$

Let α, β be the roots.

$\alpha + \beta\ \dfrac{2\sqrt{2}}{a},\ \alpha\beta = \dfrac{c}{a} = \dfrac{2}{a^2}$

$(\alpha-\beta)^2=(\alpha+\beta)^2-4\alpha\beta$

$=\frac{8}{a^2}-\frac{8}{a^2}=0$

$\alpha=\beta$

So, $\alpha=\beta=\frac{\sqrt{2}}{a}$

Hence the roots are real and equal.

2. **(c)** Let α, 3α are the roots.

$\alpha+3\alpha=\frac{-b}{a}\Rightarrow 4\alpha=\frac{-b}{a}$

$\Rightarrow \alpha=\frac{-b}{4a}$ ...(1)

$\alpha\times 3\alpha=\frac{c}{a}\Rightarrow 3\alpha^2=\frac{c}{a}$

$\frac{3b^2}{16a^2}=\frac{c}{a}$ [by (1)]

$3b^2=16ac$.

3. **(c)** The product of the roots is given by: $(a^2+18a+81)/1$. Since product is unity we get: $a^2+18a+81=1$

Thus, $a^2+18a+80=0$

Solving, we get $a=-10$ and $a=-8$.

4. **(b)** Solve this by assuming each option to be true and then check whether the given expression has equal roots for the option under check.

Thus, if we check for option (b).

$ad=bc$.

We assume $a=6$, $d=4$ $b=12$ $c=2$ (any set of values that satisfies $ad=bc$)

Then $(a^2+b^2)x^2-2(ac+bc)x+(c^2+d^2)=0$

$180x^2-120x+20=0$

We can see that this has equal roots. Thus, option (b) is a possible answer. The same way if we check for a, c and d we see that none of them gives us equal roots and can be rejected.

5. **(a)** For the roots to be opposite in sign, the product of roots should be negative.

$(c^2-4c)/2<0\Rightarrow 0<c<4$

6. **(b)** $x^2-3x+2=0$ gives its roots as $x=1$, 2. Put these values in the equation and then use the options.

7. **(b)** Here, $\alpha+\beta=-\frac{b}{a}$ and $\alpha\beta=\frac{c}{a}$

Thus, $\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}=\frac{\alpha^3+\beta^3}{\alpha\beta}$

$=\frac{(\alpha+\beta)\ (\alpha^2-\alpha\beta+\beta^2)}{\alpha\beta}$ …(1)

Now, $(\alpha^2+\beta^2-\alpha\beta)=[(\alpha+\beta)^2-2\alpha\beta-\alpha\beta]$

$=[(\alpha+\beta)^2-3\alpha\beta]$

Hence (1) becomes

$\Rightarrow \frac{(\alpha+\beta)[(\alpha+\beta)^2-3\alpha\beta)]}{\alpha\beta}=\frac{\frac{-b}{a}\left[\frac{b^2}{a^2}-\frac{3c}{a}\right]}{\frac{c}{a}}$

$=\frac{-b}{c}\left[\frac{b^2-3ac}{a^2}\right]=\frac{3abc-b^3}{a^2c}$

8. **(c)** $a+b=24$ and $a-b=8$

$\Rightarrow a=16$ and $b=8\Rightarrow ab=16\times 8=128$

A quadratic equation with roots a and b is

$x^2-(a+b)x+ab=0$ or $x^2-24x+128=0$

9. **(c)** $m+\frac{1}{m-2}=4$

$m^2-2m-3=0$

$(m-3)(m+1)=0$

$m=3$

$m-2=1$

Now $(m-2)^2+\frac{1}{(m-2)^2}$

$=1^2+\frac{1}{1^2}=2$

10. **(a)** $x^2+y^2+\frac{1}{x^2}+\frac{1}{y^2}-4=0$

$\Rightarrow x^2+\frac{1}{x^2}-2+y^2+\frac{1}{y^2}-2=0$

$\Rightarrow \left(x-\frac{1}{x}\right)^2+\left(y-\frac{1}{y}\right)^2=0$

$\Rightarrow x-\frac{1}{x}=0$

$\Rightarrow x^2-1=0\Rightarrow x=1$

Similarly,

$y=1$

$\therefore x^2+y^2=1+1=2$

11. **(c)** Given, $x+y=1$

Then, $\left(x+\frac{1}{x}\right)^2+\left(y+\frac{1}{y}\right)^2=x^2+y^2+\frac{1}{x^2}+\frac{1}{y^2}+4$

Minimum value of x^2+y^2 occur when $x=y$

$[\because x+y=1]$

Put $x = y = \frac{1}{2}$

Minimum value $= \left(\frac{5}{2}\right)^2 + \left(\frac{5}{2}\right)^2 = \frac{25}{2} = 12.5$

12. **(b)** We have $\sqrt{\frac{x}{y}} + \sqrt{\frac{y}{x}} = \frac{5}{2}$ ...(1)

and $x + y = 10$ ...(2)

Now, $\sqrt{\frac{x}{y}} + \sqrt{\frac{y}{x}} = \frac{5}{2} \Rightarrow \frac{x+y}{\sqrt{xy}} = \frac{5}{2}$

$\Rightarrow \frac{10}{\sqrt{xy}} = \frac{5}{2}$ [using eq. (2)]

$\Rightarrow \sqrt{xy} = 4 \Rightarrow xy = 16$

Thus, the given system of simultaneous equations reduces to

$x + y = 10$ and $xy = 16$

$\Rightarrow y = 10 - x$

and $xy = 16$

$\Rightarrow x(10 - x) = 16$

$\Rightarrow x^2 - 10x + 16 = 0$

$\Rightarrow (x-2)(x-8) = 0 \Rightarrow x = 2$ or $x = 8$

Now, $x = 2$ and $x + y = 10 \Rightarrow y = 8$

and $x = 8$ and $x + y = 10 \Rightarrow y = 2$

Hence, the required solution are $x = 2, y = 8$ and $x = 8, y = 2$

13. **(a)** If roots of an equation $ax^2 + bx + c = 0$ are positive, then signs of a and c should be like.

14. **(c)** Let α and β be the roots of $x^2 - (p-2)x - (p+1) = 0$

Then, $\alpha + \beta = p - 2$

and $\alpha\beta = -(p+1)$

$\therefore \alpha^2 + \beta^2 = 5$

$\Rightarrow (\alpha+\beta)^2 - 2\alpha\beta = 5$

$\Rightarrow (p-2)^2 + 2(p+1) = 5$

$\Rightarrow p^2 - 4p + 4 + 2p + 2 = 5$

$\Rightarrow p^2 - 2p + 1 = 0 \Rightarrow (p-1)^2 = 0$

$\Rightarrow p = 1$

15. **(c)** $\because$ α and β are the roots of the equation $x^2 + 6x + 1 = 0$

$\therefore \alpha + \beta = -6$ and $\alpha\beta = 1$

Now, $(\alpha - \beta)^2 = (\alpha+\beta)^2 - 4\alpha\beta$

$= (-6)^2 - 4$

$= 36 - 4 = 32$

$\Rightarrow |\alpha - \beta| = \sqrt{32} = 4\sqrt{2}$

16. **(a)** The given root is

$= \frac{1}{2 - \sqrt{-2}} = \frac{2 + \sqrt{2}i}{6}$

$\therefore$ Another root $= \frac{2 - \sqrt{2}i}{6}$

Now, find sum and product of the roots and put in $x^2 -$ (sum of the roots) x + (multiplication of the roots) = 0

17. **(b)** $S = \frac{1}{\alpha+\beta} + \frac{\alpha+\beta}{\alpha\beta} = -\frac{a}{b} - \frac{b}{c} = -\frac{(ac+b^2)}{bc}$

$P = \frac{1}{\alpha+\beta} \cdot \frac{\alpha+\beta}{\alpha\beta}$

$= \frac{1}{\alpha\beta} = \frac{a}{c}$

Put the values of P and S in $x^2 - Sx + P = 0$, we get the required result.

18. **(a)** Dividing the equation $a^3x^2 + abcx + c^3 = 0$ by c^2, we get

$a\left(\frac{ax}{c}\right)^2 + b\left(\frac{ax}{c}\right) + c = 0$

$\Rightarrow \frac{ax}{c} = \alpha, \beta$

$\Rightarrow x = \frac{c}{a}\alpha, \frac{c}{a}\beta$

$\Rightarrow x = \alpha^2\beta, \alpha\beta^2$

[$\because \frac{c}{a} = \alpha\beta$ = product of roots]

Hence, $\alpha^2\beta$ and $\alpha\beta^2$ are the roots of the equation $a^3x^2 + abcx + c^3 = 0$.

19. **(b)** Let the natural number be $= x$.

By the given condition: $x + 12 = \frac{160}{x} (x \neq 0)$

$\Rightarrow x^2 + 12x - 160 = 0 \Rightarrow x = -\frac{12 \pm \sqrt{144 + 640}}{2}$

$= -\frac{12 \pm \sqrt{784}}{2} = \frac{-12 \pm 28}{2} = -\frac{40}{2}$ or $\frac{16}{2}$

$= -10$ or 5. But x is a natural number $\therefore x = 5$.

20. **(d)** $\frac{1}{a+b+x} = \frac{1}{a} + \frac{1}{b} + \frac{1}{x} \Rightarrow \frac{1}{a+b+x} - \frac{1}{x} = \frac{1}{a} + \frac{1}{b}$

$\Rightarrow \frac{x - a - b - x}{(a+b+x)x} = \frac{b+a}{ab}$

$\Rightarrow \frac{-(a+b)}{(x^2 + ax + bx)} = \frac{(a+b)}{ab}$

$\Rightarrow \frac{-1}{x^2 + ax + bx} = \frac{1}{ab}$

$\Rightarrow x^2 + ax + bx = -ab \Rightarrow x(x+a) + b(x+a) = 0$

$\Rightarrow (x+a)(x+b) = 0 \Rightarrow x = -a$ or $x = -b$

21. **(d)** (a) is clearly true.

(b) $x^2 + 1$ is a quadratic polynomial which has no real value of x for which $x^2 + 1$ is zero.

$[\therefore x^2 \geq 0 \Rightarrow x^2 + 1 > 0$ for all real $x]$ $\therefore$ (b) is true.

(c) The quadratic polynomial

$x^2 - 2x + 1 = (x-1)^2$ has only one zero i.e. 1

$\therefore$ (c) is true.

$[\because (x-1)^2 > 0$ at $x \neq 1$ and for $x = 1, (x-1)^2 = 0]$

(d) is false [$\because$ of (b), (c)]

Hence (d) holds.

22. **(c)** For a, b negative the given expression will always be positive since, a^2, b^2 and ab are all positive.

23. **(c)** $(c + 6) = 1/2 \times 2(2c - 1) \Rightarrow c + 6 = 2c - 1 \Rightarrow c = 7$

24. **(c)** Assume any equation:

Say $x^2 - 5x + 6 = 0$

The roots are 2, 3.

We are now looking for the equation, whose roots are: $(2 + 1/3) = 2.33$ and $(3 + 1/2) = 3.5$.

Also $a = 1$, $b = -5$ and $c = 6$.

Put these values in each option to see which gives 2.33 and 3.5 as its roots.

25. **(c)** $f(x) = x^2 + ax + b$

$f(1) = f(-1) = 5$

$\Rightarrow \quad a + b = -a + b = 5$

$\Rightarrow \quad a = 0, b = 5$

26. **(a)** For both the roots: (α, β) to be positive

$\alpha + \beta > 0$ and $\alpha\beta > 0$

$$\Rightarrow \quad \frac{-b}{a} > 0 \text{ and } \frac{c}{a} > 0$$

i.e., b and a are of opposite sign and c and a are of same sign.

27. **(b)** Given quadratic equation is $x^2 - bx + 1 = 0$

It has no real roots. It means, equation has imaginary roots.

Which is possible when $B^2 - 4AC < 0$

Here, $B = -b$, $A = 1$, $C = 1$

$$\Rightarrow \quad b^2 - 4 < 0 \quad \Rightarrow \quad b^2 < 4 \quad \Rightarrow \quad -2 < b < 2$$

28. **(b)** The given equation is,

$3x^2 - 5x + p = 0$

We have, $a = 3$, $b = -5$, $c = p$

$D = b^2 - 4ac = 25 - 12p$

For Real and unequal, $D > 0$

$\therefore 25 - 12p > 0$

$$\Rightarrow 25 > 12p \Rightarrow p < \frac{25}{12}$$

29. **(b)** Let roots are $(n-1)$, n and $(n+1)$

Sum of the roots $= b$

$(n-1)n + n(n+1) + (n+1)(n-1) = b$

$\Rightarrow \quad n^2 - n + n^2 + n + n^2 - 1 = b$

$\Rightarrow \quad 3n^2 - 1 = b$

The value of b will be minimum when the value of n^2 is minimum i.e., $n^2 = 0$

Hence, minimum value of $b = -1$.

30. **(b)** Since, α, β are root of the equation

$2x^2 - 3x - 6 = 0$

$$\therefore \quad \alpha + \beta = \frac{3}{2} \text{ and } \alpha\beta = -3$$

$$\Rightarrow \quad \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$$

$$= \frac{9}{4} + 6 = \frac{33}{4}$$

Now, $(\alpha^2 + 2) + (\beta^2 + 2) = (\alpha^2 + \beta^2) + 4$

$$= \frac{33}{4} + 4 = \frac{49}{4}$$

and $(\alpha^2 + 2)(\beta^2 + 2) = \alpha^2\beta^2 + 2(\alpha^2 + \beta^2) + 4$

$$= (-3)^2 + 2\left(\frac{33}{4}\right) + 4 = \frac{59}{2}$$

So, the equation whose roots are $\alpha^2 + 2$ and $\beta^2 + 2$ is

$x^2 - x\{(\alpha^2 + 2) + (\beta^2 + 2)\} + (\alpha^2 + 2)(\beta^2 + 2) = 0$

$$\Rightarrow \quad x^2 - \frac{49}{4}x + \frac{59}{2} = 0$$

$\Rightarrow \quad 4x^2 - 49x + 118 = 0$

31. **(b)** Let first square has side x, $\therefore$ Area $= x^2$, Perimeter $= 4x$ and let second square has side y,

$\therefore$ Area $= y^2$, Perimeter $= 4y$

Let $x > y$ so that $4x > 4y$

Given, $x^2 + y^2 = 468$ ...(1)

and $4x - 4y = 24 \Rightarrow x - y = 6 \Rightarrow y = x - 6$ ...(2)

Using (2) in (1), we get $x^2 + (x-6)^2 = 468$

$\Rightarrow \quad x^2 + x^2 - 12x + 36 = 468 \Rightarrow 2x^2 - 12x - 432 = 0$

$$\Rightarrow \quad x^2 - 6x - 216 = 0 \Rightarrow x = \frac{6 \pm \sqrt{36 + 864}}{2} = \frac{6 \pm \sqrt{900}}{2}$$

$$= \frac{6 \pm 30}{2} = \frac{36}{2}, \frac{-24}{2} = 18, -12$$

But x being length cannot be negative $\therefore x = 18$

put $x = 18$ in (2), we get $y = x - 6 = 18 - 6 = 12$

$\therefore \quad$ sides of the two squares $= x, y = 18$ m, 12 m

32. **(a)** Let Puneet's age $= x$ yrs.

Let Puneet's father age $= y$ yr.

$x + y = 45 \Rightarrow y = (45 - x)$

$xy = 126$

Putting the value of y.

$(x)(45 - x) = 126$

$45x - x^2 = 126$

$x^2 - 45x + 126 = 0$

$x^2 - 42x - 3x + 126 = 0$

$x(x - 42) - 3(x - 42) = 0$

$x = 3, x = 42$

Hence, Puneet's age in 3yrs.