PROBABILITY

CONCEPT OF PROBABILITY

If you go to buy 10 kg of sugar at ₹ 40 per kg, you can easily find the exact price of your purchase is ₹ 400. On the other hand, the shopkeeper may have a good estimate of the number of kg of sugar that will be sold during the day, but it is impossible to predict the exact amount, because the number of kg of sugar that the consumers will purchase during a day is random.

There are various phenomenon in nature, leading to an outcome, which cannot be predicted in advance. For example, we cannot exactly predict that (i) a head will occur on tossing a coin, (ii) a student will clear the CAT, (iii) India will win the cricket match against Pakistan, etc. But we can measure the amount of certainty of occurrence of an outcome of a phenomenon. This amount of certainty of occurrence of an outcome of a phenomenon is called probability. For example, on tossing a coin certainty of occurrence of each of a head and a tail are the same. Hence amount certainty of occurrence of each of a head and a tail is 50% i.e., $\frac{50}{100} = \frac{1}{2}$. Therefore $\frac{1}{2}$ is the amount of certainty of occurrence of a head (or a tail) on tossing a coin and hence $\frac{1}{2}$ is the probability of occurrence of a head (or a tail) on tossing a coin. On throwing a dice (a dice is a cuboid having one of the numbers 1, 2, 3, 4, 5 and 6 on each of its six faces) certainty of occurrence of each of the numbers 1, 2, 3, 4, 5 and 6 on its top face are the same.

Therefore certainty of occurrence of each of the numbers 1, 2, 3, 4, 5 and 6 is $\frac{1}{6}$.

Therefore $\frac{1}{6}$ is the amount of certainty of occurrence of each of the numbers 1, 2, 3, 4, 5 or 6 on the top face of the dice on throwing the dice and hence $\frac{1}{6}$ is the probability of occurrence of each of the numbers 1, 2, 3, 4, 5, or 6 on the top face of the dice on tossing a dice is $\frac{1}{6}$.

BASIC TERMS

1. **An Experiment:** An action or operation resulting in two or more outcomes is called an experiment. For examples
   (i) Tossing of a coin is an experiment because there are two possible outcomes head and tail.
   (ii) Drawing a card from a pack of 52 cards is an experiment because there are 52 possible outcomes.
2. **Sample Space:** The set of all possible outcomes of an experiment is called the sample space, denoted by S. An element of S is called a sample point. For examples
   (i) In the experiment of tossing a coin, the sample space has two points corresponding to head (H) and Tail (T) i.e., $S\{H, T\}$.
   (ii) When we throw a dice then any one of the numbers 1, 2, 3, 4, 5 and 6 will come up. So the sample space,
   $S = \{1, 2, 3, 4, 5, 6\}$
3. **An Event:** Any subset of a sample space is an event. For example,
   If we throw a dice then $S = \{1, 2, 3, 4, 5, 6\}$
   Then $A = \{1, 3, 5\}$, $B\ \{2, 4, 6\}$, the null set ϕ and S itself are some events of S, because they all are subsets of set S.
4. **Impossible Event:** The null set ϕ is called the impossible event or null event. For example,
   Getting 7 when a dice is thrown is an impossible or a null event.
5. **Sure Event:** The entire sample space is called sure or certain event. For example,
   Here the event:
   Getting an odd or even number on throwing a dice is a sure event, because the event = S.
6. **Complement of an Event:** The complement of an event A is denoted by $\overline{A}$, A' or A^c, is the set of all sample points of the sample space other than the sample points in A. For example, in the experiment of tossing a fair dice,
   $S = \{1, 2, 3, 4, 5, 6\}$ If $A = \{1, 3, 5, 6\}$, then $A^c = \{2, 4\}$
   Note that $A \cup A^c = S$, $A \cap A^c = \phi$.

7. Simple (or Elementary) Event: An event is called a simple event if it is a singleton subset of the sample space S. The singleton subset means the subset having only one element. For example,

(i) When a coin is tossed, sample space $S = \{H, T\}$
Let $A = \{H\}$ = the event of occurrence of head and $B = \{T\}$ = the event of occurrence of tail.
Here A and B are simple events.

(ii) When a dice is thrown then sample space,
$S = \{1, 2, 3, 4, 5, 6\}$
Let $A = \{5\}$ = the event of occurrence of 5
$B = \{2\}$ = the event of occurrence of 2
Here A and B are simple events.

8. Compound Event: It is the joint occurrence of two or more simple events. For example,
The event of at least one head appears when two fair coins are tossed is a compound event,
$A = \{HT, TH, HH\}$

9. Equally Likely Events: A number of simple events are said to be equally likely if there is no reason for one event to occur in preference to any other event. For example,
In drawing a card from a well shuffled pack of 52 cards, there are 52 outcomes and hence 52 simple events which are equally likely because there is no reason for one event to occur in preference to any other event. For example,

MATHEMATICAL DEFINITION OF PROBABILITY

If an event A consists of m sample points of a sample space S having n elements $(0 \le m \le n)$, then the probability of occurrence of event A, denoted by $P(A)$ is defined to be $\frac{m}{n}$ i.e., $P(A) = \frac{m}{n}$

$$\because \quad 0 \le m \le n \Rightarrow 0 \le \frac{m}{n} \le 1 \Rightarrow 0 \le P(A) \le 1$$

If the event A has m elements, then A' has $(n - m)$ elements.

$$\therefore \quad P(A') = \frac{n-m}{n} = 1 - \frac{m}{n} = 1 - P(A)$$

Let $S = \{a_1, a_2, ..., a_n\}$ be the sample space

$$P(S) = \frac{n}{n} = 1, \text{ corresponding to the certain event.}$$

$$P(\phi) = \frac{0}{n} = 0, \text{ corresponding to the null event } \phi$$
(or impossible event)

If $A_i = \{a_i\}$, $i = 1, 2, ...$ or n; then A_i is the event corresponding to a single sample point a_i, then $P(A_i) = \frac{1}{n}$.

Illustration 1: Two dice are thrown at a time. Find the probability of the followings:
(i) the numbers shown are equal
(ii) the difference of numbers shown is 1

Solution: The sample space in a throw of two dice
$S = \{(1, 1), (1, 2), ..., (1, 6), (2, 1), (2, 2), ..., (2, 6), (3, 1), ..., (3, 6), (4, 1), ..., (4, 6), (5, 1), ..., (5, 6), (6, 1), ..., (6, 6)\}$
$\therefore$ total no. of outcomes, $n(S) = 36$

(i) Here E_1 = the event of showing equal number on both dice
$= \{(1, 1) (2, 2) (3, 3) (4, 4) (5, 5) (6, 6)\}$

$$\therefore n(E_1) = 6, \Rightarrow P(E_1) = \frac{n(E_1)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

(ii) Here E_2 = the event of showing numbers whose difference is 1.
$= \{(1, 2) (2, 1) (2, 3) (3, 2) (3, 4) (4, 3) (4, 5) (5, 4) (5, 6) (6, 5)\}$

$$\therefore \ n(E_2) = 10, \Rightarrow P(E_2) = \frac{n(E_2)}{n(S)} = \frac{10}{36} = \frac{15}{18}.$$

Illustration 2: If three cards are drawn from a pack of 52 cards, what is the chance that all will be queen?

Solution: If the sample space be S, then $n(S)$ = the total number of ways of drawing 3 cards out of 52 cards = ${}^{52}C_3$
Now, if A = the event of drawing three queens, then
$n(A) = {}^4C_3$

$$\therefore \quad P(E) = \frac{n(A)}{n(S)} = \frac{{}^4C_3}{{}^{52}C_3} = \frac{4}{\frac{52 \times 51 \times 50}{3 \times 2}} = \frac{1}{5525}$$

Note that in a pack of playing cards,
Total number of cards: 52(26 red, 26 black)
Four suits: Heart, Diamond, Spade, Club-13 cards of each suit
Court number of cards: 12(4 kings, 4 queens, 4 jacks)
Face number of cards: 16(4 aces, 4 kings, 4 queens, 4 jacks)

Illustration 3: Words are formed with the letters of the word PEACE. Find the probability that 2 E's come together.

Solution: Total number of words which can be formed with the letters of the word P E A C E $= \frac{5!}{2!} = 60$
Number of words in which 2 E's come together = 4! = 24

$$\therefore \quad \text{Required prob.} = \frac{24}{60} = \frac{2}{5}.$$

Illustration 4: *A* and *B* play a game where each is asked to select a number from 1 to 25. If the two numbers match, both of them win a prize. The probability that they will not win a prize in a single trial is

(a) $\frac{1}{25}$ **(b)** $\frac{24}{25}$
(c) $\frac{2}{25}$ **(d) None of these**

Solution: (b) Total number of possibilities = 25 × 25
Favourable cases for their winning = 25

$$\therefore P(\text{they win a prize}) = \frac{25}{25 \times 25} = \frac{1}{25}$$

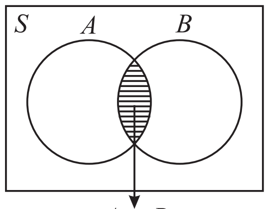
$$\therefore P(\text{they will not win a prize}) = 1 - \frac{1}{25} = \frac{24}{25}$$

ADDITION THEOREM

If A and B are any events in S, then

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

i.e., $$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$



$A \cap B$

For three events A, B and C in S, we have

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C).$$

Special Addition Rule

If A, B, and C are mutually exclusive, then $P(A \cap B)$, $P(B \cap C)$, $P(C \cap A)$, $P(A \cap B \cap C) = 0$, hence $P(A \cup B) = P(A) + P(B)$ and $P(A \cup B \cup C) = P(A) + P(B) + P(C)$

Illustration 5: A bag contains 6 white, 5 black and 4 red balls. Find the probability of getting either a white or a black ball in a single draw.

Solution: Let A = Event that we get a black ball

Two events A and B are mutually exclusive.

$$P(A) = \frac{^6C_1}{^{15}C_1} = \frac{6}{15},\ P(B) = \frac{^5C_1}{^{15}C_1} = \frac{5}{15}$$

So, $$P(A \cup B) = P(A) + P(B) = \frac{6}{15} + \frac{5}{15} = \frac{11}{15}.$$

Illustration 6: One digit is selected from first 20 positive integers. What is the probability that it is divisible by 3 or 4.

Solution:

Let A = Event that the selected number is divisible by 3

B = Event that the selected number is divisible by 4

Here, the events A and B are not mutually exclusive because 12 is divisible by both 3 and 4.

$$P(A) = \frac{6}{20},\ P(B) = \frac{5}{20},\ P(A \cap B) = \frac{1}{20}$$

$$\therefore\ P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{6}{20} + \frac{5}{20} - \frac{1}{20} = \frac{10}{20} = \frac{1}{2}.$$

Illustration 7: The probability that at least one of the events A and B occurs is 0.7 and they occur simultaneously with probability 0.2. Then $P(\bar{A}) + P(\bar{B}) =$

(a) 1.8 **(b)** 0.6

(c) 1.1 **(d)** 0.4

Solution: (c) We have $P(A \cup B) = 0.7$ and $P(A \cap B) = 0.2$

Now, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$\Rightarrow P(A) + P(B) = 0.9 \Rightarrow 1 - P(\bar{A}) + 1 - P(\bar{B}) = 0.9$$

$$\Rightarrow P(\bar{A}) + P(\bar{B}) = 1.1$$

INDEPENDENT EVENTS

Two or more events are said to be independent if occurrence or non-occurrence of any of them does not influence the probability of occurrence or non-occurrence of other events.

For example, when two cards are drawn from a pack of 52 playing cards with replacement (i.e., the first card drawn is put back in the pack and then the second card is drawn), then the event of occurrence of a king in the first draw and the event of occurrence of a king in the second draw are independent events because the occurrence or non-occurrence of a king in first draw does not influence the probability of occurrence or non-occurrence of the king in second draw. You can also see that the probability of drawing a king in the second draw is $\frac{4}{52}$ whether a king is drawn in the first draw or not. But if the two cards are drawn without replacement, then the two events are not independent, because in this case probability of drawing a king in the second draw depends on weather a king is drawn in first draw or not. If a king is drawn in first draw, then probability of drawing a king in second draw will be $\frac{3}{51}$ but if a king is not drawn in first draw, then the probability of drawing a king in second draw will be $\frac{4}{51}$.

Illustration 8: A fair coin is tossed repeatedly. If the tail appears on first four tosses, then the probability of the head appearing on the fifth toss equals

(a) $\frac{1}{2}$ **(b)** $\frac{1}{32}$

(c) $\frac{31}{32}$ **(d)** $\frac{1}{5}$

Solution: (a) The event that the fifth toss results a head is independent of the event that the first four tosses results tails.

$\therefore$ Probability of the required event = 1/2.

CONDITIONAL PROBABILITY

Let A and B be two events associated with a random experiment. Then, the probability of occurrence of A under the condition that B has already occurred and $P(B) \neq 0$, is called the conditional probability of occurrence of A when B has already occurred and it is denoted by $P(A/B)$.

Thus, $P(A/B)$ = Probability of occurrence of A, if B has already occurred and $P(B) \neq 0$

$$= \frac{P(A \cap B)}{P(B)} = \frac{\frac{n(A \cap B)}{n(S)}}{\frac{n(B)}{n(S)}} = \frac{n(A \cap B)}{n(B)}$$

Similarly, $P(B/A)$ = Probability of occurrence of B, if A has already occurred and $P(B) \neq 0$

$$= \frac{P(A \cap B)}{P(A)} = \frac{n(A \cap B)}{n(A)}$$

1. Multiplication Theorem on Probability

If A and B are two events associated with a random experiment, then

$$P(A \cap B) = P(A).\,P(B/A), \text{ if } P(A) \neq 0$$

or $$P(A \cap B) = P(B).\,P(A/B), \text{ if } P(B) \neq 0.$$

2. Multiplication Theorem for Independent Events

If A and B are independent events associated with a random experiment, then $P(A/B) = P(A)$ and $P(B/A) = P(B)$

$\therefore \quad P(A \cap B) = P(A) \cdot P(B/A) = P(A).\,P(B)$

i.e., the probability of simultaneous occurrence of two independent events is equal to the product of probability of their individual occurrence.

Extension of multiplication theorem for independent events

If $A_1, A_2, ..., A_n$ are independent events associated with a random experiment, then

$$P(A_1 \cap A_2 \cap A_3 \cap ... \cap A_n) = P(A_1)\,P(A_2)\,...\,P(A_n).$$

3. Probability of Occurrence of at Least One of the *n* Independent Events

If $p_1, p_2, p_3, ..., p_n$ be the probabilities of occurrence of n independent events $A_1, A_2, A_3, ..., A_n$ respectively, then

(i) Probability of happening none of them

$$= P(\overline{A}_1 \cap \overline{A}_2 \cap \overline{A}_3 \,..., \cap A_n)$$
$$= P(\overline{A}_1)\,P(\overline{A}_2) \cdot P(\overline{A}_3) \,...\, P(\overline{A}_n)$$
$$= (1-p_1)(1-p_2)(1-p_3)\,...\,(1-p_n)$$

(ii) Probability of happening at least one of them

$$= P(A_1 \cup A_2 \cup A_3 \,...\, \cup A_n)$$
$$= 1 - P(A_1 \cup A_2 \cup ... \cup A_n)$$
$$= 1 - P(\overline{A}_1 \cap \overline{A}_2 \cap \overline{A}_3 \,...\, \cap \overline{A}_n)$$
$$= 1 - P(\overline{A}_1)\,P((\overline{A}_2)\,P(\overline{A}_3)\,...\,P(\overline{A}_n)$$
$$= 1 - (1-p_1)(1-p_2)(1-p_3)\,...\,(1-p_n)$$

Illustration 9: A man and his wife appear for an interview for two posts. The probability of the husband's selection is $\frac{1}{7}$ and that of the wife's selection is $\frac{1}{5}$. The probability that only one of them will be selected is

(a) $\frac{6}{7}$ **(b)** $\frac{4}{35}$

(c) $\frac{6}{35}$ **(d)** $\frac{2}{7}$

Solution: (d) Probability that only husband is selected

$$= P(H)\,P(\overline{W}) = \frac{1}{7}\left(1-\frac{1}{5}\right) = \frac{1}{7} \times \frac{4}{5} = \frac{4}{35}$$

Probability that only wife is selected

$$= P(\overline{H})\,P(W) = \left(1-\frac{1}{7}\right)\left(\frac{1}{5}\right) = \frac{6}{7} \times \frac{1}{5} = \frac{6}{35}$$

$\therefore$ Probability that only one of them is selected

$$= \frac{4}{35} + \frac{6}{35} = \frac{10}{35} = \frac{2}{7}$$

Illustration 10: A bag contains 4 red and 4 blue balls. Four balls are drawn one by one from the bag, then find the probability that the drawn balls are in alternate colour.

Solution:

E_1 : Event that first drawn ball is red, second is blue and so on.

E_2 : Event that first drawn ball is blue, second is red and so on.

$$\therefore \quad P(E_1) = \frac{4}{8} \times \frac{4}{7} \times \frac{3}{6} \times \frac{3}{5} \quad \text{and} \quad P(E_2) = \frac{4}{8} \times \frac{4}{7} \times \frac{3}{6} \times \frac{3}{5}$$

$$P(E) = P(E_1) + P(E_2) = 2 \times \frac{4}{8} \cdot \frac{4}{7} \cdot \frac{3}{6} \cdot \frac{3}{5} = \frac{6}{35}$$

Illustration 11: A bag contains 5 red and 4 green balls. Four balls are drawn at random then find the probability that two balls are of red colour and two balls are of green.

Solution:

$n(S)$ = The total number of ways of drawing 4 balls out of total 9 balls = ${}^9C_4$.

If A_1 = The event of drawing 2 red balls out of 5 red balls then $n(A_1) = {}^5C_2$.

A_2 = The event of drawing 2 green balls out of 4 greens balls then $n(A_2) = {}^4C_2$.

Let A = The event of drawing 2 balls are of red colour and 2 balls are of green colour.

$\therefore \quad n(A) = n(A_1)\,.\,n(A_2) = {}^5C_2 \times {}^4C_2$

$$\therefore \quad P(A) = \frac{n(A)}{n(S)} = \frac{{}^5C_2 \times {}^4C_2}{{}^9C_4} = \frac{\dfrac{5 \times 4 \times 4 \times 3}{2 \times 2}}{\dfrac{9 \times 8 \times 7 \times 6}{4 \times 3 \times 2}} = \frac{10}{21}$$

Illustration 12: Let *A*, *B*, *C* be 3 independent events such that $P(A) = \frac{1}{3}$, $P(B) = \frac{1}{2}$, $P(C) = \frac{1}{4}$. Then find the probability of exactly 2 events occurring out of 3 events.

Solution: P (exactly two of A, B, C occur)

$$= P(A \cap B) + P(B \cap C) + P(C \cap A) - 3P(A \cap B \cap C)$$
$$= P(A)\,.\,P(B) + P(B)\,.\,P(C) + P(C)\,.\,P(A) - 3P(A)\,.\,P(B)\,.\,P(C)$$
$$= \frac{1}{3} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{4} + \frac{1}{4} \cdot \frac{1}{3} - 3\frac{1}{3} \cdot \frac{1}{2} \cdot \frac{1}{4} = \frac{1}{4}.$$

Illustration 13: A bag contains 3 red, 6 white and 7 blue balls. Two balls are drawn one by one. What is the probability that first ball is white and second ball is blue when first drawn ball is not replaced in the bag?

Solution:

Let A = Event of drawing a white ball in first draw

and B = Event of drawing a blue ball in second draw

Here A and B are dependent events.

$$P(A) = \frac{6}{16},\ P\left(\frac{B}{A}\right) = \frac{7}{15}$$

$$P(A \cap B) = P(A)\,.\,P\left(\frac{B}{A}\right) = \frac{6}{16} \times \frac{7}{15} = \frac{7}{40}.$$

Illustration 14: Three coins are tossed together. What is the probability that first shows head, second shows tail and third shows head?

Solution: Let A = The event first coin shows head

B = The event that second coin shows tail

C = The event that third coin shows head

These three events are mutually independent.

So, $P(A \cap B \cap C) = P(A) \cdot P(B) \cdot P(C) = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8}$.

Illustration 15: A problem of mathematics is given to three students A, B, and C; whose chances of solving it are 1/2, 1/3, 1/4 respectively. Then find the probability that the problem will be solved.

Solution: Obviously the events of solving the problem by A, B and C are independent.

The problem will be solved if at least one of the three students will solve the problem.

Therefore required probability

$$= 1 - \left[\left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{4}\right)\right] = 1 - \frac{1}{2} \cdot \frac{2}{3} \cdot \frac{3}{4} = \frac{3}{4}$$

Illustration 16: Two dice are thrown simultaneously. Find the probability that the sum of the number appeared on two dice is 8, if it is known that the second dice always exhibits 4.

Solution: Let A be the event of occurrence of 4 always on the second

dice = {(1, 4), (2, 4), (3, 4), (4, 4), (5, 4), (6, 4)}, $\therefore\ n(A) = 6$ and B be the event of occurrences of such numbers on both dice whose sum is 8 = {(2, 6), (3, 5), (4, 4), (5, 3), (6, 2)}

Thus, $A \cap B = \{(4, 4)\}$

$\therefore\quad n(A \cap B) = 1$

$$\therefore\quad P\left(\frac{B}{A}\right) = \frac{n(A \cap B)}{n(A)} = \frac{1}{6}.$$

Illustration 17: A coin is tossed thrice. If E be the event of showing at least two heads and F be the event of showing head in the first throw, then find $P\left(\frac{E}{F}\right)$.

Solution:

$S = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$

$E = \{HHH, HHT, HTH, THH\}$

$F = \{HHH, HHT, HTH, HTT\}$

$E \cap F = \{HHH, HHT, HTH\}$

$n(E \cap F) = 3,\ n(F) = 4$

$$\therefore \text{ Reqd prob.} = P\left(\frac{E}{F}\right) = \frac{n(E \cap F)}{n(F)} = \frac{3}{4}.$$

Practice Exercise

Level - I

1. Two dice are thrown simultaneously. The probability of obtaining a total score of seven is

(a) $\frac{1}{6}$ (b) $\frac{1}{3}$

(c) $\frac{2}{7}$ (d) $\frac{5}{6}$

2. Four balls are drawn at random from a bag containing 5 white, 4 green and 3 black balls. The probability that exactly two of them are white is

(a) $\frac{14}{33}$ (b) $\frac{7}{16}$

(c) $\frac{18}{33}$ (d) $\frac{9}{16}$

3. Two dice are tossed. The probability that the total score is a prime number is :

(a) $\frac{1}{6}$ (b) $\frac{5}{12}$

(c) $\frac{1}{2}$ (d) $\frac{7}{9}$

4. Anil can kill a bird once in 3 shots. On the assumption that he fires 3 shots, find the probability that the bird is killed.

(a) $\frac{1}{3}$ (b) $\left(\frac{1}{3}\right)^3$

(c) $\frac{19}{27}$ (d) $\frac{8}{9}$

5. If A and B are two independent events with $P(A) = 0.6$, $P(B) = 0.3$, then $P(A' \cap B')$ is equal to :

(a) 0.18 (b) 0.28

(c) 0.82 (d) 0.72

6. The probabilities that A and B will die within a year are p and q respectively, then the probability that only one of them will be alive at the end of the year is -

(a) $p+q$ (b) $p+q-pq$

(c) $p+q+pq$ (d) $p+q-2pq$

7. A pair of dice is thrown thrice. The probability of throwing doublets at least once is

(a) $\frac{1}{36}$ (b) $\frac{25}{216}$

(c) $\frac{125}{216}$ (d) None of these

8. The probability of getting number 5 in throwing a dice is

(a) 1 (b) $\frac{1}{3}$

(c) $\frac{1}{6}$ (d) $\frac{5}{6}$

9. The probability of getting head and tail alternately in three throws of a coin (or a throw of three coins), is

(a) $\frac{1}{8}$ (b) $\frac{1}{4}$

(c) $\frac{1}{3}$ (d) $\frac{3}{8}$

10. A die is thrown once. What is the probability of occurrence of an odd number on the upper face?

(a) $\frac{2}{3}$ (b) $\frac{1}{2}$

(c) $\frac{1}{4}$ (d) $\frac{1}{8}$

11. A die is thrown once. Find the probability that 3 or greater than 3 turns up.

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$

(c) $\frac{1}{4}$ (d) $\frac{2}{3}$

12. Find the probability of getting a multiple of 2 in the throw of a die.

(a) 1/2 (b) 1/4

(c) 1/3 (d) 1/6

13. India and Pakistan play a 5 match test series of hockey, the probability that India wins at least three matches is

(a) $\frac{1}{2}$ (b) $\frac{3}{5}$

(c) $\frac{4}{5}$ (d) None of these

14. The probability that a man can hit a target is 3/4. He tries 5 times. The probability that he will hit the target at least three times is

(a) $\frac{291}{364}$ (b) $\frac{371}{461}$

(c) $\frac{471}{502}$ (d) $\frac{459}{512}$

15. From eighty cards numbered 1 to 80, two cards are selected randomly. The probability that both the cards have the numbers divisible by 4 is given by

(a) $\frac{21}{316}$ (b) $\frac{19}{316}$

(c) $\frac{1}{4}$ (d) None of these

16. The probability of getting sum more than 7 when a pair of dice are thrown is

(a) $\frac{7}{36}$ (b) $\frac{5}{12}$

(c) $\frac{7}{12}$ (d) None of these

17. Two dice are thrown simultaneously then the probability of obtaining a total score of 5 is

(a) $\frac{1}{18}$ (b) $\frac{1}{12}$

(c) $\frac{1}{9}$ (d) None of these

18. The probability that the two digit number formed by digits 1, 2, 3, 4, 5 is divisible by 4 is

(a) $\frac{1}{30}$ (b) $\frac{1}{20}$

(c) $\frac{1}{5}$ (d) None of these

19. Probability of throwing 16 in one throw with three dice is

(a) $\frac{1}{36}$ (b) $\frac{1}{18}$

(c) $\frac{1}{72}$ (d) $\frac{1}{9}$

20. Of a total of 600 bolts, 20% are too large and 10% are too small. The remainder are considered to be suitable. If a bolt is selected at random, the probability that it will be suitable is

(a) $\left(\frac{1}{5}\right)$ (b) $\left(\frac{7}{10}\right)$

(c) $\left(\frac{1}{10}\right)$ (d) $\left(\frac{3}{10}\right)$

21. The probability that in the toss of two dice we obtain the sum 7 or 11 is

(a) $\frac{1}{6}$ (b) $\frac{1}{18}$

(c) $\frac{2}{9}$ (d) $\frac{23}{108}$

22. A card is drawn at random from a pack of 100 cards numbered 1 to 100. The probability of drawing a number which is a square, is

(a) $\frac{1}{10}$ (b) $\frac{1}{100}$

(c) $\frac{9}{10}$ (d) $\frac{90}{100}$

23. The alphabets of word ALLAHABAD are arranged at random. The probability that in the words so formed, all identical alphabets are found together, is

(a) 1/63 (b) 16/17

(c) 5!/9! (d) None of these

24. The probability that Krishna will be alive 10 years hence, is $\frac{7}{15}$ and that Hari will be alive is $\frac{7}{10}$. What is the probability that both Krishna and Hari will be dead 10 years hence ?

(a) $\frac{21}{150}$ (b) $\frac{24}{150}$

(c) $\frac{49}{150}$ (d) $\frac{56}{150}$

25. The probability that in the random arrangement of the letters of the word 'UNIVERSITY', the two I's does not come together is

(a) $\frac{4}{5}$ (b) 1/5

(c) 1/10 (d) 9/10

26. Among 15 players, 8 are batsmen and 7 are bowlers. Find the probability that a team is chosen of 6 batsmen and 5 bowlers:

(a) $\frac{{}^{8}C_{6} \times {}^{7}C_{5}}{{}^{15}C_{11}}$ (b) $\frac{28}{15}$

(c) $\frac{15}{28}$ (d) None of these

27. A four digit number is formed by the digits 1, 2, 3, 4 with no repetition. The probability that the number is odd is

(a) zero (b) $\frac{1}{3}$

(c) $\frac{1}{4}$ (d) None of these

28. X speaks truth in 60% and Y in 50% of the cases. The probability that they contradict each other narrating the same incident is

(a) $\frac{1}{4}$ (b) $\frac{1}{3}$

(c) $\frac{1}{2}$ (d) $\frac{2}{3}$

29. An integer is chosen at random from the numbers 1, 2, ..., 25. The probability that the chosen number is divisible by 3 or 4, is

(a) $\frac{2}{25}$ (b) $\frac{11}{25}$

(c) $\frac{12}{25}$ (d) $\frac{14}{25}$

30. The probability that a leap year will have 53 Friday or 53 Saturday, is

(a) $\frac{2}{7}$ (b) $\frac{3}{7}$

(c) $\frac{4}{7}$ (d) $\frac{1}{7}$

31. An experiment yields 3 mutually exclusive and exhaustive events A, B, C. If $P(A) = 2P(B) = 3P(C)$, then $P(A)$ is equal to

(a) $\frac{1}{11}$ (b) $\frac{2}{11}$

(c) $\frac{3}{11}$ (d) $\frac{6}{11}$

32. If $P(A \cup B) = 0.8$ and $P(A \cap B) = 0.3$, then $P(A') + P(B')$ equals to

(a) 0.3 (b) 0.5

(c) 0.7 (d) 0.9

33. Five coins whose faces are marked 2, 3 are thrown. What is the probability of obtainining a total of 12 ?

(a) $\frac{1}{16}$ (b) $\frac{3}{16}$

(c) $\frac{5}{16}$ (d) $\frac{7}{16}$

34. An aircraft has three engines A, B and C. The aircraft crashes if all the three engines fail. The probabilities of failure are 0.03, 0.02 and 0.05 for engines A, B and C respectively. What is the probability that the aircraft will not crash?

(a) 0.00003 (b) 0.90

(c) 0.99997 (d) 0.90307

35. The probability that a student passes in mathematics is 4/9 and that he passes in physics is 2/5. Assuming that passing in mathematics and physics are independent of each other, what is the probability that he passes in mathematics but fails in physics?

(a) $\frac{4}{15}$ (b) $\frac{8}{45}$

(c) $\frac{26}{45}$ (d) $\frac{19}{45}$

36. From a pack of 52 cards, two cards are drawn, the first being replaced before the second is drawn. What is the probability that the first is a diamond and the second is a king?

(a) $\frac{1}{4}$ (b) $\frac{4}{13}$

(c) $\frac{1}{52}$ (d) $\frac{4}{15}$

37. In a lottery, 16 tickets are sold and 4 prizes are awarded. If a person buys 4 tickets, what is the probability of his winning a prize?

(a) $\frac{4}{16^4}$ (b) $\frac{175}{256}$

(c) $\frac{1}{4}$ (d) $\frac{81}{256}$

38. Each of A and B tosses two coins. What is the probability that they get equal number of heads?

(a) $\frac{3}{16}$ (b) $\frac{5}{16}$

(c) $\frac{4}{16}$ (d) $\frac{6}{16}$

39. The chance of winning the race of the horse A is 1/5 and that of horse B is 1/6. What is the probability that the race will be won by A or B ?

(a) 1/30 (b) 1/3

(c) 11/30 (d) 1/15

40. What is the probability of two persons being born on the same day (ignoring date)?

(a) 1/49 (b) 1/365

(c) 1/7 (d) 2/7

41. The probabilities of two events A and B are given as $P(A) = 0.8$ and $P(B) = 0.7$. What is the minimum value of $P(A \cap B)$?

(a) 0 (b) 0.1

(c) 0.5 (d) 1

42. In tossing three coins at a time, what is the probability of getting at most one head?

(a) $\frac{3}{8}$ (b) $\frac{7}{8}$

(c) $\frac{1}{2}$ (d) $\frac{1}{8}$

43. Two balls are selected from a box containing 2 blue and 7 red balls. What is the probability that at least one ball is blue?

(a) $\frac{2}{9}$ (b) $\frac{7}{9}$

(c) $\frac{5}{12}$ (d) $\frac{7}{12}$

44. The probability of guessing a correct answer is $\frac{x}{12}$. If the probability of not guessing the correct answer is $\frac{2}{3}$, then what is x equal to?

(a) 2 (b) 3

(c) 4 (d) 6

45. A man and his wife appear for an interview for two posts. The probability of the husband's selection is $\frac{1}{7}$ and that of the wife's selection is $\frac{1}{5}$. The probability that only one of them will be selected is

(a) $\frac{6}{7}$ (b) $\frac{4}{35}$

(c) $\frac{6}{35}$ (d) $\frac{2}{7}$

46. The probability that a person will hit a target in shooting practice is 0.3. If he shoots 10 times, the probability that he hits the target is

(a) 1 (b) $1 - (0.7)^{10}$

(c) $(0.7)^{10}$ (d) $(0.3)^{10}$

47. Suppose six coins are tossed simultaneously. Then the probability of getting at least one tail is

(a) $\frac{71}{72}$ (b) $\frac{53}{54}$

(c) $\frac{63}{64}$ (d) $\frac{1}{12}$

48. In a single throw with four dice, the probability of throwing seven is

(a) $\frac{4}{6^4}$ (b) $\frac{8}{6^4}$

(c) $\frac{16}{6^4}$ (d) $\frac{20}{6^4}$

49. Six dice are thrown. The probability that different number will turn up is

(a) $\frac{129}{1296}$ (b) $\frac{1}{54}$

(c) $\frac{5}{324}$ (d) $\frac{5}{54}$

50. If two dice are tossed, find the probability of throwing a total of ten or more.

(a) $\frac{1}{6}$ (b) $\frac{1}{3}$

(c) $\frac{1}{4}$ (d) $\frac{2}{3}$

51. From a pack of 52 cards two are drawn with replacement. The probability, that the first is a diamond and the second is a king, is

(a) 1/26 (b) 17/2704

(c) 1/52 (d) None of these

52. Two cards are selected at random from a deck of 52 playing cards. The probability that both the cards are greater than 2 but less than 9 is

(a) $\frac{46}{221}$ (b) $\frac{63}{221}$

(c) $\frac{81}{221}$ (d) $\frac{93}{221}$

53. If A and B are two independent events such that $P(A)=\frac{1}{2}$ and $P(B)=\frac{1}{5}$, then which is not true?

(a) $P(A\cup B)=\frac{3}{5}$ (b) $P(A/B)=\frac{1}{4}$

(c) $P(A/A\cup B)=\frac{5}{6}$ (d) $P(A\cap B/\overline{A}\cup\overline{B})=0$

54. The probability that a man will live 10 more years is $\frac{1}{4}$ and the probability that his wife will live 10 more years is $\frac{1}{3}$. Then the probability that neither will be alive in 10 years is

(a) $\frac{5}{12}$ (b) $\frac{7}{12}$

(c) $\frac{1}{2}$ (d) $\frac{11}{12}$

55. A and B play a game where each is asked to select a number from 1 to 25. If the two numbers match, both of them win a prize. The probbility that they will not win a prize in a single trial is

(a) $\frac{1}{25}$ (b) $\frac{24}{25}$

(c) $\frac{2}{25}$ (d) None of these

56. The probability of happening an event A in one trial is 0.4. The probability that the event A happens at least once in three independent trials is –

(a) 0.936 (b) 0.216

(c) 0.904 (d) 0.784

57. Find the probability of drawing a jack or an ace from a pack of playing cards.

(a) $\frac{1}{8}$ (b) $\frac{1}{6}$

(c) $\frac{1}{3}$ (d) $\frac{2}{13}$

58. When two dice are thrown, the probability that the difference of the numbers on the dice is 2 or 3 is

(a) $\frac{7}{18}$ (b) $\frac{3}{11}$

(c) $\frac{5}{18}$ (d) $\frac{1}{2}$

59. In shuffling a pack of cards three are accidentally dropped. The probability that the missing cards are of distinct colours is

(a) $\frac{169}{425}$ (b) $\frac{165}{429}$

(c) $\frac{162}{459}$ (d) $\frac{164}{529}$

60. Four persons are selected at random out of 3 men, 2 women and 4 children. The probability that there exactly 2 children in the selection is

(a) $\frac{11}{21}$ (b) $\frac{9}{21}$

(c) $\frac{10}{21}$ (d) None of these

61. It is given that the events A and B are such that $P(A)=\frac{1}{4}, P(A\mid B)=\frac{1}{2}$ and $P(B\mid A)=\frac{2}{3}$. Then $P(B)$ is

(a) $\frac{1}{6}$ (b) $\frac{1}{3}$

(c) $\frac{2}{3}$ (d) $\frac{1}{2}$

62. A coin is tossed and a dice is rolled. The probability that the coin shows the head and the dice shows 6 is

(a) $\frac{1}{2}$ (b) $\frac{1}{6}$

(c) $\frac{1}{12}$ (d) $\frac{1}{24}$

Level - II

1. If $P(A) = 0.8$, $P(B) = 0.9$, $P(AB) = p$, which one of the following is correct?

(a) $0.72 \le p \le 0.8$ (b) $0.7 \le p \le 0.8$
(c) $0.72 < p < 0.8$ (d) $0.7 < p < 0.8$

2. A, B, C are three mutually exclusive event associated with a random experiment. Find $P(A)$ if it is given that $P(B) = 3/2\ P(A)$ and $P(C) = 1/2\ P(B)$.

(a) $\frac{4}{13}$ (b) $\frac{2}{3}$
(c) $\frac{12}{13}$ (d) $\frac{1}{13}$

3. The probability that A can solve a problem is $\frac{2}{3}$ and B can solve it is $\frac{3}{4}$. If both attempt the problem, what is the probability that the problem gets solved?

(a) $\frac{11}{12}$ (b) $\frac{7}{12}$
(c) $\frac{5}{12}$ (d) $\frac{9}{12}$

4. A dice is thrown 6 times. If 'getting an odd number' is a 'success', the probability of 5 successes is

(a) $\frac{1}{10}$ (b) $\frac{3}{32}$
(c) $\frac{5}{6}$ (d) $\frac{25}{26}$

5. A bag contains 5 white and 3 black balls, and 4 are successively drawn out and not replaced. What's the chance of getting different colours alternatively?

(a) $\frac{1}{6}$ (b) $\frac{1}{5}$
(c) $\frac{1}{4}$ (d) $\frac{1}{7}$

6. A bag contains 5 white and 7 black balls and a man draws 4 balls at random. The odds against these being all black is

(a) 7 : 92 (b) 92 : 7
(c) 92 : 99 (d) 99 : 92

7. The letters of the word SOCIETY are placed at random in a row. The probability that the three vowels come together is

(a) $\frac{1}{6}$ (b) $\frac{1}{7}$
(c) $\frac{2}{7}$ (d) $\frac{5}{6}$

8. Course materials are sent to students by a distance education institution. The probability that they will send a wrong programme's study material is $\frac{1}{5}$. There is a probability of $\frac{3}{4}$ that the package is damaged in transit, and there is a probability of $\frac{1}{3}$ that there is a short shipment. What is the probability that the complete material for the course arrives without any damage in transit ?

(a) $\frac{4}{5}$ (b) $\frac{8}{60}$
(c) $\frac{8}{15}$ (d) $\frac{4}{20}$

9. A coin is tossed 5 times. What is the probability that head appears an odd number of times?

(a) $\frac{2}{5}$ (b) $\frac{1}{5}$
(c) $\frac{1}{2}$ (d) $\frac{4}{25}$

10. Two dice are tossed. The probability that the total score is a prime number is

(a) $\frac{1}{6}$ (b) $\frac{5}{12}$
(c) $\frac{1}{2}$ (d) $\frac{7}{9}$

11. The probability that the sum of the square of the two numbers, which show up when two fair dice are thrown, is even is

(a) $\frac{3}{7}$ (b) $\frac{4}{7}$
(c) $\frac{5}{7}$ (d) None of these

12. There are 5 pairs of shoes in a cupboard from which 4 shoes are picked at random. The probability that there is at least one pair is

(a) $\frac{8}{21}$ (b) $\frac{11}{21}$
(c) $\frac{13}{21}$ (d) $\frac{12}{31}$

13. The fair dice are thrown. The probability that the number appear are not all distinct is

(a) $\frac{5}{9}$ (b) $\frac{4}{9}$
(c) $\frac{1}{6}$ (d) $\frac{5}{6}$

14. Two dice are thrown simultaneously. What is the probability of obtaining a multiple of 2 on one of them and a multiple of 3 on the other

(a) $\frac{5}{36}$ (b) $\frac{11}{36}$

(c) $\frac{1}{6}$ (d) $\frac{1}{3}$

15. Two dice are thrown at a time, find the probability that the sums of the numbers on the upper faces of the dice are equal to 7.

(a) $\frac{1}{8}$ (b) $\frac{1}{4}$

(c) $\frac{1}{3}$ (d) $\frac{1}{6}$

16. One card is drawn from a well-shuffled pack of 52 cards. What is the probability, that it is not the ace of hearts ?

(a) $\frac{51}{52}$ (b) $\frac{1}{52}$

(c) $\frac{1}{12}$ (d) $\frac{1}{2}$

17. A dice is thrown twice. The probability of getting 4, 5 or 6 in the first throw and 1, 2, 3 or 4 in the second throw is

(a) 1/3 (b) 2/3

(c) 1/2 (d) 1/4

18. Ram and Shyam appear for an interview for two vacancies in an organisation for the same post. The probabilities of their selection are 1/6 and 2/5 respectively. What is the probability that none of them will be selected?

(a) 5/6 (b) 1/5

(c) 1/2 (d) 3/5

19. A class consists of 80 students, 25 of them are girls and 55 are boys. If 10 of them are rich and the remaining poor and also 20 of them are intelligent then the probability of selecting an intelligent rich girl is

(a) $\frac{5}{128}$ (b) $\frac{25}{128}$

(c) $\frac{5}{512}$ (d) None of these

20. If the probability of A to fail in an examination is 0.2 and that for B is 0.3, then probability that either A or B is fail, is :

(a) 0.5 (b) 0.44

(c) 0.8 (d) 0.25

21. The probability of choosing at random a number that is divisible by 6 or 8 from among 1 to 90 is equal to

(a) $\frac{1}{6}$ (b) $\frac{1}{30}$

(c) $\frac{11}{80}$ (d) $\frac{23}{90}$

22. In single cast with two dice the odds against drawing 7 is

(a) 5 (b) $\frac{1}{5}$

(c) 6 (d) $\frac{1}{6}$

23. From a group of 7 men and 4 women a committee of 6 persons is formed. What is the probability that the committee will consist of exactly 2 women?

(a) $\frac{5}{11}$ (b) $\frac{3}{11}$

(c) $\frac{4}{11}$ (d) $\frac{2}{11}$

24. Two numbers a and b are chosen at random from the set of first 30 natural numbers. The probability that $a^2 - b^2$ is divisible by 3 is:

(a) $\frac{37}{87}$ (b) $\frac{47}{87}$

(c) $\frac{17}{29}$ (d) None of these

25. An article manufactured by a company consists of two parts X and Y. In the process of manuifacture of the part X, 9 out of 100 parts may be defective. Similarly, 5 out of 100 are likely to be defective int he manufacture of the part. Y. Calculate the probability that the assembled product will not be defective.

(a) 0.6485 (b) 0.6565

(c) 0.8645 (d) None of these

26. If $P(A) = 3/7$, $P(B) = 1/2$ and $P(A' \cap B') = 1/14$, then are A and B are mutually exclusive events?

(a) No (b) Yes

(c) Either yes or no (d) Cannot be determined

27. One bag contains 4 white balls and 2 black balls. Another bag contains 3 white balls and 5 black balls. If one ball is drawn from each bag, determine the probability that one ball is white and another is black.

(a) 6/24 (b) 5/24

(c) 7/24 (d) 13/24

28. The probability that A can solve a problem is $\frac{2}{3}$ and B can solve it is $\frac{3}{4}$. If both attempt the problem, what is the probability that the problem gets solved?

(a) $\frac{11}{12}$ (b) $\frac{7}{12}$

(c) $\frac{5}{12}$ (d) $\frac{9}{12}$

29. Atul can hit a target 3 times in 6 shots, Bhola can hit the target 2 times in 6 shots and Chandra can hit the 4 times in 4 shots. What is the probability that at least 2 shots (out of 1 shot taken by each one of them) hit the target ?

(a) $\frac{1}{2}$ (b) $\frac{2}{3}$

(c) $\frac{1}{3}$ (d) $\frac{5}{6}$

30. Suppose six coins are tossed simultaneously. Then the probability of getting at least one tail is :

(a) $\frac{71}{72}$ (b) $\frac{53}{54}$

(c) $\frac{63}{64}$ (d) $\frac{1}{12}$

31. Seven digits from the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9 are written in a random order. The probability that this seven digit number is divisible by 9 is

(a) $\frac{2}{9}$ (b) $\frac{7}{36}$

(c) $\frac{1}{9}$ (d) $\frac{7}{12}$

32. A committee of 5 Students is to be chosen from 6 boys and 4 girls. Find the probability that the committee contains exactly 2 girls.

(a) 10/21 (b) 11/21

(c) 12/21 (d) 13/21

33. 4 gentlemen and 4 ladies take seats at random round a table. The probability that they are sitting alternately is

(a) 4/35 (b) 1/70

(c) 2/35 (d) 1/35

34. Two cards are drawn one by one from a pack of cards. The probability of getting first card an ace and second a coloured one is (before drawing second card, first card is not placed again in the pack) :

(a) 1/26 (b) 5/52

(c) 5/221 (d) 4/13

35. Seven people seat themselves indiscriminately at round table. The probability that two distinguished persons will be next to each other is

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$

(c) $\frac{1}{4}$ (d) $\frac{2}{3}$

36. Let $0 < P(A) < 1$, $0 < P(B) < 1$ and

$P(A \cup B) = P(A) + P(B) - P(A)P(B)$, then :

(a) $P(B / A) = P(B) - P(A)$

(b) $P(A' \cup B') = P(A') + P(B')$

(c) $P(A \cap B) = P(A')\,P(B')$

(d) None of these

37. Eleven books, consisting of five Engineering books, four Mathematics books and two Physics books, are arranged in a shelf at random. What is the probability that the books of each kind are all together?

(a) $\frac{5}{1155}$ (b) $\frac{2}{1155}$

(c) $\frac{3}{1155}$ (d) $\frac{1}{1155}$

38. 12 persons are seated around a round table. What is the probability that two particular persons sit together?

(a) $\frac{2}{11}$ (b) $\frac{1}{6}$

(c) $\frac{3}{11}$ (d) $\frac{3}{15}$

39. Two small squares on a chess board are choosen at random. Find the probability that they have a common side:

(a) $\frac{1}{12}$ (b) $\frac{1}{18}$

(c) $\frac{2}{15}$ (d) $\frac{3}{14}$

40. A bag contains 7 blue balls and 5 yellow balls. If two balls are selected at random, what is the probability that **none** is yellow? [*SBI PO-2013*]

(a) $\frac{5}{35}$ (b) $\frac{5}{22}$

(c) $\frac{7}{22}$ (d) $\frac{7}{33}$

(e) $\frac{7}{66}$

41. A die is thrown twice. What is the probability of getting a sum 7 from both the throws? [*SBI PO-2013*]

(a) $\frac{5}{18}$ (b) $\frac{1}{18}$

(c) $\frac{1}{9}$ (d) $\frac{1}{6}$

(e) $\frac{5}{36}$

DIRECTIONS (Qs. 42-46) : ***Study the given information carefully to answer the questions that follow.***

An urn contains 4 green, 5 blue, 2 red and 3 yellow marbles.

42. If four marbles are drawn at random, what is the probability that two are blue and two are red? [*IBPS-PO-2011*]

(a) $\frac{10}{1001}$ (b) $\frac{9}{14}$

(c) $\frac{17}{364}$ (d) $\frac{2}{7}$

(e) None of these

43. If eight marbles are drawn at random, what is the probability that there are equal number of marbles of each colour ? [*IBPS-PO-2011*]

(a) $\frac{4}{7}$ (b) $\frac{361}{728}$

(c) $\frac{60}{1001}$ (d) $\frac{1}{1}$

(e) None of these

44. If two marbles are drawn at random, what is the probability that both are red or at least one is red ? [*IBPS-PO-2011*]

(a) $\frac{26}{91}$ (b) $\frac{1}{7}$

(c) $\frac{199}{364}$ (d) $\frac{133}{191}$

(e) None of these

45. If three marbles are drawn at random, what is the probability that at least one is yellow ? [*IBPS-PO-2011*]

(a) $\frac{1}{3}$ (b) $\frac{199}{364}$

(c) $\frac{165}{364}$ (d) $\frac{3}{11}$

(e) None of these

46. If three marbles are drawn at random, what is the probability that none is green ? [*IBPS-PO-2011*]

(a) $\frac{2}{7}$ (b) $\frac{253}{728}$

(c) $\frac{10}{21}$ (d) $\frac{14}{91}$

(e) None of these

Level-I

1. **(a)** When two are thrown then there are 6×6 exhaustive cases $\therefore n = 36$. Let A denote the event "total score of 7" when 2 dice are thrown then $A = [(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)]$.
Thus there are 6 favourable cases.

$\therefore m = 6$ By definition $P(A) = \dfrac{m}{n}$

$\therefore P(A) = \dfrac{6}{36} = \dfrac{1}{6}$.

2. **(a)** $\because$ No of ways of drawing 2 white balls from 5 white balls = ${}^5C_2$.
Also, No of ways of drawing 2 other from remaining 7 balls = ${}^7C_2$
Total number of balls = 12

Hence, required probability $= \dfrac{{}^5C_2 \times {}^7C_2}{{}^{12}C_4} = \dfrac{14}{33}$

3. **(b)** Total no. of outcomes when two dice are thrown = $n(S) = 36$ and the possible cases for the event that the sum of numbers on two dice is a prime number, are (1, 1), (1, 2), (1, 4), (1, 6), (2, 1), (2, 3), (2, 5), (3, 2), (3, 4), (4, 1), (4, 3), (5, 1), (5, 6), (6, 1), (6, 5)
Number of outcomes favouring the event = $n(A) = 15$

Required probability $= \dfrac{n(A)}{n(S)} = \dfrac{15}{36} = \dfrac{5}{12}$

4. **(c)** $P(A) = \dfrac{1}{3}$, $P(\bar{A}) = \dfrac{2}{3}$

P (bird killed) = $1 - P$ (none of 3 shots hit)

$= 1 - \dfrac{2}{3} \times \dfrac{2}{3} \times \dfrac{2}{3} = \dfrac{19}{27}$.

5. **(b)** Since, A and B are independent events
$\therefore A'$ and B' are also independent events

$\Rightarrow P(A' \cap B') = P(A').P(B')$

$= (0.4)(0.7) = 0.28$

$[\because P(A') = 1 - P(A),\ P(B') = 1 - P(B)]$

6. **(d)**

7. **(d)** Doublets occur when the numbers thrown are (1, 1), (2, 2), ..., (6, 6). Therefore the probability of a doublet occurring in single throw $= \dfrac{6}{36} = \dfrac{1}{6}$.

The probability of a doublet not occurring at all in three throws $= \left(\dfrac{5}{6}\right)^3 = \dfrac{125}{216}$.

Required probability $= 1 - \dfrac{125}{216} = \dfrac{91}{216}$.

8. **(c)** Required probability = 1/6.

9. **(b)** Total probable ways = 8
Favourable number of ways = HTH, THT

Hence required probability $= \dfrac{2}{8} = \dfrac{1}{4}$

10. **(b)** Any of the six numbers 1,2,3,4,5,6 may appear on the upper face. $\therefore n = 6$
Number of odd numbers = 3, since the odd numbers are 1, 3, 5
$\therefore m = 3$.
$\therefore$ The required probability

$= \dfrac{\text{number of favourable cases}}{\text{number of all cases}} = \dfrac{m}{n} = \dfrac{3}{6} = \dfrac{1}{2}$

11. **(d)** n = Number of all cases = 6
m = Number of favourable cases = 4 (since the numbers that appear are 3, 4, 5, 6)

$\therefore$ The required probability $= p = \dfrac{m}{n} = \dfrac{4}{6} = \dfrac{2}{3}$

12. **(a)** $S = (1, 2, 3, 4, 5, 6)$ $\therefore n(S) = 6$
Let A be the event that the die shows a multiple of 2.
$A = \{2, 4, 6\}$ $\therefore n(A) = 3$

$P(A) = \dfrac{n(A)}{n(S)} = \dfrac{3}{6} = \dfrac{1}{2}$

13. **(a)** India win atleast three matches

$= {}^5C_3\left(\dfrac{1}{2}\right)^5 + {}^5C_4\left(\dfrac{1}{2}\right)^5 + {}^5C_5\left(\dfrac{1}{2}\right)^5 = \left(\dfrac{1}{2}\right)^5(16)$

$= \left(\dfrac{1}{2}\right)$

14. **(d)** Required probability

$= {}^5C_3\left(\dfrac{3}{4}\right)^3\left(\dfrac{1}{4}\right)^2 + {}^5C_4\left(\dfrac{3}{4}\right)^4.\left(\dfrac{1}{4}\right) + {}^5C_5\left(\dfrac{3}{4}\right)^5 = \dfrac{459}{512}$

15. **(b)** Total no. of numbers divisible by 4 between 1 to 80
$80 = 4 + (n - 1)4$
$80 = 4n$
$\Rightarrow n = 20$

$\therefore$ Required probability $= \dfrac{{}^{20}C_2}{{}^{80}C_2} = \dfrac{19}{316}$

16. **(b)** Here $n(S) = 6^2 = 36$

Let E be the event "getting sum more than 7" *i.e.* sum of pair of dice = 8, 9, 10, 11, 12

$$i.e.,\ E = \begin{Bmatrix} (2,6), & (3,5), & (4,4), & (5,3), & (6,2), \\ (3,6), & (4,5), & (5,4), & (6,3), & \\ (4,6), & (5,5), & (6,4), & & \\ (5,6), & (6,5), & (6,6) & & \end{Bmatrix}$$

$\therefore\ n(E) = 15$

$\therefore$ Required prob $= \dfrac{n(E)}{n(S)} = \dfrac{15}{36} = \dfrac{5}{12}$

17. **(c)** Number of sample points on throwing two dice

$= 6 \times 6 = 36$

The possible outcomes are (1, 4), (2, 3), (3, 2), (4, 1)

The probability of obtaining a total score of 5 is

$= \dfrac{4}{6 \times 6} = \dfrac{1}{9}$.

18. **(c)** Given digits are 1, 2, 3, 4, 5

Total no. of 2 digits numbers formed = $(5)^2$ = 25

Favourable cases are 12, 24, 32, 44, 52

No. of favourable cases = 5

$\therefore$ Required Probability = $\dfrac{5}{25} = \dfrac{1}{5}$

19. **(a)** Total no. of cases = $6^3 = 216$

16 can appear on three dice in following ways

(6, 6, 4), (6, 5, 5), (6, 4, 6), (4, 6, 6), (5, 5, 6), (5, 6, 5).

$\therefore$ No. of favourable cases = 6

Hence, the required probability $= \dfrac{6}{6^3} = \dfrac{1}{36}$

20. **(b)** Total number of bolts = 600

Number of too large bolts = 20% of 600

$= \dfrac{20 \times 600}{100} = 120$

Number of too small bolts = 10% of 600 = 60

Number of suitable bolts = 600 – 120 – 60 = 420

Thus required probability = $\dfrac{420}{600} = \dfrac{7}{10}$

21. **(c)** Total possible outcomes = 36

E = Event of getting sum 7

= {(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)}

F = Event of getting sum 11

= {(6,5), (5,6)}

Total no. of favourable cases = 6 + 2 = 8

Now required probability

$= \dfrac{\text{Total favourable cases}}{\text{Total outcomes}} = \dfrac{8}{36} = \dfrac{2}{9}$

22. **(a)** $n(S) = 100$

E = square numbers from 1 to 100.

= 1, 4, 9, 16, 25, 36, 49, 64, 81, 100

$n(E) = 10$

$\therefore$ Required probability $= \dfrac{n(E)}{n(S)} = \dfrac{10}{100} = \dfrac{1}{10}$

23. **(a)** (AAAA), (LL), HBD

$$P = \frac{5!}{\frac{9!}{4!\,2!}} = \frac{5! \times 4! \times 2!}{9!} = \frac{24 \times 2}{9 \times 8 \times 7 \times 6} = \frac{1}{63}$$

24. **(b)** The probability that Krishna will be alive 10 years hence, is $\dfrac{7}{15}$

So, probability that Krishna will be dead 10 years hence, the

$= 1 - \dfrac{7}{15} = \dfrac{8}{15}$

Also, probability that Hari will be alive 10 years hence is $\dfrac{7}{10}$

So, the probability that Hari will be dead 10 years hence, $= 1 - \dfrac{7}{10} = \dfrac{3}{10}$

So, the probability that both Krishna and Hari will be dead 10 years hence

$= \dfrac{8}{15} \times \dfrac{3}{10} = \dfrac{24}{150}$

25. **(a)** Total no. of arrangements of the letters of the word UNIVERSITY is $\dfrac{10!}{2!}$.

No. of arrangements when both I's are together = 9!

So. the no. of ways in which 2 I's do not together

$= \dfrac{10!}{2!} - 9!$

$\therefore$ Required probability

$$= \frac{\frac{10!}{2!} - 9!}{\frac{10!}{2!}} = \frac{10! - 9!\,2!}{10!}$$

26. **(a)** Total no. of players = 15

Total no. of batsmen = 8

Total no. of bowlers = 7

Total no. of players in the team = 11

$\therefore$ No. of ways to choose a team = ${}^{15}C_{11}$

$\therefore$ No. of way to choose 6 batsmen and 5 bowler

$= {}^{8}C_6 \times {}^{7}C_5$

$\therefore$ Required Probability = $\dfrac{{}^{8}C_6 \times {}^{7}C_5}{{}^{15}C_{11}}$

27. **(d)** Total number of numbers = 4! = 24

For odd nos. 1 or 3 has to be at unit's place

If 1 is at unit place, then total number of numbers = 3! = 6

And if 3 is at units place, then total number of numbers = 3! = 6

$\therefore$ Total number of odd number = 6 + 6 = 12

$\therefore$ Required probability = $\frac{12}{24}=\frac{1}{2}$

28. **(c)** Required Probability = $P(X).P(\bar{Y})+P(\bar{X}).P(Y)$

$=\frac{60}{100}\times\frac{50}{100}+\frac{40}{100}\times\frac{50}{100}=\frac{1}{2}$

29. **(c)** $P(3\cup 4)=P(c)+P(d)-P(3\cap 4)$

$=\frac{8}{25}+\frac{6}{25}-\frac{2}{25}=\frac{12}{25}$

30. **(b)** In a leap year there are 366 days in which 52 weeks and two days. The combination of 2 days may be : Sun-Mon, Mon-Tue, Tue-Wed, Wed-Thu, Thu-Fri, Fri-Sat, Sat-Sun.

$P(53\text{ Fri})=\frac{2}{7}$; $P(53\text{ Sat})=\frac{2}{7}$

and $P(53\text{ Fri and }53\text{ Sat})=\frac{1}{7}$

$\therefore\ P(53\text{ Fri or Sat})=P(53\text{ Fri})+P(53\text{ Sat})-P(53\text{ Fri and Sat})$

$=\frac{2}{7}+\frac{2}{7}-\frac{1}{7}=\frac{3}{7}$

31. **(d)** Clearly, $P(A\cup B\cup C)=1$

$\Rightarrow\ P(A)+P(B)+P(C)=1$

$\Rightarrow\ P(A)+\frac{1}{2}P(A)+\frac{1}{3}P(A)=1$

$\Rightarrow\ \frac{11}{6}P(A)=1$

$\Rightarrow\ P(A)=\frac{6}{11}$

32. **(d)** Now, $P(A'\cap B')=P(A'\cup B')$

$=\ 1-P(A\cup B)=1-0.8=0.2$

and $P(A'\cup B')=1-P(A\cap B)=1-0.3=0.7$

But $P(A'\cup B')=P(A')+P(B')-P(A'\cap B')$

$\Rightarrow\ 0.7=P(A')+P(B')-0.2$

$\Rightarrow\ P(A')+P(B')=0.9.$

33. **(c)** Let E be the event of total of 12.

$E=$ (2, 2, 2, 3, 3), (2, 2, 3, 3, 2), (2, 3, 3, 2, 2), (3, 3, 2, 2, 2), (3, 2, 3, 2, 2), (3, 2, 2, 3, 2), (3, 2, 2, 2, 3), (2, 3, 2, 3, 2), (2, 3, 2, 2, 3), (2, 2, 3, 2, 3)

$n(E)=10$

Sample sapce contain total possibility = $2^5=32$

Hence, $n(s)=32$

So, $P(E)=\frac{n(E)}{n(S)}=\frac{10}{32}=\frac{5}{16}$

34. **(c)** Since, probabilities of failure for engines A, B and C $P(A)$, $P(B)$ and $P(C)$ are 0.03, 0.02 and 0.05 respectively.

The aircraft will crash only when all the three engine fail. So, probability that it crashes = $P(A).P(B).P(C)$ $=0.03\times 0.02\times 0.05=0.00003$

Hence, the probability that the aircraft will not crash, $=1-0.00003=0.99997$

35. **(a)** Probability of passing in mathematics $=\frac{4}{9}$

Probability of passing in physics $=\frac{2}{5}$

Probability of failure in physics $=1-\frac{2}{5}=\frac{3}{5}$

Given that both the events are independent.

Required probability $=\frac{4}{9}\times\frac{3}{5}=\frac{4}{15}$

36. **(c)** Probability of getting a diamond, $P(D)=\frac{13}{52}=\frac{1}{4}$

and probability to king, $P(K)=\frac{4}{52}=\frac{1}{13}$

So, required probability = $P(D).P(K)$

$=\frac{1}{4}\times\frac{1}{13}=\frac{1}{52}$

37. **(c)** 16 tickets are sold and 4 prizes are awarded. A person buys 4 tickets, then required probability $=\frac{4}{16}=\frac{1}{4}$

38. **(b)** If both get one head then it is $\frac{1}{4}\times\frac{1}{4}$

and if both get two heads then it is $\frac{1}{2}\times\frac{1}{2}$

$\Rightarrow$ Prob (getting same number of heads) $=\frac{1}{4}\times\frac{1}{4}+\frac{1}{2}\times\frac{1}{2}$

$=\frac{1}{16}+\frac{1}{4}=\frac{5}{16}$

39. **(c)** Let $P(A)$ be the probability that the race will be won by A and $P(B)$ be the probability that the race will be won by B.

$\therefore\ P(A)=\frac{1}{5}$ and $P(B)=\frac{1}{6}$

$\therefore$ Probability that the race will be won by

A or $B=P(A)+P(B)=\frac{1}{5}+\frac{1}{6}=\frac{11}{30}$

40. **(b)** Required probability $= \frac{365}{365} \times \frac{1}{365} = \frac{1}{365}$.

41. **(c)** As we know $P(A \cup B) \le 1$

$\therefore\ P(A) + P(B) - P(A \cap B) \le 1$

$\Rightarrow\ 0.8 + 0.7 - P(A \cap B) \le 1$

$\Rightarrow\ P(A \cap B) \ge 1.5 - 1$

$\Rightarrow\ P(A \cap B) \ge 0.5$

Hence, the minimum value of $P(A \cap B)$ is 0.5.

42. **(c)** Possible samples are as follows

$\{HHH, HTH, HHT, THH, TTH, THT, HTT, TTT\}$

Let A be the event of getting one head.

Let B be the event of getting no head.

Favourable outcome for

$A = \{TTH, THT, HTT\}$

Favourable outcome for

$B = \{TTT\}$

Total no. of outcomes = 8

$\therefore\ P(A) = \frac{3}{8}, P(B) = \frac{1}{8}$

$\therefore$ Required probability = Probability of getting one head + Probability of getting no head

$= P(A) + P(B) = \frac{3}{8} + \frac{1}{8} = \frac{4}{8} = \frac{1}{2}$

43. **(a)** No. of blue balls =2

No. of red balls = 7

Total no. of balls = 9

Required probability

= P (one ball is blue) + P (both ball is blue)

$= \frac{2}{9} \times \frac{7}{8} + \frac{2}{9} \times \frac{1}{8} = \frac{14}{72} + \frac{2}{72} = \frac{16}{72} = \frac{2}{9}$

44. **(c)** Given probability of guessing a correct answer $= \frac{x}{12}$

and probability of not guessing the correct answer $= \frac{2}{3}$

As we know

P (occurence of an event) + P (non-occurence of an event) = 1

$\therefore\ \frac{x}{12} + \frac{2}{3} = 1 \Rightarrow \frac{x+8}{12} = 1 \Rightarrow x = 12 - 8 = 4$

45. **(d)** Probability that only husband is selected

$= P(H)P(\bar{W}) = \frac{1}{7}\left(1 - \frac{1}{5}\right) = \frac{1}{7} \times \frac{4}{5} = \frac{4}{35}$

Probability that only wife is selected

$= P(\bar{H})P(W) = \left(1 - \frac{1}{7}\right)\left(\frac{1}{5}\right) = \frac{6}{7} \times \frac{1}{5} = \frac{6}{35}$

$\therefore$ Probability that only one of them is selected

$= \frac{4}{35} + \frac{6}{35} = \frac{10}{35} = \frac{2}{7}$

46. **(b)** The probability that the person hits the target = 0.3

$\therefore$ The probability that he does not hit the target in a trial = $1 - 0.3 = 0.7$

$\therefore$ The probability that he does not hit the target in any of the ten trials = $(0.7)^{10}$

$\therefore$ Probability that he hits the target

= Probability that at least one of the trials succeeds

$= 1 - (0.7)^{10}$.

47. **(c)** If six coins are tossed, then the total no. of outcomes $= (2)^6 = 64$

Now, probability of getting no tail $= \frac{1}{64}$

Probability of getting at least one tail

$= 1 - \frac{1}{64} = \frac{63}{64}$

48. **(d)** Total of seven can be obtained in the following ways

1, 1, 1, 4 in $\frac{4!}{3!} = 4$ ways

[there are four objects, three repeated]

Similarly,

1, 1,2, 3 in $\frac{4!}{2!} = 12$ ways

1, 2,2, 2 in $\frac{4!}{3!} = 4$ ways

Hence, required probability $= \frac{4+12+4}{6^4} = \frac{20}{6^4}$

[$\because$ Exhaustive no. of cases $= 6 \times 6 \times 6 \times 6 = 6^4$]

49. **(c)** The number of ways of getting the different number 1, 2, ..., 6 in six dice = 6 !.

Total number of ways $= 6^6$

Hence, required probability $= \frac{6!}{6^6}$

$= \frac{1 \times 2 \times 3 \times 4 \times 5 \times 6}{6^6} = \frac{5}{324}$

50. **(a)** Here the number of favourable cases, consists of throwing 10,11 or 12 with the two dice. The number of ways in which a sum of 10 can be thrown are (4,6), (5,5), (6,4) i.e. 3 ways. The number of ways in which a total of 11 can be thrown are (5, 6), (6,5) i.e. 2 ways. The number of ways in which a total of 12 can be thrown in (6, 6) i.e. 1 way.

m = number of favourable cases = 3 + 2 + 1 = 6

n = Total number of cases = 6 × 6 = 36

$\therefore$ Probability $= p = \frac{m}{n} = \frac{6}{36} = \frac{1}{6}$

51. (c) Required probability

$= P\text{ (Diamond)} . P\text{ (King)} = \frac{13}{52}.\frac{4}{52} = \frac{1}{52}$

52. (a) The cards are of four colours and the number of cards of given description is 24.

The probability $= \frac{24}{52}.\frac{23}{51} = \frac{46}{221}$.

53. (b) Since A and B are independent

$\therefore\ P(A \cap B) = P(A).P(B)$

and $P(A/B) = P(A)$

Thus, $P(A/B) = \frac{1}{2}$

Hence, option (b) is not true.

54. (c) The probability that a man will not live 10 more years = 3/4 and the probability that his wife will not live 10 more years = 2/3. Then the probability that neither will be alive in 10 years = 3/4 × 2/3 = 1/2

55. (b) Total number of possibilities = 25 × 25

Favourable cases for their winning = 25

$\therefore P$ (they win a prize) $= \frac{25}{25 \times 25} = \frac{1}{25}$

$\therefore P$ (they will not win a prize) $= 1 - \frac{1}{25}$

56. (d) Here $P(A) = 0.4$ and $P(\bar{A}) = 0.6$

Probability that A does not happen at all = $(0.6)^3$

Thus required probability = $1 - (0.6)^3 = 0.784$

57. (d) As there are four jacks and four aces, the number of favourable cases = 8

$\therefore$ The required probability $p = \frac{8}{52} = \frac{2}{13}$

58. (a) The favourable cases are (1, 3), (2, 4), (3,5), (4,6) and (1, 4), (2, 5), (3, 6) and their reversed cases like (3, 1)......

Total number of favourable cases = 2 × 7

$\therefore p = \frac{14}{36} = \frac{7}{18}$

59. (a) The first card can be one of the 4 colours, the second can be one of the three and the third can be one of the two. The required probability is therefore

$4 \times \frac{13}{52} \times 3 \times \frac{13}{51} \times 2 \times \frac{13}{50} = \frac{169}{425}$.

60. (c) Total number of ways in which 4 persons can be selected out of 3 + 2 + 4 = 9 persons = ${}^9C_4 = 126$

Number of ways in which a selection of 4 contains exactly 2 children = ${}^4C_2 \times {}^5C_2 = 60$

$\therefore$ reqd. prob. $= \frac{60}{126} = \frac{10}{21}$

61. (b) $P(A) = 1/4,\ P(A/B) = \frac{1}{2},\ P(B/A) = 2/3$

By conditional probability,

$P(A \cap B) = P(A)\,P(B/A) = P(B)P(A/B)$

$\Rightarrow \frac{1}{4} \times \frac{2}{3} = P(B) \times \frac{1}{2} \Rightarrow P(B) = \frac{1}{3}$

62. (c) Probability of getting a head on tossing a coin $(P_1) = \frac{1}{2}$.

Probability of getting a six on rolling a dice $(P_2) = \frac{1}{6}$.

These two events are independent.

So the probability that the coin shows the head and the dice shows 6 is given by

$P = P_1 \times P_2 = \frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$.

Level-II

1. (b) We know,

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow\ 0.8 + 0.9 - p \le 1$

$\Rightarrow\ 1.7 - p \le 1$

$\Rightarrow\ 0.7 \le p$

Now, $P(A) < P(B)$

$\therefore\ P(A \cap B) \le P(A)$

$\Rightarrow\ p \le 0.8$

Hence, $0.7 \le p \le 0.8$

2. (a) $P(A) + P(B) + P(C) = 1 \to 2\,P(B)/3 + P(B) + P(B)/2 = 1 \to 13P(B)/6 = 1 \to P(B) = 6/13$. Hence, $P(A) = 4/13$

3. (a) The probability that A cannot solve the problem

$= 1 - \frac{2}{3} = \frac{1}{3}$

The probability that B cannot solve the problem

$= 1 - \frac{3}{4} = \frac{1}{4}$

The probability that both A and B cannot solve the problem $= \frac{1}{3} \times \frac{1}{4} = \frac{1}{12}$

$\therefore$ The probability that at least one of A and B can solve the problem $= 1 - \frac{1}{12} = \frac{11}{12}$

$\therefore$ The probability that the problem is solved $= \frac{11}{12}$

4. (b) Let A be the event of getting an odd number.

Here, $n(S) = 6$ and

$n(A) = 3$

Probability of getting an odd number $= \frac{3}{6} = \frac{1}{2}$

Hence, probability of not getting an odd number

$=1-\frac{1}{2}=\frac{1}{2}$

Required probability of 5 successes

$= {}^6C_5\times\left(\frac{1}{2}\right)^5\times\frac{1}{2}=\frac{3}{32}$

5. **(d)** Total number of balls = 8. Let the first drawn ball is white, so required probability = $\frac{5}{8}\times\frac{3}{7}\times\frac{4}{6}\times\frac{2}{5}=\frac{1}{14}$.

But here we had started with a white ball. When we start with a black ball, the required probability

$=\frac{3}{8}\times\frac{5}{7}\times\frac{2}{6}\times\frac{4}{5}=\frac{1}{14}$.

Since these two cases are mutually exclusive.

Total probability = $\frac{1}{14}+\frac{1}{14}=\frac{2}{14}=\frac{1}{7}$.

6. **(b)** There are 7 + 5 = 12 balls in the bag and the number of ways in which 4 balls can be drawn is ${}^{12}C_4$ and the number of ways of drawing 4 black balls (out of seven) is ${}^7C_4$.

Hence, P (4 black balls)

$=\frac{{}^7C_4}{{}^{12}C_4}=\frac{7.6.5.4}{1.2.3.4}\times\frac{1.2.3.4}{12.11.10.9}=\frac{7}{99}$

Thus the odds against the event 'all black balls' are

$(1-\frac{7}{99}):\frac{7}{99}$: i.e., $\frac{92}{99}:\frac{7}{99}$ or $92:7$.

7. **(b)** The word 'SOCIETY' contains seven distinct letters and they can be arranged at random in a row in ${}^7P_7$ ways, *i.e.,* in 7! = 5040 ways.

Let us now consider those arrangements in which all the three vowels come together. So in this case we have to arrange four letters. S,C,T,Y and a pack of three vowels in a row which can be done in ${}^5P_5$ i.e. 5! = 120 ways.

Also, the three vowels in their pack can be arranged in ${}^3P_3$ i.e. 3! = 6 ways.

Hence, the number of arrangements in which the three vowels come together is 120 × 6 = 720

$\therefore$ The probability that the vowels come together $=\frac{720}{5040}=\frac{1}{7}$

8. **(b)** Probability (sending a correct programme)

$=1-\frac{1}{5}=\frac{4}{5}$

Probability (the packet is not damaged) $=1-\frac{3}{4}=\frac{1}{4}$

Probability (there is no short shipment) $=1-\frac{1}{3}=\frac{2}{3}$

Required probability $=\frac{4}{5}\times\frac{1}{4}\times\frac{2}{3}=\frac{2}{15}=\frac{8}{60}$

9. **(c)** Probability of occurence of head in a toss of a coin is 1/2.

Required probability = Prob [Head appears once] + Prob. [Head appears thrice] + Prob. [Head appears five times]

$= {}^5C_1\left(\frac{1}{2}\right)^5+{}^5C_3\left(\frac{1}{2}\right)^5+{}^5C_5\left(\frac{1}{2}\right)^5$

$=\left(\frac{1}{2}\right)^5[5+10+1]=\frac{16}{32}=\frac{1}{2}$

10. **(b)** Total no. of outcomes when two dice are thrown = $n(S)$ = 36 and the possible cases for the event that the sum of numbers on two dice is a prime number, are (1, 1), (1, 2), (1, 4), (1, 6), (2, 1), (2, 3), (2, 5), (3, 2), (3, 4), (4, 1), (4, 3), (5, 1), (5, 6), (6, 1), (6, 5)

Number of outcomes favouring the event = $n(A)$ = 15

Required probability $=\frac{n(A)}{n(S)}=\frac{15}{36}=\frac{5}{12}$

11. **(d)** Out of 36 possible outcomes the ones which are favourable for the event are

(i) When the numbers are both even and

(ii) When the numbers are both odd. There are six doublets and the pairs. (1, 3), (1, 5), (2, 4), (2, 6) etc. Which make a total of 6 × 3 = 18. The required probability is 1/2.

12. **(c)** There are 5 pairs of shoes and 4 shoes can be picked in 10 × 9 × 8 × 7 ways. Number of ways in which 4 shoes can be picked such that no two are alike = 10 × 8 × 6 × 4.

The required probability $=1-\frac{10\times8\times6\times4}{10\times9\times8\times7}=\frac{13}{21}$.

13. **(b)** Out of the 6^3 possible outcomes 6.5.5 outcomes will have all distinct numbers.

The probability $=1-\frac{6.5.4}{6^3}=\frac{4}{9}$.

14. **(b)** Favourable cases for one are there *i.e.,* 2, 4 and 6 and for other are two i.e., 3, 6.

Hence required probability $=\left[\left(\frac{3\times2}{36}\right)2-\frac{1}{36}\right]=\frac{11}{36}$

[As same way happen when dice changes numbers among themselves]

15. **(d)** If a die is thrown, there are 6 equally likely and mutually exclusive cases. Since two dice are thrown, the total number of ways = 6 × 6 = 36. If a sum of 7 is to be obtained from the numbers appearing on the two upper faces, the numbers in the two dice can be (1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6,1), which are six in number.

$\therefore$ Number of favourable cases = m = 6

Total number of cases = 36

$\therefore$ The required probability = $p=\frac{m}{n}=\frac{6}{36}=\frac{1}{6}$

16. **(a)** The ace of hearts can be drawn in only 1 way
($\because$ in a pack of cards there is only one ace of heart)

$P(A)$ = Probability of drawing the ace of hearts $= \dfrac{1}{52}$.

Hence the probability of not drawing an ace of hearts

$$= P(\bar{A}) = 1 - P(A) = 1 - \frac{1}{52} = \frac{51}{52}$$

17. **(a)** Let $P(A)$ and $P(B)$ be the probability of the events of getting 4, 5 or 6 in the first throw and 1, 2, 3 or 4 in the second throw respectively, then

$$P(A \text{ and } B) = P(A).P(B) = \frac{1}{2} \times \frac{2}{3} = \frac{1}{3}$$

18. **(c)** Required probability $= \left(1 - \dfrac{1}{6}\right) \times \left(1 - \dfrac{2}{5}\right) = \dfrac{5}{6} \times \dfrac{3}{5} = \dfrac{1}{2}$

19. **(c)** Total 80, Girls = 25, Boys = 55
10 R, 70 P, 20 I

$$\frac{1}{4} \times \frac{1}{8} \times \frac{25}{80} = \frac{5}{512}$$

20. **(b)** Given $P(A_f) = 0.2$ and $P(B_f) = 0.3$

Since, A and B are independent events

$\therefore \quad P(A \cap B) = P(A).P(B)$

$$= (0.2) \times (0.3) = 0.06$$

$\therefore \quad$ Required prob $= P(A \cup B)$

$$= P(A) + P(B) - P(A \cap B)$$

$$= 0.2 + 0.3 - 0.06 = 0.44$$

21. **(d)** Nos. divisible by 6 are 6, 12, 18, ..., 90.
Nos. divisible by 8 are 8, 16, 24, ..., 88.
Now, total no. divisible by 6 = 15
and total no. divisible by 8 = 11
Now, the no. divisible by both 6 and 8 are 24, 48, 72.
So, total no. divisible by both 6 and 8 = 3
$\therefore \quad$ Probability (number divisible by 6 or 8)

$$= \frac{15 + 11 - 3}{90} = \frac{23}{90}$$

22. **(a)** Let $E = \{(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)\}$

$$\therefore P(E) = \frac{6}{6 \times 6} = \frac{1}{6}$$

So, odds against drawing 7

$$= \frac{P(\bar{E})}{P(E)} = \frac{1 - \frac{1}{6}}{\frac{1}{6}} = \frac{5}{1}$$

23. **(a)** ${}^6C_2 \times [(7/11) \times (6/10) \times (5/9) \times (4/8) \times (4/7) \times (3/6)]$
= 5/11.

24. **(b)** Out of 30 numbers 2 numbers can be chosen in ${}^{30}C_2$ ways. So, exhaustive number of cases = ${}^{30}C_2 = 435$
Since $a^2 - b^2$ is divisible by 3 if either a and b are divisible by numbers, of cases = ${}^{10}C_2 + {}^{20}C_2 = 235$

Hence, required probability $= \dfrac{235}{435} = \dfrac{47}{87}$

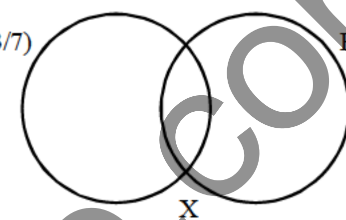
25. **(c)** Required probability
= P (X not defective and Y not defective)

$= P(\bar{X})P(\bar{Y})$

$= \{1 - P(X)\}\{1 - P(Y)\}$

$$= \frac{91}{100} \times \frac{95}{100} = \frac{8645}{10000} = 0.8645$$

26. **(b)**



Also $P(A \cup B) = 1 - P(A' \cap B')$
$(3/7) + (1/2) - x = 13/14 \rightarrow x = 0$
Thus, there is no interference between A and B as $P(A \cup B) = x = 0$. Hence, A and B are mutually exclusive.

27. **(d)** Probability that first ball is white and second black
= (4/6) × (5/8) = 5/12
Probability that first ball is black and second white
= (2/6) × (3/8) = 1/8
These are mutually exclusive events hence the required probability

$$P = \frac{5}{12} + \frac{1}{8} = \frac{13}{24}.$$

28. **(a)** The probability that A cannot solve the problem

$$= 1 - \frac{2}{3} = \frac{1}{3}$$

The probability that B cannot solve the problem

$$= 1 - \frac{3}{4} = \frac{1}{4}$$

The probability that both A and B cannot solve the problem $= \dfrac{1}{3} \times \dfrac{1}{4} = \dfrac{1}{12}$

$\therefore$ The probability that at least one of A and B can solve the problem $= 1 - \dfrac{1}{12} = \dfrac{11}{12}$

$\therefore$ The probability that the problem is solved $= \dfrac{11}{12}$

29. (b) Chandra hits the target 4 times in 4 shots. Hence, he hits the target definitely.
The required probability, therefore, is given by.
P (both Atul and Bhola hit) + P (Atul hits, Bhola does not hit) + P (Atul does not hit, Bhola hits)

$$= \frac{3}{6}\times\frac{2}{6}+\frac{3}{6}\times\frac{4}{6}+\frac{3}{6}\times\frac{2}{6} = \frac{1}{6}+\frac{1}{3}+\frac{1}{6}=\frac{4}{6}=\frac{2}{3}$$

30. (c) If six coins are tossed, then the total no. of outcomes $= (2)^6 = 64$

Now, probability of getting no tail $= \frac{1}{64}$

Probability of getting at least one tail

$$= 1-\frac{1}{64}=\frac{63}{64}$$

31. (c) A number is divisible by 9, if the sum of its digits is divisible by 9. Here $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 = 45$ is divisible by 9.
$\therefore$ the two numbers to be removed should be such that their sum is 9.
$\therefore$ they can be any one of the following pairs (1, 8), (2, 7), (3, 6), (4, 5).
Hence the number of favourable cases = 4
Total number of cases of removing two numbers $= {}^9C_2$

$\therefore$ Required probability $= \frac{4}{{}^9C_2} = \frac{4}{36} = \frac{1}{9}$.

32. (a) 5 Students can be selected from 10 in ${}^{10}C_5$ ways.

$$\therefore n(S) = {}^{10}C_5 = \frac{10!}{5!.5!} = \frac{10\times9\times8\times7\times6}{5\times4\times3\times2} = 252$$

Let A be the event that the committee includes exactly 2 girls and 3 boys. The two girls. can be selected in ${}^4C_2$ ways and the 3 boys can be selected in ${}^6C_3$ ways.
$\therefore\ n(A) = {}^4C_2 \times {}^6C_3 = 6 \times 20 = 120$

$$\therefore P(A) = \frac{n(A)}{n(S)} = \frac{120}{252} = \frac{10}{21}$$

33. (d) $n(S) = 7!$, $n(E) = (3!) \times (4!)$

$$\therefore P(E) = \frac{(3!)\times(4!)}{7!} = \frac{6}{7\times6\times5} = \frac{1}{15} = \frac{1}{35}$$

34. (c) Let A be the event of getting first card an ace and B be the event of getting second a coloured one. Since, both the events associated with a random experiment.
(i.e. condition of probability)
Therefore, the probability of getting first card an ace

$$P(A) = \frac{4}{52} = \frac{1}{13}$$

and probability of drawing a coloured one in second draw

$$P(B/A) = \frac{15}{51} = \frac{5}{17}$$

(since one card has already been drawn)
Hence, by conditional probability,

$$P(B/A) = \frac{P(A\cap B)}{P(A)}$$

$$\Rightarrow \frac{5}{17} = \frac{P(A\cap B)}{\frac{1}{13}}$$

$$\Rightarrow P(A\cap B) = \frac{5}{17}\times\frac{1}{13} = \frac{5}{221}$$

35. (a) Seven people can seat themselves at a round table in 6! ways. The number of ways in which two distinguished persons will be next to each other = 2 (5) !, Hence, the required probability

$$= \frac{2(5)!}{6!} = \frac{1}{3}$$

36. (d) Given $P(A) + P(B) - P(A)\,P(B) = P(A \cup B)$
Comparing with
$P(A) + P(B) - P(A \cap B) = P(A \cup B)$
we get $P(A \cap B) = P(A).P(B)$
$\therefore$ A and B independent events.

37. (d) $\frac{(5!\times 4!\times 2!\times3!)}{11!} = \frac{24\times2\times6}{11\times10\times9\times8\times7\times6} = 1/1155.$

38. (a)

$$P = \frac{\text{Total no. of ways in which two people sit together}}{\text{Total no. of ways}}$$

$= (10! \times 2!)/11!$

39. (b) The common side could be horizontal or vertical. Accordingly, the number of ways the event can occur is.
$n(E) = 8 \times 7 + 8 \times 7 = 112$
$n(S) = {}^{64}C_2$

Required probability $= \frac{2\times8\times7\times2}{64\times63} = \frac{1}{18}$

40. (c) Total balls = 12
Blue balls = 7
None of two balls are yellow i.e., both balls are blue.

$\therefore$ P (both blue balls) $= \frac{7}{12}\times\frac{6}{11} = \frac{7}{22}$

41. (d) Total possible outcomes when A die is thrown twice = 36
Outcome for getting a sum 7 from both throwns = 6{(2, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)}

$\therefore$ P(E) $= \frac{6}{36} = \frac{1}{6}$

42. (a) According to question,

$$n(S) = {}^{14}C_4 = \frac{14!}{(14-4)!4!} = \frac{14!}{10!4!} \left[\because {}^nC_r = \frac{n!}{(n-r)!r!}\right]$$

$$= \frac{14 \times 13 \times 12 \times 11}{4 \times 3 \times 2 \times 1} = 1001$$

and $n(E) = {}^5C_2 \times {}^2C_2 = \frac{5!}{(5-2)!2!} \times \frac{2!}{(2-2)!2!}$

$$= \frac{5 \times 4}{2 \times 1} \times \frac{2 \times 1}{1 \times 2 \times 1} = 10$$

$\therefore$ Required probability $= \frac{n(E)}{n(S)} = \frac{10}{1001}$

43. (c) According to question,

$$n(S) = {}^{14}C_8 = \frac{14!}{(14-8)!8!} \times \frac{14!}{6!8!}$$

$$= \frac{14 \times 13 \times 12 \times 11 \times 10 \times 9}{6 \times 5 \times 4 \times 3 \times 2 \times 1} = 3003$$

and $n(E) = {}^4C_2 \times {}^5C_2 \times {}^2C_2 \times {}^3C_2$

$$= \frac{4!}{(4-2)!2!} \times \frac{5!}{(5-2)!2!} \times \frac{2!}{(2-2)!2!} \times \frac{3!}{(3-2)!2!}$$

$$= \frac{4!}{2!2!} \times \frac{5!}{3!2!} \times \frac{2!}{0!2!} \times \frac{3!}{1!2!}$$

$$= \frac{4 \times 3}{2 \times 1} \times \frac{5 \times 4}{2 \times 1} \times \frac{1}{1} \times \frac{3}{1} = 180$$

$\therefore$ Required probability $= \frac{n(E)}{n(S)} = \frac{180}{3003} = \frac{60}{1001}$

44. (e) According to question,

$$n(S) = {}^{14}C_2 = \frac{14!}{(14-2)!2!} = \frac{14 \times 13}{2 \times 1} = 91$$

$\therefore$ Probability of at least one red ball

$$= 1 - \frac{{}^{12}C_2}{{}^{14}C_2} = 1 - \frac{66}{91} = \frac{91-66}{91} = \frac{25}{91}$$

45. (b) According to question,

$$n(S) = {}^{14}C_3 = \frac{14!}{(14-3)!3!} = \frac{14 \times 13 \times 12}{3 \times 2 \times 1} = 364$$

$\therefore$ Required probability

$$= 1 - \frac{{}^{11}C_3}{{}^{14}C_3} = 1 - \frac{165}{364} = \frac{364-165}{364} = \frac{199}{364}$$

46. (e) According to question,

$$n(S) = {}^{14}C_3 = \frac{14!}{(14-3)!3!} = \frac{14 \times 13 \times 12}{3 \times 2 \times 1} = 364$$

and $n(E) = {}^{10}C_3 = \frac{10!}{(10-3)!3!} = \frac{10 \times 9 \times 8}{3 \times 2 \times 1} = 120$

$\therefore$ Required probability $= \frac{n(E)}{n(S)} = \frac{120}{364} = \frac{30}{91}$