# PERMUTATIONS AND COMBINATIONS

## FUNDAMENTAL PRINCIPLE OF COUNTING

### Multiplication Principle

If an operation can be performed in '$m$' different ways; followed by a second operation performed in '$n$' different ways, then the two operations in succession can be performed in $m \times n$ ways. This can be extended to any finite number of operations.

**Illustration 1: A person wants to go from station *P* to station *R* via station *Q*. There are 4 routes from *P* to *Q* and 5 routes from *Q* to *R*. In how many ways can he travel from *P* to *R* ?**

**Solution:** He can go from $P$ to $Q$ in 4 ways and $Q$ to $R$ in 5 ways.

So number of ways of travel from $P$ to $R$ is $4 \times 5 = 20$.

**Illustration 2: A college offers 6 courses in the morning and 4 in the evening. Find the possible number of choices with the student if he wants to study one course in the morning and one in the evening.**

**Solution:** The college has 6 courses in the morning out of which the student can select one course in 6 ways.

In the evening the college has 4 courses out of which the student can select one in 4 ways.

Hence the required number of ways = $6 \times 4 = 24$.

**Illustration 3: In how many ways can 5 prizes be distributed among 4 boys when every boy can take one or more prizes ?**

**Solution:** First prize may be given to any one of the 4 boys, hence first prize can be distributed in 4 ways.

Similarly every one of second, third, fourth and fifth prizes can also be given in 4 ways.

$\therefore$ The number of ways of their distribution

$= 4 \times 4 \times 4 \times 4 \times 4 = 4^5 = 1024$

### Addition Principle

If an operation can be performed in '$m$' different ways and another operation, which is independent of the first operation, can be performed in '$n$' different ways. Then either of the two operations can be performed in $(m + n)$ ways. This can be extended to any finite number of independent operations.

**Illustration 4: A college offers 6 courses in the morning and 4 in the evening. Find the number of ways a student can select exactly one course, either in the morning or in the evening.**

**Solution:** The college has 6 courses in the morning out of which the student can select one course in 6 ways.

In the evening the college has 4 courses out of which the student can select one in 4 ways.

Hence the required number of ways = $6 + 4 = 10$.

**Illustration 5: A person wants to leave station *Q*. There are 4 routes from station *Q* to *P* and 5 routes from *Q* to *R*. In how many ways can he travel from the station *Q* ?**

**Solution:** He can go from $Q$ to $P$ in 4 ways and $Q$ to $R$ in 5 ways. To go from $Q$ to $P$ and $Q$ to $R$ are independent to each other. Hence the person can leave station $Q$ in $4 + 5 = 9$ ways.

## FACTORIALS

If $n$ is a natural number then the product of all natural numbers upto $n$ is called factorial $n$ and it is denoted by $n$ ! or $\underline{|n}$

Thus, $n! = n(n-1)(n-2) \ldots.. 3.2.1$

Note that $0! = 1 = 1!$

$$n! = n(n-1)!$$
$$= n(n-1)(n-2)!$$
$$= n(n-1)(n-2)(n-3)!, \text{ etc.}$$

For example $6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1$

But $4! = 4 \times 3 \times 2 \times 1$

$\therefore$ $6! = 6 \times 5 \times 4!$ or $6 \times 5 \times 4 \times 3!$

Remember that

$0! = 1, 1! = 1, 2! = 2, 3! = 6, 4! = 24, 5! = 120, 6! = 720$, etc.

## MEANING OF PERMUTATION AND COMBINATION

Each of the different arrangements which can be made by taking some or all of a number of things is called a permutation. Note that in an arrangement, the order in which the things arranged is considerable i.e., arrangement $AB$ and $BA$ of two letters $A$ and $B$ are different because in $AB$, $A$ is at the first place and $B$ is at the second place from left whereas in $BA$, $B$ is at the first place and $A$ is at the second place.

The all different arrangements of three letters $A$, $B$ and $C$ are $ABC$, $ACB$, $BCA$, $BAC$, $CAB$ and $CBA$.

Here each of the different arrangements $ABC$, $ACB$, $BCA$, $BAC$, $CAB$ and $CBA$ is a permutation and number of different arrangement i.e. 6 is the number of permutations.

$ABC$, $ACB$, $BCA$, $BAC$, $CAB$ and $CBA$ are different arrangements of three letters $A$, $B$ and $C$, because in each arrangement, order in which the letters arranged, is considered. But if the order in which the things are arranged is not considered; then $ABC$, $ACB$, $BCA$, $BAC$, $CAB$ and $CBA$ are not different but the same. Similarly $AB$ and $BA$ are not different but the same.

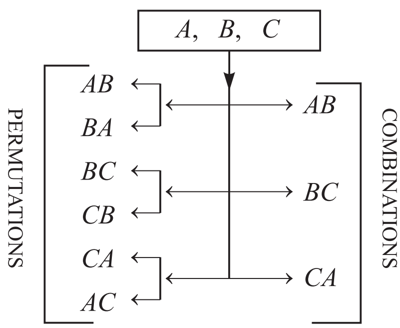
Each of the different selections or groups which can be made by some or all of a number of given things without reference to the order of things in any selection or group is called a combination.

As in selection order in which things are selected is not considered; hence, selections of two letters $AB$ and $BA$ out of three letters $A$, $B$ and $C$ are the same. Similarly selections of $BC$ and $CB$ are the same.

Also selections of $CA$ and $AC$ are the same.

Hence selection of two letters out of the three letters $A$, $B$ and $C$ can be made as $AB$, $BC$ and $CA$ only.

As in arrangements, order in which things are arranged is considered. Hence all arrangements of two letters out of the three letters $A$, $B$ and $C$ are $AB$, $BA$, $BC$, $CB$, $CA$ and $AB$.




Number of permutations (or arrangements) of two letters out of three letters $A$, $B$ and $C = 6$.

Number of combinations (or groups) of two letters out of three letters $A$, $B$ and $C = 3$.

Permutations of three different letters $A$, $B$ and $C$ taken two at a time is also understood as selections of any two different letters $AB$, $BC$ or $CA$ out of $A$, $B$ and $C$, then the selected two letters arranged in two ways as

$AB, BA$ ;  $BC, CB$  or  $CA, AC$

Hence using multiplication principle, number of permutations of three different letters $A$, $B$ and $C$ taken two at a time

= (Number of ways to select any two different letters out of the three given letters) × (Number of arrangements of two selected letters)

$= 3 \times 2 = 6$

Thus permutations means selection of some or all of the given things at a time and then arrangements of selected things. In most of the problems, it is mentioned that the problem is of permutation or combination but in some problems it is not mentioned. In the case where it is not mentioned that problem given is of permutation or combination, you can easily identify the given problem is of permutation or combination using the following classifications of problems:

### Problems of Permutations

**(i)** Problems based on arrangements
**(ii)** Problems based on standing in a line
**(iii)** Problems based on seated in a row
**(iv)** Problems based on digits
**(v)** Problems based on arrangement letters of a word
**(vi)** Problems based on rank of a word (in a dictionary)

### Problems of Combinations

**(i)** Problems based on selections or choose
**(ii)** Problems based on groups or committee
**(iii)** Problems based on geometry

If in any problem, it is neither mentioned that the problem is of permutation or combination nor does the problem fall in the categories mentioned above for the problems of permutations or problems of combinations, then do you think whether arrangement (i.e. order) is meaningful or not? If arrangement (i.e., order) is considerable in the given problem, then the problem is of permutation otherwise it is of combination. This will be more clear through the following illustrations:

Suppose you have to select three batsmen out of four batsmen $B_1$, $B_2$, $B_3$ and $B_4$, you can select three batsmen $B_1\, B_2\, B_3$, $B_2\, B_3\, B_4$, $B_3\, B_4\, B_1$ or $B_4\, B_1\, B_2$.

Here order of selections of three batsmen in any group of three batsmen is not considerable because it does not make any difference in the match.

Hence in the selection process; $B_2\, B_3\, B_4$, $B_2\, B_4\, B_3$, $B_3\, B_2\, B_4$, $B_3\, B_4\, B_2$, $B_4\, B_2\, B_3$ and $B_4\, B_3\, B_2$ all are the same.

But for batting, the order of batting is important.

Therefore for batting; $B_2\, B_3\, B_4$, $B_2\, B_4\, B_3$, $B_3\, B_2\, B_4$, $B_3\, B_4\, B_2$, $B_4\, B_2\, B_3$ and $B_4\, B_3\, B_2$, are different because $B_2\, B_3\, B_4$ means batsman $B_2$ batting first then batsman $B_3$ and then batsman $B_4$ whereas $B_2\, B_4\, B_3$ means batsman $B_2$ batting first then batsman $B_4$ and then batsman $B_3$.

## COUNTING FORMULA FOR LINEAR PERMUTATIONS

### Without Repetition

1. Number of permutations of $n$ different things, taking $r$ at a time is denoted by ${}^nP_r$ or $P(n, r)$, which is given by

$${}^nP_r = \frac{n!}{(n-r)!}\ (0 \le r \le n)$$

$$= n(n-1)\,(n-2)\ .......\ (n-r+1),$$

where $n$ is a natural number and $r$ is a whole number.

2. Number of arrangements of $n$ different objects taken all at a time is ${}^nP_n = n\,!$

**Note:**

$${}^nP_1 = n,\quad {}^nP_r = n.\,{}^{n-1}P_{r-1},\quad {}^nP_r = (n-r+1).\,{}^nP_{r-1},$$

$${}^nP_n = {}^nP_{n-1}$$

**Illustration 6: Find the number of ways in which four persons can sit on six chairs.**

**Solution:** ${}^6P_4 = 6.5.4.3 = 360$

### With Repetition

1. Number of permutations of $n$ things taken all at a time, if out of $n$ things $p$ are alike of one kind, $q$ are alike of second kind, $r$ are alike of a third kind and the rest $n - (p + q + r)$ are all different is

$$\frac{n!}{p!\,q!\,r!}$$

2. Number of permutations of $n$ different things taken $r$ at a time when each thing may be repeated any number of times is $n^r$.

**Illustration 7: Find the number of words that can be formed out of the letters of the word COMMITTEE taken all at a time.**

**Solution:** There are 9 letters in the given word in which two T's, two M's and two E's are identical. Hence the required number of words $= \frac{9!}{2!\,2!\,2!} = \frac{9!}{(2!)^3} = \frac{9!}{8} = 45360$

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## NUMBER OF LINEAR PERMUTATIONS UNDER CERTAIN CONDITIONS

1. Number of permutations of $n$ different things taken all together when $r$ particular things are to be placed at some $r$ given places $= {}^{n-r}P_{n-r} = (n-r)!$
2. Number of permutations of $n$ different things taken $r$ at a time when $m$ particular things are to be placed at $m$ given places $= {}^{n-m}P_{r-m}$
3. Number of permutations of $n$ different things, taken $r$ at a time, when a particular thing is to be always included in each arrangement, is $r\,.\,{}^{n-1}P_{r-1}$
4. Number of permutation of $n$ different things, taken $r$ at a time, when $m$ particular thing is never taken in each arrangement is ${}^{n-m}P_r$.
5. Number of permutations of $n$ different things, taken all at a time, when $m$ specified things always come together is
$$m! \times (n - m + 1)!$$
6. Number of permutations of $n$ different things, taken all at a time, when $m$ specific things never come together is
$$n! - m! \times (n - m + 1)!$$

**Illustration 8: How many different words can be formed with the letters of the word 'JAIPUR' which start with '$A$' and end with '$I$'?**

**Solution:** After putting $A$ and $I$ at their respective places (only in one way) we shall arrange the remaining 4 different letters at 4 places in 4! ways. Hence the required number $= 1 \times 4! = 24$.

**Illustration 9: How many different 3 letter words can be formed with the letters of word 'JAIPUR' when $A$ and $I$ are always to be excluded?**

**Solution:** After leaving $A$ and $I$, we are remained with 4 different letters which are to be used for forming 3 letters words. Hence the required number $= {}^4P_3 = 4 \times 3 \times 2 = 24$.

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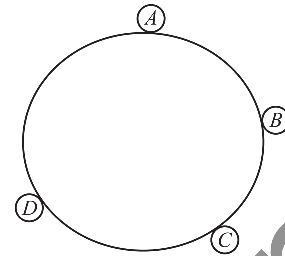
## CIRCULAR PERMUTATIONS

### 1. Arrangement Around a Circular Table

In circular arrangements, there is no concept of starting point (i.e. starting point is not defined). Hence number of circular permutations of $n$ different things taken all at a time is $(n-1)!$ if clockwise and anti-clockwise order are taken as different.




In the case of four persons $A$, $B$, $C$ and $D$ sitting around a circular table, then the two arrangements $ABCD$ (in clockwise direction) and $ADCB$ (the same order but in anti-clockwise direction) are different.

Hence the number of arrangements (or ways) in which four different persons can sit around a circular table $= (4 - 1)! = 3! = 6$.

### 2. Arrangement of Beads or Flowers (All Different) Around a Circular Necklace or Garland

The number of circular permutations of $n$ different things taken all at a time is $\frac{(n-1)!}{2}$, if clockwise and anti-clockwise order are taken as the same.

If we consider the circular arrangement, if necklace made of four precious stones $A$, $B$, $C$ and $D$; the two arrangements $ABCD$ (in clockwise direction) and $ADCB$ (the same but in anti-clockwise direction) are the same because when we take one arrangement $ABCD$ (in clockwise direction) and then turn the necklace around (front to back), then we get the arrangement $ADCB$ (the same but in anti-clockwise direction). Hence the two arrangements will be considered as one arrangement because the order of the stones is not changing with the change in the side of observation. So in this case, there is no difference between the clockwise and anti-clockwise arrangements.

Therefore number of arrangements of four different stones in the necklace $= \frac{(n-1)!}{2}$.

### 3. Number of Circular Permutations of $n$ Different Things Taken $r$ at a Time

Case I: If clockwise and anti-clockwise orders are taken as different, then the required number of circular permutations $= \frac{{}^nP_r}{r}$.

Case II: If clockwise and anti-clockwise orders are taken as same, then the required number of circular permutations $= \frac{{}^nP_r}{2r}$.

### 4. Restricted Circular Permutations

When there is a restriction in a circular permutation then first of all we shall perform the restricted part of the operation and then perform the remaining part treating it similar to a linear permutation.

**Illustration 10: In how many ways can 5 boys and 5 girls be seated at a round table so that no two girls may be together ?**

**Solution:** Leaving one seat vacant between two boys, 5 boys may be seated in 4! ways. Then at remaining 5 seats, 5 girls can sit in 5! ways. Hence the required number = 4! × 5!

**Illustration 11: In how many ways can 4 beads out of 6 different beads be strung into a ring ?**

**Solution:** In this case a clockwise and corresponding anticlockwise order will give the same circular permutation. So the required number $= \dfrac{^6P_4}{4.2} = \dfrac{6.5.4.3}{4.2} = 45$ .

**Illustration 12: Find the number of ways in which 10 persons can sit round a circular table so that none of them has the same neighbours in any two arrangements.**

**Solution:** 10 persons can sit round a circular table in 9! ways. But here clockwise and anti-clockwise orders will give the same neighbours. Hence the required number of ways $= \dfrac{1}{2}9!$.

## COUNTING FORMULA FOR COMBINATION

### 1. Selection of Objects Without Repetition

The number of combinations or selections of $n$ different things taken $r$ at a time is denoted by $^nC_r$ or $C\ (n, r)$ or $C\begin{pmatrix} n \\ r \end{pmatrix}$

where $^nC_r = \dfrac{n!}{r!(n-r)!}$; $(0 \le r \le n)$

$$= \frac{n(n-1)(n-2)...(n-r+1)}{r(r-1)(r-2)....2.1};$$

where $n$ is a natural number and $r$ is a whole number.

#### Some Important Results

**(i)** $^nC_n = 1$ , $^nC_0 = 1$ **(ii)** $^nC_r = \dfrac{^nP_r}{r!}$

**(iii)** $^nC_r = {^nC_{n-r}}$ **(iv)** $^nC_x = {^nC_y} \Rightarrow x + y = n$

**(v)** $^nC_r + {^nC_{r-1}} = {^{n+1}C_r}$ **(vi)** $^nC_r = \dfrac{n}{r}\,.\,{^{n-1}C_{r-1}}$

**(vii)** $^nC_1 = {^nC_{n-1}} = n$

**Illustration 13: If $^{20}C_r = {^{20}C_{r-10}}$, then find the value of $^{18}C_r$**

**Solution:** $^{20}C_r = {^{20}C_{r-10}} \Rightarrow r + (r - 10) = 20 \Rightarrow r = 15$

$$\therefore\ ^{18}C_r = {^{18}C_{15}} = {^{18}C_3} = \frac{18.17.16}{1.2.3} = 816$$

**Illustration 14: How many different 4-letter words can be formed with the letters of the word 'JAIPUR' when *A* and *I* are always to be included ?**

**Solution:** Since $A$ and $I$ are always to be included, so first we select 2 letters from the remaining 4, which can be done in $^4C_2 = 6$ ways. Now these 4 letters can be arranged in 4! = 24 ways, so the required number = 6 × 24 = 144.

**Illustration 15: How many combinations of 4 letters can be made of the letters of the word 'JAIPUR' ?**

**Solution:** Here 4 things are to be selected out of 6 different things.

So the number of combinations $= {^6C_4} = \dfrac{6.5.4.3}{4.3.2.1} = 15$

### 2. Selection of Objects With Repetition

The total number of selections of $r$ things from $n$ different things when each thing may be repeated any number of times is $^{n+r-1}C_r$

### 3. Restricted Selection

**(i)** Number of combinations of $n$ different things taken $r$ at a time when $k$ particular things always occur is $^{n-k}C_{r-k}$.

**(ii)** Number of combinations of $n$ different things taken $r$ at a time when $k$ particular things never occur is $^{n-k}C_r$.

### 4. Selection From Distinct Objects

Number of ways of selecting at least one thing from $n$ different things is

$^nC_1 + {^nC_2} + {^nC_3} + .....+ {^nC_n} = 2^n - 1$.

This can also be stated as the total number of combination of $n$ different things is $2^n - 1$.

**Illustration 16: Ramesh has 6 friends. In how many ways can he invite one or more of them at a dinner ?**

**Solution:** He can invite one, two, three, four, five or six friends at the dinner. So total number of ways of his invitation

$$= {^6C_1} + {^6C_2} + {^6C_4} + {^6C_5} + {^6C_6} = 2^6 - 1 = 63$$

### 5. Selection From Identical Objects

**(i)** The number of combination of $n$ identical things taking $r$ $(r \le n)$ at a time is 1.

**(ii)** The number of ways of selecting any number $r$ $(0 \le r \le n)$ of things out of $n$ identical things is $n + 1$.

**(iii)** The number of ways to select one or more things out of $(p + q + r)$ things; where $p$ are alike of first kind, $q$ are alike of second kind and $r$ are alike of third kind $= (p + 1)\ (q + 1)\ (r + 1) - 1$.

**Illustration 17: There are *n* different books and *p* copies of each in a library. Find the number of ways in which one or more than one books can be selected.**

**Solution:** Required number of ways

$$= (p + 1)(p + 1)......n \text{ terms} - 1 = (p + 1)^n - 1$$

**Illustration 18: A bag contains 3 one ₹ coins, 4 five ₹ coins and 5 ten ₹ coins. How many selection of coins can be formed by taking atleast one coin from the bag?**

**Solution:** There are 3 things of first kind, 4 things of second kind and 5 things of third kind, so the total number of selections = (3 + 1) (4 + 1) (5 + 1) – 1 = 119

## DIVISION AND DISTRIBUTION OF OBJECTS

**1.** The number of ways in which $(m + n)$ different things can be divided into two groups which contain $m$ and $n$ things respectively is

$$^{m+n}C_m\ {^nC_n} = \frac{(m+n)!}{m!n!},\ m \ne n$$

**Particular case:**
When $m = n$, then total number of ways is

$\frac{(2m)!}{(m!)^2}$, when order of groups is considered and

$\frac{(2m)!}{2!(m!)^2}$, when order of groups is not considered.

**2.** The number of ways in which $(m + n + p)$ different things can be divided into three groups which contain $m$, $n$ and $p$ things respectively is

$$^{m+n+p}C_m \cdot {}^{n+p}C_p \cdot {}^pC_p = \frac{(m+n+p)!}{m!n!p!}, m \neq n \neq p$$

**Particular case:**
When $m = n = p$, then total number of ways is

$\frac{(3m)!}{(m!)^3}$, when order of groups is considered and

$\frac{(3m)!}{3!(m!)^3}$, when order of groups is not considered.

**3.** (i) Total number of ways to divide $n$ identical things among $r$ person is $^{n+r-1}C_{r-1}$

(ii) Also total number of ways to divide $n$ identical things among $r$ persons so that each gets atleast one is $^{n-1}C_{r-1}$.

**Illustration 19: In how many ways 20 identical mangoes may be divided among 4 persons if each person is to be given at least one mango?**

**Solution:** If each person is to be given at least one mango, then number of ways will be $^{20-1}C_{4-1} = {}^{19}C_3 = 969$.

**Illustration 20: In how many ways can a pack of 52 cards be divided in 4 sets, three of them having 17 cards each and fourth just one card?**

**Solution:** Since the cards are to be divided into 4 sets, 3 of them having 17 cards each and 4th just one card, so number of ways

$= \frac{52!}{1!51!} \cdot \frac{51!}{(17!)^3 3!} = \frac{52!}{(17!)^3 3!}$.

## IMPORTANT RESULTS ABOUT POINTS

**1.** If there are $n$ points in a plane of which $m$ $(< n)$ are collinear, then

(i) Total number of different straight lines obtained by joining these $n$ points is $^nC_2 - {}^mC_2 + 1$.

(ii) Total number of different triangles formed by joining these $n$ points is $^nC_3 - {}^mC_3$

**2.** Number of diagonals of a polygon of $n$ sides is $^nC_2 - n$ *i.e.*, $\frac{n(n-3)}{2}$.

**3.** If $m$ parallel lines in a plane are intersected by a family of other $n$ parallel lines, then total number of parallelograms so formed is $^mC_2 \times {}^nC_2$ *i.e.*, $\frac{mn(m-1)(n-1)}{4}$.

**4.** Given $n$ points on the circumference of a circle, then

(i) Number of straight lines obtained by joining these $n$ points $= {}^nC_2$

(ii) Number of triangles obtained by joining these $n$ points $= {}^nC_3$

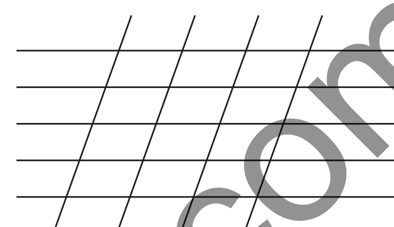
(iii) Number of quadrilaterals obtained by joining these $n$ points $= {}^nC_4$

**Illustration 21: There are 10 points in a plane and 4 of them are collinear. Find the number of straight lines joining any two of them.**

**Solution:** Total number of lines $= {}^{10}C_2 - {}^4C_2 + 1 = 40$.

**Illustration 22: If 5 parallel straight lines are intersected by 4 parallel straight lines, then find the number of parallelograms thus formed.**

**Solution:**



Number of parallelograms $= {}^5C_2 \times {}^4C_2 = 60$.

## FINDING THE RANK OF A WORD

We can find the rank of a word out of all the words with or without meaning formed by arranging all the letters of a given word in all possible ways when these words are listed as in a dictionary. You can easily understand the method to find the above mentioned rank by the following illustrations.

**Illustration 23: If the letters of the word RACHIT are arranged in all possible ways and these words (with or without meaning) are written as in a dictionary, then find the rank of this word RACHIT.**

**Solution:** The order of the alphabet of RACHIT is $A, C, H, I, R, T$.

The number of words beginning with $A$ (i.e. the number of words in which $A$ comes at first place) is $^5P_5 = 5!$.

Similarly, number of words beginning with $C$ is 5!, beginning with $H$ is 5! and beginning with $I$ is also 5!.

So before $R$, four letters $A, C, H, I$ can occur in $4 \times (5!) = 480$ ways.

Now the word RACHIT happens to be the first word beginning with $R$. Therefore the rank of this word RACHIT = 480 + 1 = 481.

**Illustration 24: The letters of the word MODESTY are written in all possible orders and these words (with or without meaning) are listed as in a dictionary then find the rank of the word MODESTY.**

**Solution:**

The order of the alphabet of MODESTY is D, E, M, O, S, T, Y.

Number of words beginning with $D$ is $^6P_6 = 6!$

Number of words beginning with $E$ is $^6P_6 = 6!$

Number of words beginning with $MD$ is $^5P_5 = 5!$

Number of words beginning with $ME$ is $^5P_5 = 5!$

Now the first word start with $MO$ is MODESTY.

Hence rank of the word MODESTY

= 6! + 6! + 5! + 5! + 5! + 1

= 720 + 720 + 120 + 120 + 1

= 1681.

# Practice Exercise

## Level - I

**1.** The sum of all the four digit even numbers which can be formed by using the digits 0, 1, 2, 3, 4 and 5 if repetition of digits is allowed is

(a) 1765980 (b) 1756980
(c) 1769580 (d) 1759680

**2.** How many words beginning with vowels can be formed with the letters of the word EQUATION?

(a) 25200 (b) 15200
(c) 25300 (d) 35200

**3.** The number of words that can be formed out of the letters of the word COMMITTEE is

(a) $\frac{9!}{(2!)^3}$ (b) $\frac{9!}{(2!)^2}$
(c) $\frac{9!}{2!}$ (d) 9!

**4.** If ${}^{10}P_r = 720$, then $r$ is equal to

(a) 4 (b) 2
(c) 3 (d) 1

**5.** Number of ways in which 12 different balls can be divided into groups of 5, 4 and 3 balls are

(a) $\frac{12!}{5!4!}$ (b) $\frac{12!}{5!4!3!}$
(c) $\frac{12!}{5!4!3!3!}$ (d) None of these

**6.** How many different letter arrangements can be made from the letter of the word EXTRA in such a way that the vowels are always together?

(a) 48 (b) 60
(c) 40 (d) 30

**7.** In how many ways can a committee of 5 made out 6 men and 4 women containing atleast one woman?

(a) 246 (b) 222
(c) 186 (d) None of these

**8.** How many integers greater than 5000 can be formed with the digit 7, 6, 5, 4 and 3, using each digit at most once?

(a) 72 (b) 144
(c) 84 (d) 192

**9.** Every body in a room shakes hands with every else. If total number of hand-shaken is 66, then number of persons in the room is

(a) 11 (b) 12
(c) 13 (d) 14

**10.** The number of words from the letters of the words BHARAT in which $B$ and $H$ will never come together, is

(a) 360 (b) 240
(c) 120 (d) None of these

**11.** A bag contains 3 black, 4 white and 2 red balls, all the balls being different. The number of at most 6 balls containing balls of all the colours is

(a) 42(4!) (b) $2^6 \times 4!$
(c) $(2^6 - 1)(4!)$ (d) None of these

**12.** How many different ways are possible to arrange the letters of the word "MACHINE" so that the vowels may occupy only the odd positions?

(a) 800 (b) 125
(c) 348 (d) 576

**13.** If ${}^nP_r = {}^nP_{r+1}$ and ${}^nC_r = {}^nC_{r-1}$, then the values of $n$ and $r$ are

(a) 4, 3 (b) 3, 2
(c) 4, 2 (d) None of these

**14.** If ${}^nP_r = 720\ {}^nC_r$, then r is equal to

(a) 3 (b) 7
(c) 6 (d) 4

**15.** In how many ways a hockey team of eleven can be elected from 16 players?

(a) 4368 (b) 4267
(c) 5368 (d) 4166

**16.** In how many ways can twelve girls be arranged in a row if two particular girls must occupy the end places?

(a) $\frac{10!}{2!}$ (b) 12!
(c) $10! \times 2!$ (d) $\frac{12!}{2!}$

**17.** To fill a number of vacancies, an employer must hire 3 programmers from among 6 applicants, and 2 managers from among 4 applicants. What is the total number of ways in which she can make her selection?

(a) 1,490 (b) 132
(c) 120 (d) 60

**18.** A father has 2 apples and 3 pears. Each weekday (Monday through Friday) he gives one of the fruits to his daughter. In how many ways can this be done?

(a) 120 (b) 10
(c) 24 (d) 12

**19.** If a secretary and a joint secretary are to be selected from a committee of 11 members, then in how many ways can they be selected?

(a) 110 (b) 55
(c) 22 (d) 11

**20.** On a railway route there are 20 stations. What is the number of different tickets required in order that it may be possible to travel from every station to every other station?

(a) 40 (b) 380
(c) 400 (d) 420

**21.** If $P(32, 6) = kC(32, 6)$, then what is the value of $k$?

(a) 6 (b) 32
(c) 120 (d) 720

**22.** How many straight lines can be formed from 8 non-collinear points on the $X$-$Y$ plane?

(a) 28 (b) 56
(c) 18 (d) 19860

**23.** A man has 3 shirts, 4 trousers and 6 ties. What are the number of ways in which he can dress himself with a combination of all the three?

(a) 13 (b) 72
(c) 13!/3!·4!·6! (d) 3!·4!·6!

**24.** If $({}^{28}C_{2r} : {}^{24}C_{2r-4}) = 225 : 11$. Find the value of $r$.

(a) 10 (b) 11
(c) 7 (d) 9

**25.** How many numbers can be formed with the digits 1, 6, 7, 8, 6, 1 so that the odd digits always occupy the odd places.

(a) 15 (b) 12
(c) 18 (d) 20

**26.** There are 20 people among whom two are sisters. Find the number of ways in which we can arrange them around a circle so that there is exactly one person between the two sisters.

(a) 18! (b) 2!19!
(c) 19! (d) None of these

**27.** In a company, each employee gives a gift to every other employee. If the number of gifts is 61, then the number of employees in the company is :

(a) 11 (b) 13
(c) 12 (d) 8

**28.** There are three rooms in a hotel: one single, one double and one for four persons. How many ways are there to house seven persons in these rooms?

(a) 7!/1!2!4! (b) 7!
(c) 7!/3 (d) 7!/3!

**29.** The digits, from 0 to 9 are written on 10 slips of paper (one digit on each slip) and placed in a box. If three of the slips are drawn and arranged, then the number of possible different arrangements is

(a) 1000 (b) 720
(c) 810 (d) None of these

**30.** The number of ways in which 7 different books can be given to 5 students if each can receive none, one or more books is

(a) $5^7$ (b) $7^5$
(c) ${}^{11}C_5$ (d) 12!

**31.** In how many ways can 13 different alphabets ($a, b, c, \ldots m$) be arranged so that the alphabets f and g never come together?

(a) 13 ! – 12 ! (b) 13 ! – 12! / 2!
(c) 13 ! – 2 × 12 ! (d) None of these

**32.** Number of ways in which the letters of word GARDEN can be arranged with vowels in alphabetical order, is

(a) 360 (b) 240
(c) 120 (d) 480

**33.** If 5 parallel straight lines are intersected by 4 parallel straight lines, then the number of parallelograms thus formed is

(a) 20 (b) 60
(c) 101 (d) 126

**34.** The number of ways in which a couple can sit around a table with 6 guests if the couple take consecutive seat is

(a) 1440 (b) 720
(c) 5040 (d) None of these

**35.** How many different words beginning with O and ending with E can be formed with the letters of the word ORDINATE, so that the words are beginning with O and ending with E?

(a) 8! (b) 6!
(c) 7! (d) 7!/2!

**36.** How many 6 digit number can be formed from the digits 1, 2, 3, 4, 5, 6 which are divisible by 4 and digits are not repeated?

(a) 192 (b) 122
(c) 140 (d) 242

**37.** In how many ways can the eight directors, the vice-chairman and the chairman of a firm be seated at a round-table, if the chairman has to sit between the vice-chairman and the director?

(a) 9! × 2 (b) 2 × 8!
(c) 2 × 7! (d) None of these

**38.** In how many different ways can the letters of the word 'CREAM' be arranged ? [*SBI Clerk-June-2012*]

(a) 720 (b) 240
(c) 360 (d) 504
(e) None of these

**39.** Which of the following words can be written in 120 different ways? [*IBPS Clerk-2012*]

(a) STABLE (b) STILL
(c) WATER (d) NOD
(e) DARE

# Level - II

**1.** 5 men and 6 women have to be seated in a straight row so that no two women are together. Find the number of ways this can be done.

(a) 48400 (b) 39600
(c) 9900 (d) 86400

**2.** The total number of ways in which 8 men and 6 women can be arranged in a line so that no 2 women are together is

(a) 48 (b) ${}^8P_8.{}^9P_6$
(c) 8! (84) (d) ${}^8C_8.{}^9C_8$

**3.** The number of different ways in which 8 persons can stand in a row so that between two particular person $A$ and $B$ there are always two person, is

(a) 60 (5!) (b) 15(4!) × (5!)
(c) 4! × 5! (d) None of these

**4.** From 6 boys and 7 girls a committee of 5 is to be formed so as to include atleast one girl. The number of ways this can be done is

(a) ${}^{13}C_4$ (b) ${}^6C_4 . {}^7C_1$
(c) $7 . {}^6C_4$ (d) ${}^{13}C_5 - {}^6C_1$

**5.** How many different nine digit numbers can be formed from the number 223355888 by rearranging its digits so that the odd digits occupy even positions?

(a) 16 (b) 36
(c) 60 (d) 180

**6.** If two dices are tossed simultaneously, the number of elements in the resulting sample space is

(a) 6 (b) 8
(c) 36 (d) 24

**7.** In how many ways can 7 persons stand in the form of a ring?

(a) $P(7, 2)$ (b) 7 !
(c) 6 ! (d) $\frac{7!}{2}$

**8.** In a football championship 153 matches were played. Every team played one match with each other team. How many teams participated in the championship?

(a) 21 (b) 18
(c) 17 (d) 15

**9.** If $P(77, 31) = x$ and $C(77, 31) = y$, then which one of the following is correct?

(a) $x = y$ (b) $2x = y$
(c) $77x = 31y$ (d) $x > y$

**10.** In how many ways can 12 papers be arranged if the best and the worst paper never come together?

(a) 12!/2! (b) 12! – 11!
(c) (12! – 11!)/2 (d) 12! – 2.11!

**11.** If a team of four persons is to be selected from 8 males and 8 females, then in how many ways can the selections be made to include at least one male.

(a) 1550 (b) 1675
(c) 1725 (d) 1750

**12.** Letters of the word DIRECTOR are arranged in such a way that all the vowels come together. Find out the total number of ways for making such arrangement.

(a) 4320 (b) 2720
(c) 2160 (d) 1120

**13.** 4 boys and 2 girls are to be seated in a row in such a way that the two girls are always together. In how many different ways can they be seated?

(a) 1200 (b) 7200
(c) 148 (d) 240

**14.** In how many ways can 7 Englishmen and 7 Americans sit down at a round table, no 2 Americans being in consecutive positions?

(a) 3628800 (b) 2628800
(c) 3628000 (d) 3328800

**15.** How many numbers greater than one million can be formed with 2, 3, 0, 3, 4, 2, 3? (repetitions not allowed)

(a) 720 (b) 360
(c) 120 (d) 240

**16.** 5 Indian and 5 American couples meet at a party & shake hands . If no wife shakes hands with her husband and no Indian wife shakes hands with a male, then the number of hand shakes that takes place in the party is

(a) 95 (b) 110
(c) 135 (d) 150

**17.** The total number of ways in which letters of the word ACCOST can be arranged so that the two C's never come together will be

(a) 120 (b) 360
(c) 240 (d) 6 ! – 2 !

**18.** In how many ways can a term of 11 cricketers be chosen from 6 bowlers. 4 wicket keepers and 11 batsmen to give a majority of bastemen if at least 4 bowlers are to be included and there is one wicket keeper?

(a) 27730 (b) 27720
(c) 17720 (d) 26720

**19.** Three dice are rolled. The number of possible outcomes in which at least one die shows 5 is

(a) 215 (b) 36
(c) 125 (d) 91

**20.** The sides $AB$, $BC$, $CA$ of a traingle $ABC$ have 3, 4 and 5 interior points respectively on them. The total number of triangles that can be constructed by using these points as vertices is

(a) 220 (b) 204
(c) 205 (d) 195

**21.** If all permutations of the letters of the word AGAIN are arranged as in dictionary, then fiftieth word is

(a) NAAGI (b) NAGAI
(c) NAAIG (d) NAIAG

**22.** All the words that can be formed using alphabets $A$, $H$, $L$, $U$ and $R$ are written as in a dictionary (no alphabet is repeated). Rank of the word RAHUL is

(a) 71 (b) 72
(c) 73 (d) 74

**23.** How many new words are possible from the letters of the word PERMUTATION?

(a) 11!/2! (b) (11!/2!) – 1
(c) 11! – 1 (d) None of these

**24.** There are five boys and three girls who are sitting together to discuss a management problem at a round table. In how many ways can they sit around the table so that no two girls are together?

(a) 1220 (b) 1400
(c) 1420 (d) 1440

**25.** How many 6-digit numbers have all three digits either all odd or all even?

(a) 31,250 (b) 28,125
(c) 15,625 (d) None of these

**26.** Out of 10 consonants and four vowels, the number of words that can be formed using six consonants and three vowels is

(a) $^{10}P_6 \times {}^6P_3$ (b) $^{10}C_6 \times {}^6C_3$
(c) $^{10}C_6 \times {}^4C_3 \times 9!$ (d) $^{10}P_6 \times {}^4P_3$

**27.** The number of 5 digit numbers that can be made using the digits 1 and 2 and in which at least one digit is different, is

(a) 30 (b) 31
(c) 32 (d) None of these

**28.** A class photograph has to be taken. The front row consists of 6 girls who are sitting. 20 boys are standing behind. The two corner positions are reserved for the 2 tallest boys. In how many ways can the students be arranged?

(a) 18! × 1440 (b) 6! × 1440
(c) 18! × 2! × 1440 (d) None of these

**29.** $A$,$B$,$C$ and $D$ are four towns any three of which are non-colinear. Then the number of ways to construct three roads each joining a pair of towns so that the roads do not form a triangle is

(a) 7 (b) 8
(c) 9 (d) More than 9

**30.** There are 10 points on a line and 11 points on another line, which are parallel to each other. How many triangles can be drawn taking the vertices on any of the line?

(a) 1,050 (b) 2,550
(c) 150 (d) 1,045

**31.** How many motor vehicle registration number plates can be formed with the digits 1, 2, 3, 4, 5 (No digits being repeated) if it is given that registration number can have 1 to 5 digits?

(a) 100 (b) 120
(c) 325 (d) 205

**32.** How many different 9-digit numbers can be formed from the number 223355888 by rearranging its digits so that the odd digits occupy even positions?

(a) 120 (b) $9!(2!)^3.3!$
(c) $(4!)(2!)^3.(3!)$ (d) None of these

**33.** There are 5 different Jeffrey Archer books, 3 different Sidney Sheldon books and 6 different John Grisham books. The number of ways in which at least one book can be given away is

(a) $2^{10} – 1$ (b) $2^{11} – 1$
(c) $2^{12} – 1$ (d) $2^{14} – 1$

**34.** How many natural numbers not more than 4300 can be formed with the digits 0, 1, 2, 3, 4 (if repetitions are allowed)?

(a) 574 (b) 570
(c) 575 (d) 569

**35.** The sides of a triangle have 4, 5 and 6 interior points marked on them respectively. The total number of triangles that can be formed using any of these points

(a) 371 (b) 415
(c) 286 (d) 421

**36.** Total number of ways in which six '+' and four '–' sings can be arranged in a line such that no two '–' sings occur together, is

(a) 35 (b) 18
(c) 15 (d) 42

**37.** In how many ways can 5 prizes be distributed among 4 boys when every boy can take one or more prizes?

(a) 1024 (b) 625
(c) 120 (d) 600

**38.** Three dice are rolled. The number of possible outcomes in which at least one die shows 5 is

(a) 215 (b) 36
(c) 125 (d) 91

**39.** Find the number of numbers between 300 and 3000 that can be formed with the digits 0, 1, 2, 3, 4 and 5, no digit being repeated. [*SBI PO-2011*]

(a) 120 (b) 160
(c) 240 (d) 60
(e) None of these

## Level-I

**1. (c)** □□□□

Last place can be filled by 0, 2, 4

So total sum = $5 \times 6 \times 6 (0 + 2 + 4) + 5 \times 6 \times 3 \times 10 (0 + 1 + 2 + 3 + 4 + 5) + 5 \times 6 \times 3 \times 100 (0 + 1 + 2 + 3 + 4 + 5) + 6 \times 6 \times 3 \times 1000 (0 + 1 + 2 + 3 + 4 + 5)$

$= 180 \times 6 + 900 \times 15 + 9000 \times 15 + 10800 \times 15$

$= 1080 + 13500 + 135000 + 1620000 = 1769580$

**2. (a)** There are 8 letters in the word EQUATION.

| A/E/I/O/U | | | | | | | |
|---|---|---|---|---|---|---|---|
| 5 ways | $^7P_7 = 7! = 5040$ | | | | | | |

$\therefore$ Reqd. no. $= 5 \times 5040 = 25200$

**3. (a)** There are 9 letters in the given word in which two T's, two M's and two E's are identical. Hence the required number of words $= \dfrac{9!}{2!2!2!} = \dfrac{9!}{(2!)^3}$

**4. (c)** Given, $^{10}P_r = 720$

$\therefore \dfrac{10!}{(10-r)!} = 720$

$\therefore 10 \times 9 \times 8 \times \ldots$ to $r$ factors $= 720 = 10 \times 9 \times 8$

$\therefore r = 3$

**5. (b)** $\dfrac{12!}{5!4!3!}$

**6. (a)** Considering the two vowels *E* and *A* as one letter, the total no. of letters in the word 'EXTRA' is 4 which can be arranged in $^4P_4$, i.e. 4! ways and the two vowels can be arranged among themselves in 2! ways.

$\therefore$ reqd. no. $= 4! \times 2! = 4 \times 3 \times 2 \times 1 \times 2 \times 1 = 48$

**7. (a)** A committee of 5 out of 6 + 4= 10 can be made in $^{10}C_5 = 252$ ways.

If no woman is to be included, then number of ways $= {}^5C_5 = 6$

$\therefore$ the required number $= 252 - 6 = 246$

**8. (d)** 4 digit number | 3 | 4 | 3 | 2 | = 72,

5 digit number = 120

Total = 192

**9. (b)** If number of persons be $n$, then total number of hand-shaken $= {}^nC_2 = 66$

$\Rightarrow n(n-1) = 132 \Rightarrow (n + 11)(n = 12) = 0$

$\therefore n = 12 \qquad (\because n \neq -11)$

**10. (b)** There are 6 letters in the word BHARAT, 2 of them are identical.

Hence total number of words with these letter = 360

Also the number of words in which *B* and *H* come together = 120

$\therefore$ The required number of words = 360 – 120 = 240

**11. (a)** The required number of selections

$= {}^3C_1 \times {}^4C_1 \times {}^2C_1 ({}^6C_3 + {}^6C_2 + {}^6C_0) = 42 \times 4!$

**12. (d)** MACHINE has 4 consonants and 3 vowels.

The vowels can be placed in position no. 1, 3, 5, 7

$\Rightarrow$ Total number of ways possible $= {}^4P_3 = 24$.

For each of these 24 ways the 4 consonants can occupy the other 4 places in $^4P_4$ ways

$\Rightarrow$ Total $= 24 \times 24 = 576$

**13. (b)** We have, $^nP_r = {}^nP_{r+1}$

$\Rightarrow \dfrac{n!}{(n-r)!} = \dfrac{n!}{(n-r-1)!} \Rightarrow \dfrac{1}{(n-r)} = 1$

or $n - r = 1$ ...(1)

Also, $^nC_r = {}^nC_{r-1} \Rightarrow r + r - 1 = n$

$\Rightarrow 2r - n = 1$ ...(2)

Solving (1) and (2), we get $r = 2$ and $n = 3$

**14. (c)** $^nP_r = 720\,{}^nC_r$

or $\dfrac{n!}{(n-r)!} = \dfrac{720(n!)}{(n-r)!r!}$

$\Rightarrow r! = 720 = 1 \times 2 \times 3 \times 4 \times 5 \times 6!$

or $r = 6$

**15. (a)** Total number of ways $= {}^{16}C_{11} = \dfrac{16!}{11! \times 5!} = 4368.$

$= \dfrac{16 \times 15 \times 14 \times 13 \times 12}{5 \times 4 \times 3 \times 2 \times 1} = 4368.$

**16. (c)** Two particular girls can be arranged in 2! ways and remaining 10 girls can be arranged in 10! ways.

Required no. of ways $= 2! \times 10!$

**17. (c)** Required no. of the ways $= {}^6C_3 \times {}^4C_2 = 20 \times 6 = 120$

**18. (b)** Required number of ways $= \dfrac{5!}{2!3!} = 10$.

**19. (b)** Selection of 2 members out of 11 has $^{11}C_2$ number of ways

$^{11}C_2 = 55$

**20. (b)** From each railway station, there are 19 different tickets to be issued. There are 20 railway station

So, total number of tickets $= 20 \times 19 = 380$.

**21. (d)** Since $^{32}P_6 = k\,{}^{32}C_6$

$\Rightarrow \dfrac{32!}{(32-6)!} = k.\dfrac{32!}{6!(32-6)!}$

$\Rightarrow k = 6! = 720$

**22. (a)** For a straight line we just need to select 2 points out of the 8 points available. $^8C_2$ would be the number of ways of doing this.

**23. (b)** $^3C_1 \times {}^4C_1 \times {}^6C_1 = 72$

**24. (c)** At $r = 7$, the value becomes
(28!/14! × 14!) /(24!/10! × 14!) → 225 : 11

**25. (c)** The digits are 1, 6, 7, 8, 7, 6, 1. In this seven-digit no. there are four odd places and three even places OEOEOEO. The four odd digits 1, 7, 7, 1 can be arranged in four odd places in [4!/2!×2] = 6 ways [as 1 and 7 are both occurring twice].
The even digits 6, 8, 6 can be arranged in three even places in 3!/2! = 3 ways.
Total no. of ways = 6 × 3 = 18

**26. (d)** First arrange the two sisters around a circle in such a way that there will be one seat vacant between them. [This can be done in 2! ways since the arrangement of the sisters is not circular.]
Then, the other 18 people can be arranged on 18 seats in 18! ways.

**27. (c)** Let the total number of employees in the company be $n$.

Total number of gifts = $^nC_2 = \frac{n(n-1)}{2} = 61$

$\Rightarrow\ n^2 - n - 132 = 0$ or $(n+11)(n-12) = 0$
or $n = 12$ [– 11 is rejected]

**28. (a)** Choose 1 person for the single room & from the remaining choose 2 people for the double room & from the remaining choose 4 people for the 4 persons room $\rightarrow {}^7C_1 \times {}^6C_2 \times {}^4C_4$.

**29. (b)** $^{10}P_3 = 720$

**30. (a)** Ist book can be given to any of the five students. Similarly other six books also have 5 choices. Hence the total number of ways is $5^7$.

**31. (c)** Total possible arrangements = $^{13}P_{13}$ = 13!
Total number in which $f$ and $g$ are together
= 2 × $^{12}P_{12}$ = 2 × 12!

**32. (a)** Order of vowels of fixed

∴ required number of ways are $\frac{6!}{2!}$

**33. (b)** Number of parallelograms = $^5C_2 \times {}^4C_2 = 60$.

**34. (a)** A couple and 6 guests can be arranged in (7 – 1) ways. But in two people forming the couple can be arranged among themselves in 2! ways.
∴ the required number of ways = 6! × 2! = 1440

**35. (b)** 6! ways, O fixed 1st and E fixed in last.

**36. (a)** For the number to be divisible by 4, the last two digits must be any of 12, 24, 16, 64, 32, 36, 56 and 52. The last two digit places can be filled in 8 ways. Remaining 3 places in $^4P_3$ ways. Hence no. of 5 digit nos. which are divisible by 4 are 24 × 8 = 192.

**37. (b)** Let the vice-chairman and the chairman from 1 unit along with the eight directors, we now have to arrange 9 different units in a circle.
This can be done in 8! ways.
At the same time, the vice-chairman & the chairman can be arranged in two different ways. Therefore, the total number of ways = 2 × 8!.

**38. (e)** C R E A M
1 2 3 4 5
Required number of ways = 5!
= 5 × 4 × 3 × 2 × 1 = 120

**39. (c)** (a) The word STABLE has six distinct letters.
∴ Number of arrangements = 6 !
= 6 × 5 × 4 × 3 × 2 × 1 = 720
(b) The word STILL has five letters in which letter ‘L’ comes twice.
∴ Number of arrangements
$= \frac{5!}{2} = 60$
(c) The word WATER has five distinct letters.
∴ Number of arrangements = 5 ! = 5 × 4 × 3 × 2× 1 = 120
(d) The word ‘NOD’ has 3 distinct letters.
∴ Number of arrangements = 3 ! = 6
(e) Number of arrangements = 4 ! = 24

## Level-II

**1. (d)** Total seats = 5 + 6 = 11.
Arrangement will be : *W M W M W M W M W M W*
⇒ Total possible arrangements will be :
$^6P_6 \times {}^5P_5 = 86400$.

**2. (b)** 8 men can sit in a row in $^8P_8$ ways. Then for the 6 women, there are 9 seats to sit
∴ the women can sit in $^9P_6$ ways
∴ total number of ways = $^8P_8 \cdot {}^9P_6$

**3. (a)** The number of 4 persons including $A$, $B$ = $^6C_2$
Considering these four as a group, number of arrangements with the other four = 5!
But in each group the number of arrangements
= 2! × 2!
∴ The required number of ways = $^6C_2$ × 5! × 2! × 2!

**4. (d)** From total 13 members 5 can be select as $^{13}C_5$
For at least one girl in the committee, number of ways are $^{13}C_5 - {}^6C_1$

**5. (c)** $X$ - $X$ - $X$ - $X$ - $X$. The four digits 3, 3, 5,5 can be arranged

at (–) places in $\frac{4!}{2!2!}$ = 6 ways.

The five digits 2, 2, 8, 8, 8 can be arranged at ($X$) places

in $\frac{5!}{2!3!}$ ways = 10 ways

Total no. of arrangements = $6 \times 10 = 60$ ways

**6.** **(c)** Number of elements in the sample space
$= 6 \times 6 = 36$
(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)

**7.** **(c)** Number of ways in which 7 persons can stand in the form of a ring $= (7 - 1)! = 6!$

**8.** **(b)** Let total no. of team participated in a championship be $n$.
Since, every team played one match with each other team.

$\therefore\ {}^nC_2 = 153 \Rightarrow \dfrac{n!}{2!(n-2)!} = 153$

$\Rightarrow \dfrac{n(n-1)(n-2)!}{2!(n-2)!} = 153 \Rightarrow \dfrac{n(n-1)}{2} = 153$

$\Rightarrow n(n-1) = 306$
$\Rightarrow n^2 - n - 306 = 0$
$\Rightarrow n^2 - 18n + 17n - 306 = 0$
$\Rightarrow n(n-18) + 17(n-18) = 0$
$\Rightarrow n = 18, -17$
$n$ cannot be negative
$\therefore n \neq -17$
$\Rightarrow n = 18$

**9.** **(d)** As we know
$P(n, r) = r!\ C(n, r)$
$\therefore$ From the question, we have
$x = r!\,(y)$
Here $r = 31$
$\therefore\ x = (31)!.\ y.$

**10.** **(d)** All arrangements – Arrangements with best and worst paper together $= 12! - 2! \times 11!$.

**11.** **(d)** $1\text{ m} + 3f = {}^8C_1 \times {}^8C_3 = 8 \times 56 = 448$
$2\text{ m} + 2f = {}^8C_2 \times {}^8C_2 = 28 \times 28 = 784$
$3\text{ m} + 1f = {}^8C_3 \times {}^8C_1 = 56 \times 8 = 448$
$4\text{ m} + 8f = {}^8C_4 \times {}^8C_0 = 70 \times 1 = 70$
Total = 1750

**12.** **(c)** Taking all vowels (IEO) as a single letter (since they come together) there are six letters among which there are two R.
Hence no. of arrangements $= \dfrac{6!}{2!} \times 3! = 2160$
There vowels can be arranged in 3! ways among themselves, hence multiplied with 3!.

**13.** **(d)** Assume the 2 given students to be together (i.e. one). Now these are five students.
Possible ways of arranging them are $= 5! = 120$
Now they (two girls) can arrange themselves in 2! ways.
Hence total ways $= 120 \times 2 = 240$

**14.** **(a)** Putting l Englishman in a fixed position, the remaining 6 can be arranged in 6! 720 ways, For each such arrangement, there are 7 positions for the 7 Americans and they can be arranged in 7! ways.
Total number of arrangements $= 7! \times 6! = 3628800$

**15.** **(b)** Required number is greater than 1 million (7 digits).
From given digits, total numbers which can be formed $= 7!$
Number starting from zero $= 6!$
$\Rightarrow$ Required number $= 7! - 6!$
$\because$ Repetition not allowed, so required answer
$= \dfrac{7! - 6!}{2!3!} = 360$

**16.** **(c)** Total number of hand shakes $= {}^{20}C_2$ of those no Indian female shakes hand with male
$\Rightarrow 5 \times 10 = 50$ hand shakes
No American wife shakes hand with her husband
$= 5 \times 1 = 5$ hand shakes
$\Rightarrow$ total number of hand shakes occurred
$= {}^{20}C_2 - (50 + 5) = 190 - 55 = 135$

**17.** **(c)** Total number of ways to permute 6 alphabets 2 of which are common $= 6!/2! = 360$.
(1) Treat the two C's as one
$\Rightarrow$ Number of possible ways $= {}^5P_5 = 120$
(b) Number of ways = Total arrangements – Number of arrangements in which they always come together $= 360 - 120 = 240$.

**18.** **(b)** 1 wicket keeper from 4 can be selected in
${}^4C_1 = \dfrac{4!}{3!.1!} = 4$ ways
If 4 bowlers are chosen then remaining 6 batsmen - can be chosen in ${}^{11}C_6$.

${}^6C_4.\ {}^{11}C_6 = \dfrac{6!}{4!.2!} \times \dfrac{11!}{3!.1!} = \dfrac{5 \times 6}{2} \times \dfrac{11\times10\times9\times8\times7}{5\times4\times3\times2}$

$= 15 \times 14 \times 33 = 6930$

If we choose 5 bowlers then we have to choose 5 batsmen
$\therefore$ there is no majority.
$\therefore$ Total number of ways $= 4 \times 6930 = 27720$.

**19.** **(d)** Required number of possible outcomes
= Total number of possible outcomes –
Number of possible outcomes in which 5 does not appear on any dice. (hence 5 possibilities in each throw)
$= 6^3 - 5^3 = 216 - 125 = 91$

**20.** **(c)** We have in all 12 points. Since, 3 points are used to form a traingle, therefore the total number of traingles including the triangles formed by collinear points on $AB$, $BC$ and $CA$ is ${}^{12}C_3 = 220$. But this includes the following :

The number of traingles formed by 3 points on $AB = {}^3C_3 = 1$

The number of triangles formed by 4 points on $BC = {}^4C_3 = 4$.

The number of triangles formed by 5 points on $CA = {}^5C_3 = 10$.

Hence, required number of traingles $= 220 - (10 + 4 + 1) = 205$.

**21.** **(c)** Starting with the letter $A$, and arranging the other four letters, there are $4! = 24$ words. These are the first 24 words. Then starting with $G$, and arranging $A$, $A$, $I$, and $N$ in different ways, there are $\frac{4!}{2!1!1!} = \frac{24}{2} = 12$ words.

Hence, total 36 words.

Next, the 37th word starts with I. There are 12 words starting with I. This accounts up to the 48th word. The 49th word is NAAGI. The 50th word is NAAIG.

**22.** **(d)** No. of words starting with $A$ are $4\ ! = 24$

No. of words starting with $H$ are $4\ ! = 24$

No. of words starting with $L$ are $4\ ! = 24$

These account for 72 words

Next word is RAHLU and the $74^{th}$ word RAHUL

**23.** **(b)** Number of 11 letter words formed from the letter P, E, R, M, U, T, A, I, O, N = 11!/2!.

Number of new words formed = total words – 1 = 11!/2! – 1.

**24.** **(d)** We have no girls together, let us first arrange the 5 boys and after that we can arrange the girls in the space between the boys.

Number of ways of arranging the boys around a circle $= [5 - 1]! = 24$.

Number of ways of arranging the girls would be by placing them in the 5 spaces that are formed between the boys. This can be done in ${}^5P_3$ ways = 60 ways.

Total arrangements $= 24 \times 60 = 1440$.

**25.** **(b)** When all digits are odd

$5 \times 5 \times 5 \times 5 \times 5 \times 5 = 5^6$

When all digits are even

$4 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 = 4 \times 5^5$

$5^6 + 4 \times 5^5 = 28125$

**26.** **(c)** Six consonants and three vowels can be selected from 10 consonants and 4 vowels in ${}^{10}C_6 \times {}^4C_3$ ways. Now, these 9 letters can be arranged in 9! ways. So, required number of words $= {}^{10}C_6 \times {}^4C_3 \times 9!$.

**27.** **(a)** Total number of numbers without restriction $= 2^5$

Two numbers have all the digits equal. So, the required numbers $= 2^5 - 2 = 30$.

**28.** **(a)** Two tallest boys can be arranged in 2! ways. Rest 18 can be arranged in 18! ways.

Girls can be arranged in 6! ways.

Total number of ways of arrangement $= 2! \times 18! \times 6! = 18! \times 2 \times 720 = 18! \times 1440$

**29.** **(d)** To construct 2 roads, three towns can be selected out of 4 in $4 \times 3 \times 2 = 24$ ways.

Now if the third road goes from the third town to the first town, a triangle is formed, and if it goes to the fourth town, a triangle is not formed. So, there are 24 ways to form a triangle and 24 ways of avoiding a triangle.

**30.** **(d)** For a triangle, two points on one line and one on the other has to be chosen.

No. of ways $= {}^{10}C_2 \times {}^{11}C_1 + {}^{11}C_2 \times {}^{10}C_1 = 1{,}045$ .

**31.** **(c)** Single digit numbers = 5

Two digit numbers $= 5 \times 4 = 20$

Three digit numbers $= 5 \times 4 \times 3 = 60$

Four digit numbers $= 5 \times 4 \times 3 \times 2 = 120$

Five digit numbers $= 5 \times 4 \times 3 \times 2 \times 1 = 120$

Total $= 5 + 20 + 60 + 120 + 120 = 325$

**32.** **(d)** The odd digits have to occupy even positions. This can be done in $\frac{4!}{2!2!} = 6$ ways

The other digits have to occupy the other positions.

This can be done in $\frac{5!}{3!2!} = 10$ ways

Hence total number of rearrangements possible $= 6 \times 10 = 60$.

**33.** **(d)** For each book we have two options, give or not give. Thus, we have a total of $2^{14}$ ways in which the 14 books can be decided upon. Out of this, there would be 1 way in which no book would be given. Thus, the number of ways is $2^{14} - 1$.

**34.** **(c)** The condition is that we have to count the number of natural numbers not more than 4300.

The total possible numbers with the given digits $= 5 \times 5 \times 5 \times 5 = 625 - 1 = 624$.

Subtract from this the number of natural number greater than 4300 which can be formed from the given digits $= 1 \times 2 \times 5 \times 5 - 1 = 49$.

Hence, the required number of numbers $= 624 - 49$.

**35.** **(d)** You can form triangles by taking 1 point from each side, or by taking 2 points from any 1 side and the third point from either of the other two sides.

This can be done in: $4 \times 5 \times 6 = {}^4C_2 \times {}^{11}C_1 + {}^5C_2 \times {}^{10}C_1 + {}^6C_2 \times {}^9C_1 = 120 + 66 + 100 + 135 = 421$

**36.** **(a)** First we write six '+' sings at alternate places i.e., by leaving one place vacant between two successive '+' sings. Now there are 5 places vacant between these sings and these are two places vacant at the ends. If we write 4 '–' sings these 7 places then no two '–' will come together. Hence total number of ways ${}^7C_4 = 35$

**37.** **(a)** First prize may be given to any one of the 4 boys, hence first prize can be distributed in 4 ways.

Similarly every one of second, third, fourth and fifth prizes can also be given in 4 ways.

∴ the number of ways of their distribution

$$= 4 \times 4 \times 4 \times 4 \times 4 = 4^5 = 1024.$$

**38.** **(d)** Required number of possible outcomes

= Total number of possible outcomes – Number of possible outcomes in which 5 does not appear on any dice

$= 6^3 - 5^3 = 91.$

**39.** **(e)**

| 3 | 5 | 4 |
|---|---|---|
| (3 or 4 or 5) | | |

$3 \times 5 \times 4 = 60$

| 2 | 5 | 4 | 3 |
|---|---|---|---|
| (1 or 2) | | | |

$2 \times 5 \times 4 \times 3 = 120$

Total = 120 + 60 = 180