Electromagnetic Fields

Index

Basics of Field Theory

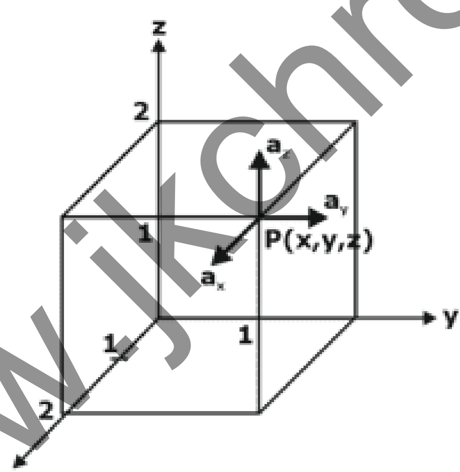
1. Cartesian Coordinate System.

In the Cartesian system, the 3 base vectors are $\hat{a}_x$, $\hat{a}_y$ and $\hat{a}_z$. Any point in space can be written in the form, $\vec{P} = x_1\hat{a}_x + y_1\hat{a}_y + z_1\hat{a}_z$

where (x_1, y_1, z_1) are the coordinates of the point P in the Cartesian space which is the intersection of the three planes $x = x_1$, $y = y_1$, $z = z_1$. The distance of the point from the origin is given by,

$$|\vec{P}| = \sqrt{x_1^2 + y_1^2 + z_1^2}$$

The figure below depicts point P in the Cartesian Coordinate System. As you can see x_1,y_1 and z_1 can also be understood as the perpendicular distance of point P from the YZ, XZ and XY plane,



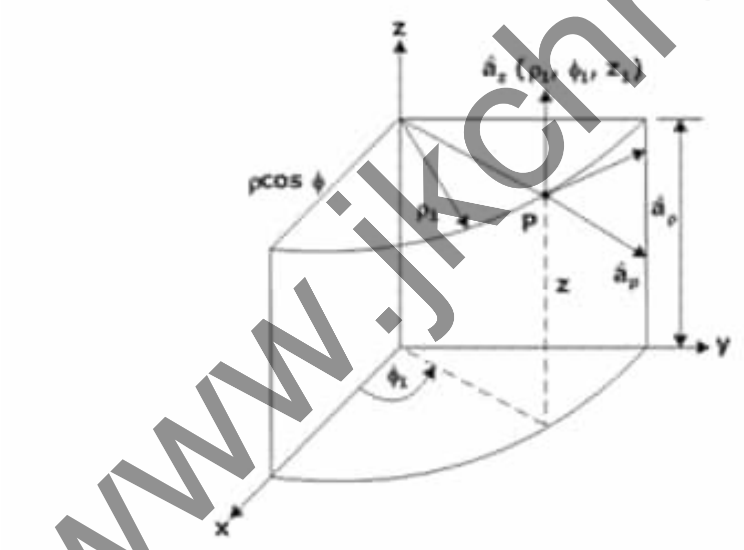
- The differential length, differential surface and differential volume are given by,

$$\vec{dl} = dx\hat{a}_x + dy\hat{a}_y + dz\hat{a}_z$$
$$\vec{dS}_x = dydz\hat{a}_x$$
$$\vec{dS}_y = dxdz\hat{a}_y$$
$$\vec{dS}_z = dxdy\hat{a}_z$$
$$dV = dxdydz$$

2. Cylindrical Coordinate System.

In the cylindrical system, the three base vectors are $\hat{a}_r$, $\hat{a}_\phi$ and $\hat{a}_z$. Any point in space can be written in the form, $\vec{P} = r_1\hat{a}_r + \phi_1\hat{a}_\phi + z_1\hat{a}_z$,

where (r_1, φ_1, z_1) are the coordinates of the point P in the Cylindrical Space. The point P is the intersection of a circular cylinder surface $r = r_1$, a half-plane containing z-axis and making an angle $\varphi = \varphi_1$ with the XZ plane and a plane $z = z_1$ parallel to the XY plane. φ_1 is measured from the positive x-axis and the base vector $\hat{a}_\varphi$ is tangential to the cylindrical surface.



The distance of the point from the origin is given by,

$$|\vec{P}| = \sqrt{r_1^2 + z_1^2}$$

- The values of the 3 coordinates vary as follows,

$$r \in [0, \infty)$$
$$\phi \in [0, 2\pi)$$
$$z \in (-\infty, \infty)$$

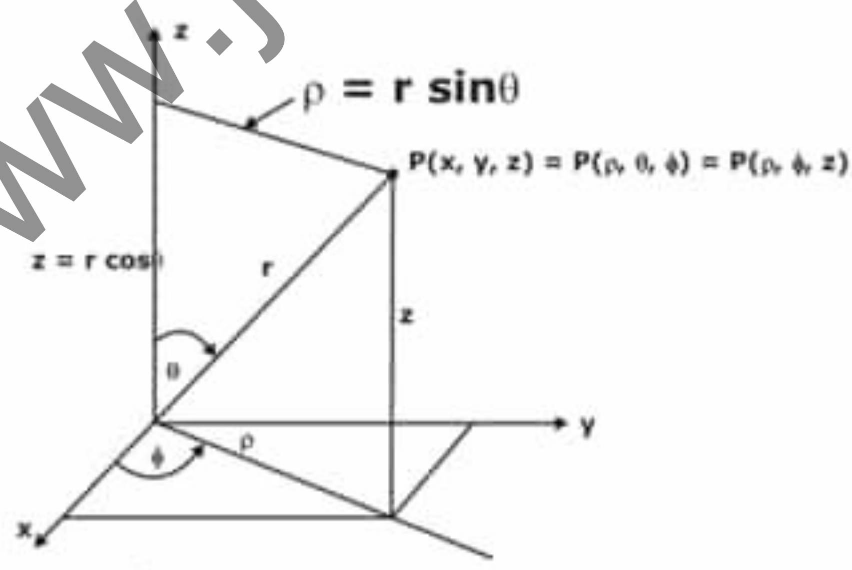
- the differential length, differential surface and differential volume are given by,

$$\vec{dl} = dr\hat{a}_r + rd\phi\hat{a}_\phi + dz\hat{a}_z$$
$$\vec{dS}_r = rd\phi dz\hat{a}_r$$
$$\vec{dS}_\phi = drdz\hat{a}_\phi$$
$$\vec{dS}_z = rd\phi dr\hat{a}_z$$
$$dV = rdrd\phi dz$$

3. Spherical Coordinate System.

In the Spherical system, the 3 bases are ˆar, ˆaθ and ˆaφ. Any point in space can be written in the form, $\vec{P} = r_1\hat{a}_r + \theta_1\hat{a}_\theta + \phi_1\hat{a}_\phi$

where (r, θ, φ) are the coordinates of the point P in the Spherical Space. A point $P(r_1, \theta_1, \varphi_1)$ in the spherical coordinates are specified as the intersection of the following three surfaces: a spherical surface centred at the origin and has a radius r_1, a right circular cone with its apex at origin and half angle θ_1 and a half-plane containing z-axis and making an angle φ_1 with the XZ plane,



- The distance of the point from the origin is given by, $|\vec{P}| = r_1$
- The values of the 3 coordinates vary as follows,

$$r \in [0, \infty)$$
$$\theta \in [0, \pi]$$
$$\phi \in [0, 2\pi)$$

- the differential length, differential surface and differential volume are given by,

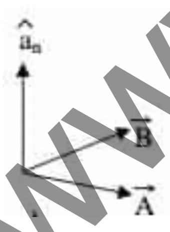
$$\vec{dl} = dr\hat{a}_r + rd\theta\hat{a}_\theta + r\sin\theta d\phi\hat{a}_\phi$$
$$\vec{dS}_r = r^2 \sin\theta d\theta d\phi\hat{a}_r$$
$$\vec{dS}_\theta = r\sin\theta dr d\phi\hat{a}_\theta$$
$$\vec{dS}_\phi = r dr d\theta\hat{a}_\phi$$
$$dV = r^2 \sin\theta dr d\theta d\phi$$

3. Scalar and Vector Products

- **Dot Product:** is also called scalar product. Let 'θ' be the angle between vectors A and B.

$$\vec{A} \cdot \vec{B} = |A| |B| \cos\theta$$

- **Cross product:** is also called vector product.



$$\vec{A} \times \vec{B} = |A| |B| \sin\theta \, \hat{a}_n$$

S = |S| ân where |S| = |A| |B| Sinθ, To find the direction of S, consider a right threaded screw being rotated from A to B. i.e. perpendicular to the plane containing the vectors A and B. therefore, $\vec{A} \times \vec{B} = -(\vec{B} \times \vec{A})$

Cartesian coordinate to Cylindrical-coordinate Conversion:

Point transformation,

$$\rho = \sqrt{x^2 + y^2}$$

$$\phi = \tan^{-1}\frac{y}{x}$$

$$z = z$$

or

$$x = \rho\cos\phi$$

$$y = \rho\sin\phi$$

$$z = z$$

The relationship between are vector transformation,

The relationship between $\hat{a}_x, \hat{a}_y, \hat{a}_z$ and $\hat{a}_\rho, \hat{a}_\phi, \hat{a}_z$ are vector transformation,

$$\hat{a}_x = \cos\phi\ \hat{a}_\rho - \sin\phi\ \hat{a}_\phi$$

$$\hat{a}_y = \sin\phi\ \hat{a}_\rho + \cos\phi\ \hat{a}_\phi$$

$$\hat{a}_z = \hat{a}_z$$

or $\hat{a}_\rho = \cos\phi\,\hat{a}_x + \sin\phi\,\hat{a}_y$

$$\hat{a}_\phi = -\sin\phi\,\hat{a}_x + \cos\phi\,\hat{a}_y$$

$$\hat{a}_z = \hat{a}_z$$

Finally, the relationship between (A_x, A_y, A_z) and (A_ρ, A_ϕ, A_z) are

$$\begin{vmatrix} A_\rho \\ A_\phi \\ A_z \end{vmatrix} = \begin{vmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} A_x \\ A_y \\ A_z \end{vmatrix}$$

$$\begin{vmatrix} A_x \\ A_y \\ A_z \end{vmatrix} = \begin{vmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} A_\rho \\ A_\phi \\ A_z \end{vmatrix}$$

Cartesian coordinate to Spherical coordinate:

Point transformation,

$$r = \sqrt{x^2 + y^2 + z^2}$$

$$\theta = \tan^{-1} \frac{\sqrt{x^2 + y^2 + z^2}}{z}$$

$$\phi = \tan^{-1} \frac{y}{x}$$

Or

$$x = r \sin\theta \cos\phi$$

$$y = r \sin\theta \sin\phi$$

$$z = r \cos\theta$$

The relationship between $\hat{a}_x, \hat{a}_y \hat{a}_z$ and $\hat{a}_r, \hat{a}_\theta, \hat{a}_\phi$ are

$$\hat{a}_x = \sin\theta\cos\phi\ \hat{a}_r + \cos\theta\cos\phi\ \hat{a}_\theta - \sin\phi\hat{a}_\phi$$

$$\hat{a}_y = \sin\theta\sin\phi\ \hat{a}_r + \cos\theta\sin\phi\ \hat{a}_\theta + \cos\phi\hat{a}_\phi$$

$$\hat{a}_z = \cos\phi\hat{a}_r - \sin\phi\hat{a}_\phi$$

$$\hat{a}_r = \sin\theta\cos\phi\hat{a}_x + \sin\theta\sin\phi\hat{a}_y + \cos\phi\hat{a}_z$$

$$\hat{a}_\theta = \cos\theta\cos\phi\hat{a}_x + \cos\theta\sin\phi\hat{a}_y - \sin\phi\hat{a}_z$$

$$\hat{a}_\phi = -\sin\phi\hat{a}_x + \cos\phi\hat{a}_y$$

Finally, the relationship between (A_x, A_y, A_z) and $\left(A_r, A_\theta, A_\phi\right)$ are

Vector transformation,

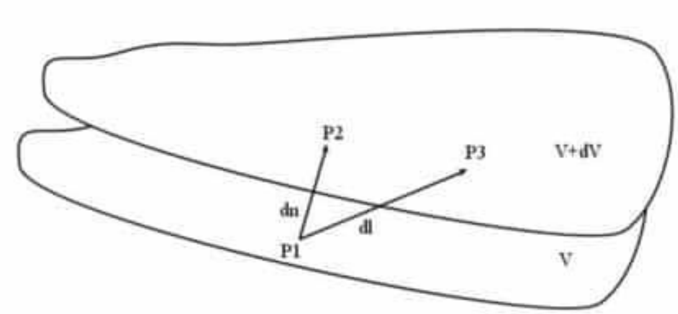
$$\begin{vmatrix} A_r \\ A_\theta \\ A\phi \end{vmatrix} = \begin{vmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{vmatrix} \begin{vmatrix} A_x \\ A_y \\ A_z \end{vmatrix}$$

$$\begin{vmatrix} A_x \\ A_y \\ Az \end{vmatrix} = \begin{vmatrix} \sin\theta\cos\phi & \cos\theta\cos\phi & -\sin\phi \\ \sin\theta\cos\phi & \cos\theta\sin\phi & \cos\phi \\ \cos\phi & -\sin\phi & 0 \end{vmatrix} \begin{vmatrix} A_r \\ A_\theta \\ A_\phi \end{vmatrix}$$

3. Gradient, Curl & Divergence

- **Gradient:** Gradient of a scalar field V, is a vector that represents both magnitude and direction of maximum space rate of change of V. The gradient of V, ∇V, will always be perpendicular to a constant V surface. If = ∇V, then V is said to be the scalar potential of

$\vec{A}$



The vector that represents the magnitude and direction of the maximum space rate of increase of a scalar is the gradient of that scalar. $grad\, V = \hat{a}_n \frac{dV}{dn}$

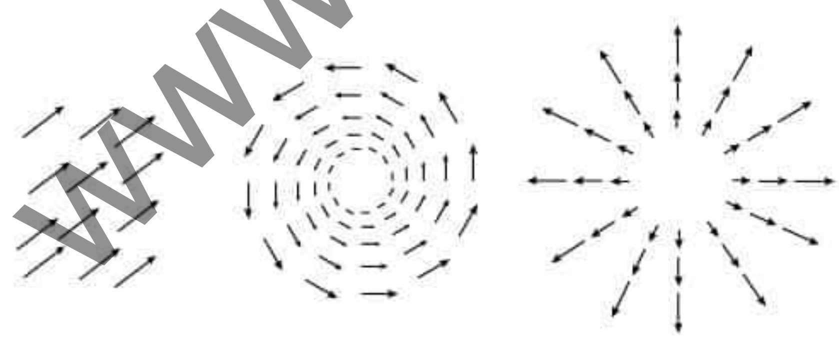
It can be shown that the gradient operator in the three coordinate systems are

$$\nabla V = \frac{\partial V}{\partial x}\alpha_z + \frac{\partial V}{\partial y}\alpha_y + \frac{\partial V}{\partial z}\alpha_z \quad \text{(Rectangular coordinates)}$$

$$\nabla V = \frac{\partial V}{\partial \rho}\alpha_\rho + \frac{1\partial V}{\rho\partial\phi}\alpha\phi + \frac{\partial V}{\partial z}\alpha_z \quad \text{(Cylindrical coordinates)}$$

$$\nabla V = \frac{\partial V}{\partial r}\alpha_r + \frac{1\partial V}{r\partial\theta}\alpha\theta + \frac{1}{r\sin\theta}\frac{\partial V}{\partial\phi}\alpha_\phi \quad \text{(Spherical coordinates)}$$

- **A divergence of a Vector Field:** The spatial derivatives of a vector field are represented through divergence and curl. It is usually convenient to represent vector field variations in space as field lines or flux lines whose directions indicate the direction of these lines.



- Divergence of a vector field $\vec{A}$ at a given point P is the outward flux per unit volume, as the volume shrinks about P. Divergence of a vector field is a scalar $div\vec{A} = \vec{\nabla}.\vec{A} = \lim_{\Delta V \to 0} \frac{\oint_S \vec{A}.\vec{dS}}{\Delta V}$
- The divergence operator in the 3 coordinate systems:

$$\nabla.A = \left(\frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}\right) \quad \text{(Rectangular)}$$

$$\nabla.A = \frac{1}{\rho}\frac{\partial(\rho A_\rho)}{\partial \rho} + \frac{1}{\rho}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z} \quad \text{(Cylindrical)}$$

$$\nabla.A = \frac{1}{r^2}\frac{\partial(r^2 A_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(\sin\theta A_\theta)}{\partial \theta} + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial \phi} \quad \text{(Spherical)}$$

The divergence theorem relates the divergence of a vector field $\vec{A}$ to the surface integral over a surface. It is given by $\oint_S \vec{A}.\vec{dS} = \int_V \vec{\nabla}.\vec{A}dV$ where S is the surface and V in the volume enclosed by the surface S.

- **Curl:** Similar to a flow source, vector fields can also exist as 'vortex sources' which causes circulation of a vector field around it, If $\vec{A}$ is a force acting on an object, circulation would be the work done by the force in moving the object once around the contour. The curl of $\vec{A}$ is a vector whose magnitude is the maximum circulation of $\vec{A}$ per unit area, as the area tends to zero. The direction of the curl is the normal direction of the area when the area is oriented to make the circulation maximum.

$$curl\vec{A} = \vec{\nabla} \times \vec{A} = \lim_{\Delta S \to 0} \left[\frac{\oint_c \vec{A}.\vec{dl}}{\Delta S}\hat{a}_n\right]_{max}$$

- **Stokes Theorem** relates the curl of a vector field $\vec{A}$ to the line integral of $\vec{A}$ over a contour C. It is given by,

$$\oint_C \vec{A}.\vec{dl} = \int_S (\vec{\nabla} \times \vec{A}).\vec{dS}$$

- Given below is the ∇ operator in the 3 coordinate systems

$$\nabla \times A = \begin{bmatrix} a_x & a_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{bmatrix} \qquad \text{(Rectangular coordinates)}$$

$$\nabla \times A = \left(\frac{1}{\rho}\right) \begin{bmatrix} a_x & \rho a_\phi & a_z \\ \frac{\partial}{\partial \rho} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ A_\rho & \rho A\phi & A_z \end{bmatrix} \qquad \text{(Cylindrical coordinates)}$$

$$\nabla \times A = \left(\frac{1}{r^2 \sin\theta}\right) \begin{vmatrix} a_\rho & r a_\theta & r\sin\theta a_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_r & rA_\theta & r\sin\theta A_\phi \end{vmatrix} \qquad \text{(Spherical coordinates)}$$

3. Laplacian Operation

The Laplacian of a scalar field V in different coordinate systems is defined as

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} \qquad \text{(Rectangular coordinates)}$$

$$\nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial V}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2} \qquad \text{(Cylindrical coordinates)}$$

$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial V}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial V}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 V}{\partial \phi^2} \qquad \text{(Spherical coordinates)}$$

- **Laplacian of a Vector function ($\vec{A}$):**

Let $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$

$$\nabla^2 \vec{A} = \left(\frac{\partial^2 A_x}{\partial x^2} + \frac{\partial^2 A_x}{\partial y^2} + \frac{\partial^2 A_x}{\partial z^2}\right)\hat{i} + \left(\frac{\partial^2 A_y}{\partial x^2} + \frac{\partial^2 A_y}{\partial y^2} + \frac{\partial^2 A_y}{\partial z^2}\right)\hat{j} + \left(\frac{\partial^2 A_z}{\partial x^2} + \frac{\partial^2 A_z}{\partial y^2} + \frac{\partial^2 A_z}{\partial z^2}\right)\hat{k}$$

Laplacian of a vector function is a vector function.

Electrostatics

In this article, you will find the Study Notes on **Electrostatics** which will cover the topics such as **Force of interaction between two charged particles, Electric Field due to Infinite Line Charge, Electric Field due to Uniformly Charged Ring,**

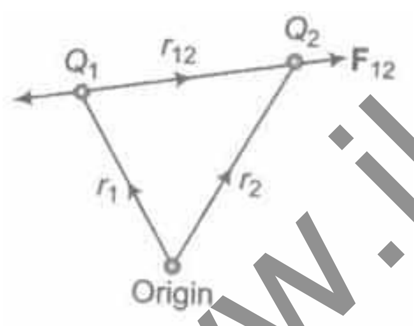
Electric Field due to Infinite Sheet of Charge, Superposition Principle of Fields, Electric Flux, Electric Potential, Dipole Moment, Electric flux lines and Electric field due to Dipole etc

- Coulomb's Law describes the electrostatic interaction between two charged particles.
- It can be derived by combining the equation for the electric field around a spherical charge.
- According to Coulomb's law, the force acting between two point charges is:
    - directly proportional to the magnitude of each charge,
    - inversely proportional to the square of the separation between their centres, and
    - directed along the separation vector connecting their centres.

- The force acting between two electric charges is radial, inverse-square, and proportional to the product of the charges.
- It is an inverse-square law, given by:

(OR)

$F_{12} = (k_e Q_1 Q_2) / r^2$



- where Q_1 and Q_2 are the magnitudes of the two charges respectively and r is the distance between them, F_{12} is the force on particle 1 from particle 2, k_e is called as Coulomb's constant (k_e = 8.99×109 N m^2 C^{-2}), and ε_0 is vacuum permittivity (8.85 × 10^{-12} C^2/Nm^2).
- $F_{12} = -F_{21}$ and $|F_{12}| = |F_{21}|$, it means that force acting on charge Q_2 due to Q_1 is always equal to the force acting on charge Q_1 due to Q_2 magnitude but opposite in direction.
- The signs of Q_1 and Q_2 must be taken into account means for the same polarity charges $Q_1Q_2 > 0$ and for opposite polarity charges $Q_1Q_2 < 0$.
- Coulomb's force obeys the law of superposition.

- Note that when both particles have the same sign of charge then the force is in the same direction as the unit vector and the particle is repelled.
- The SI unit of electric charge is the coulomb (C)

Force per unit charge is called electric field intensity

line charge:

$$E = \frac{\rho_L}{2\pi \varepsilon_0 \rho} a_\rho \text{.....}$$

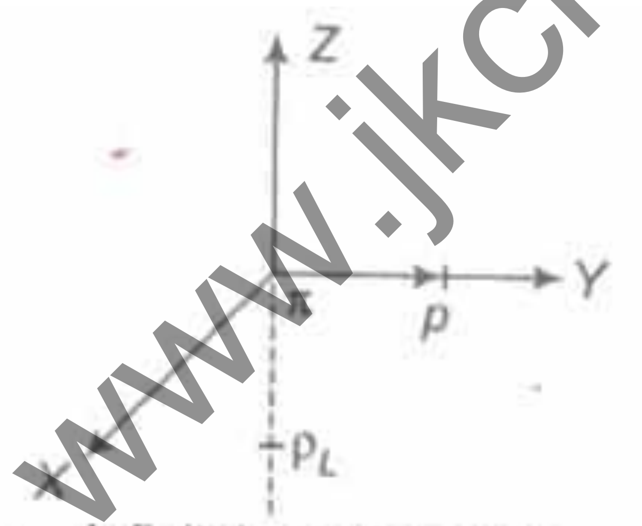
Where ρ_L is line charge density in C/m and r is radial distance.

surface charge :

$$E = \frac{\rho_S}{2\varepsilon_0} a_n \text{.....}$$

Where ρ_S is surface charge density in C/m2 and a_n is unit vector normal to the plane containing the sheet.

Electric Field due to Infinite Line Charge



Infinitely straight line carrying a charge ρ_L

Consider an infinitely long straight line carrying uniformly line charge having density ρ_L C/m.

The electric field intensity at point P.

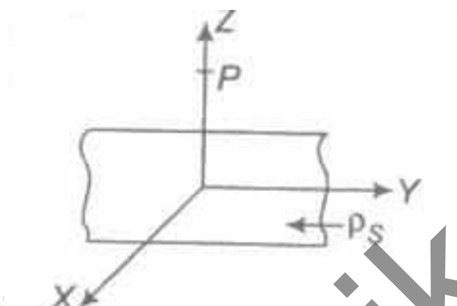
$$E = \frac{\rho_L}{2\pi \varepsilon_0 r} a_y$$

Electric Field due to Uniformly Charged Ring

Consider a charged circular ring of radius r placed in XY plane with centre at the origin. Carrying a charge uniformly along its circumference. The charge density is ρ_L C/m.

Electric Field due to Infinite Sheet of Charge

Consider an infinite sheet of charge having uniform surface charge density ρ_s C/m^2, placed in XY plane. We want to find E at point P present at the z-axis.



Electric field due to infinite sheet of charge

$$E = \frac{\rho_s}{2\varepsilon_0} a_z$$

Superposition Principle of Fields

The electric field of a point charge is a linear function of the value of the charge. The fields of more than one point charge are linearly superimposable by vector addition. This is the principle of superposition applied to the electric field and states that the total resultant field at a point is the vector sum of the individual components of the field at the point.

Electric Flux

It may be defined as

$$\psi = \int D \cdot dS$$

Where D is electric flux density and it can be given as

D = ε_0 E = Electric flux density (C/m^2)

where, ε_0 = Permittivity of the vacuum = 8.854 × 10^{-12} F/m

Electric Potential

The Work was done per unit charge or potential energy per unit charge is known as potential difference.

$$V_{AB} = \frac{W_{AB}}{Q} = -\int_A^B E \cdot dl = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{R_B} - \frac{1}{R_A}\right] = V_B - V_A$$

Electric field intensity can be given as the negative potential gradient of electric field i.e,

E = -∇V

The electric field is irrotational or conservative in nature i.e., ∇ × *E = 0*

Note: Electric dipole is formed when two points charged of equal magnitudes and opposite polarities are separated by a small distance.

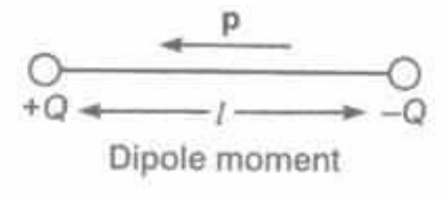
Dipole Moment

p = Q d Coulomb-meter

where, d is the distance vector from –Q to Q.

Electric field Intensity at a Point due to a Dipole

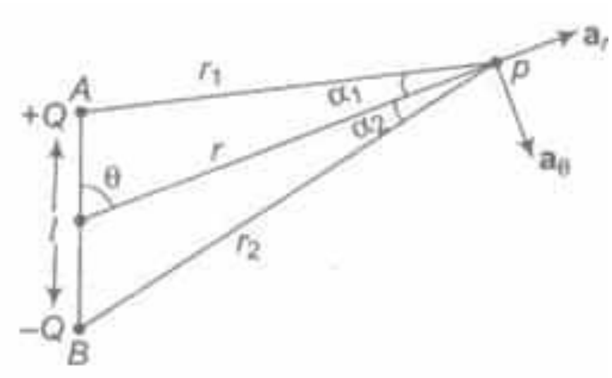
An electric dipole or simply a dipole consists of two point charges of equal magnitude and opposite sign, separated by a very small distance.



Dipole moment

Field due to Dipole

The field due to a dipole at a very large distance from the dipole.



Field due to dipole

$$E = \frac{p}{4\pi\varepsilon_0 r^3}[2\cos\theta a_r + \sin\theta a_\theta]$$

where, p = Q*l*

and a_r, a_θ = Unit vectors

S.No.	Charge	Electric Field	Potential
1	One point charge (monopole)	$E \propto \frac{1}{r^2}$	$V \propto \frac{1}{r}$
2	Two-point charge (dipole)	$E \propto \frac{1}{r^3}$	$V \propto \frac{1}{r^2}$
3	Three-point charge (tripole)	$E \propto \frac{1}{r^4}$	$V \propto \frac{1}{r^3}$
4	Four-point charge (quadruple)	$E \propto \frac{1}{r^5}$	$V \propto \frac{1}{r^4}$

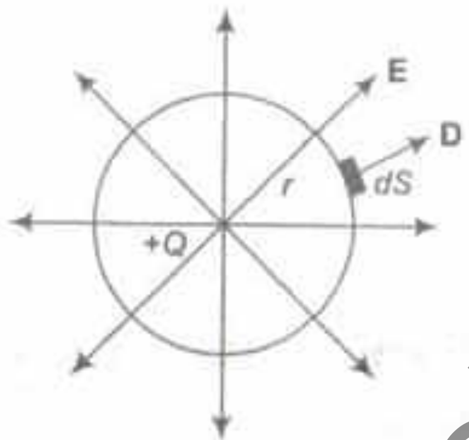
Electric Flux

The electric field at any distance r from a point charge in free space.

$$E = \frac{Q}{4\pi\varepsilon_0 r^2} a,$$

Newton/Coulomb

$$\varepsilon_0 E = \frac{Q}{4\pi r^2} = D$$



Electric field from a point charge

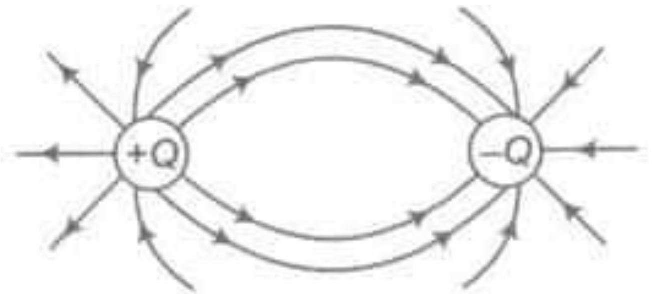
With E as a vector in free space, $\varepsilon_0 E$ is designated by a symbol D; called electric flux density

$D = \varepsilon_0 E$

The integral of the normal component of the vector D over a surface is defined as the electric flux over the surface.

Electric Flux Lines

An electric flux line is an imaginary path or line which is drawn in such a way that its direction at any point is the same as the direction of the electric field at that point. The electric flux density (D) is always tangential to electric flux lines. Electric flux lines are also called electric lines of force.



Electric flux lines

Electric field emanates (or originates) from a positive and terminates (or ends) on a negative charge.

An equipotential surface is a surface on which potentially remains the same throughout the surface and there is no potential difference. The flux line or force line for an equipotential surface is known as an equipotent line on equipotential surfaces or equipotential lines, the potential difference between any points A and B is always zero.

Electrostatic Energy Density

$$W_E = \frac{1}{2} D \cdot E = \frac{1}{2} \varepsilon E^2 = \frac{D^2}{2\varepsilon_0}$$

Introduction to Gauss's law for a conductor and example on Gauss's Law Applications and Divergence of the flux density, electric field and potential.

- The total of the electric flux out of a closed surface is equal to the charge enclosed divided by the permittivity.
- Total electric flux y through any closed surface is equal to the total charge enclosed by that surface.
- Gauss's Law helps us understand the behaviour of electric fields inside the conductors.
- The Gauss law also helps us understand the distribution of electric charge placed on a conductor.
- The area integral of the electric field over any closed surface is equal to the net charge enclosed in the surface divided by the permittivity of space.
- Gauss' law is a form of one of Maxwell's equations.

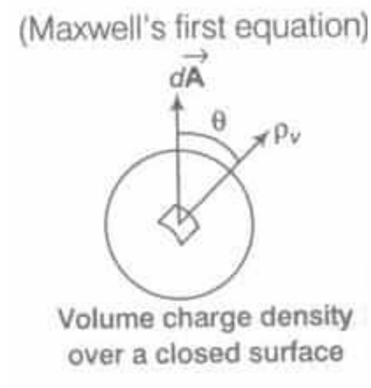
$$Q_{enc} = \oint_S D \cdot dS = \psi$$

$$\psi = \int_S D.ds = \int_V \rho_V dV,$$

$$\nabla \cdot D = \rho_V$$

(Maxwell's first equation)

- Net flux through a surface is equal to the net charge enclosed by the volume occupied by the surface.



Volume charge density over a closed surface

Where, r_V = Volume charge density

- Total charge enclosed:
  $Q_{enc} = \int_V \rho_V dv$
- Gauss's law is an alternative statement of Coulomb's law. Proper application of the divergence theorem to Coulomb's law results in Gauss's law.
- Coulomb's law is applicable in finding the electric field due to any charge configuration but Gauss's law is applicable when charge distribution is symmetrical.

Divergence of the Flux Density

Let *E* be a simple solid region and *S* is the boundary surface of *E* with positive orientation.

Let D be a vector field whose components have continuous first-order partial derivatives.

Then, the divergence of a vector field D is defined at any point as

$$divD = \lim_{\Delta V \to 0} \frac{\oint_s D \cdot dS}{\Delta V}$$

$$div\, D = \nabla \cdot D = \rho_V$$

$$div\, E = \nabla \cdot E = \frac{\rho_V}{\varepsilon}$$

(Gauss's law in differential form)

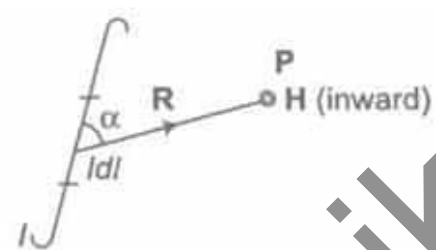
$$\oint_s D.dS = \int_V (\nabla \cdot D)\, dV$$

(Gauss's law in integral form)

Magnetostatics

Biot-Savart's Law

It states that the differential magnetic field intensity dH produced at a point P by the differential current element *I*d*l* is proportional to the product *I*d*l* and the sine of the angle a between the element and the line joining P to the element and is inversely proportional to the square of the distance R between P and the element.



Magnetic field intensity at point P

$$dH = \frac{Idl \times a_R}{4\pi R^2}\ Ampere / metre$$

- Different current distributions are related as *I*d*l* = K dS = J dV

Introduction & Ampere's Law in differential and integral form.

Ampere's law allows the calculation of magnetic fields.

Ampere's circuital law states that the line integral of the magnetic field (circulation of H) around a closed path is the net current enclosed by this path.

Ampere's law in differential form:

$$\overline{\nabla} \times \overline{B} = \mu_0 \overline{J}$$

Ampere's law in integral form:

$$\int_{Surface} \left[\overline{\nabla} \times \overline{B}\right] \bullet d\overline{a} = \oint_{Line} \overline{B} \bullet d\overline{l} = \mu_0 \int_{Surface} \overline{J} \bullet d\overline{a} = \mu_0 I$$

Ampere’s Circuit Law:

$$\oint H \cdot dl = I_{enc} = \int J \cdot dS$$

The configurations that can be handled by Ampere's law are:

- Infinite straight lines
- Infinite planes
- Infinite solenoids
- Toroids

Electromagnetic field

A time-varying magnetic field produces an electromotive force (or emf) which may establish a current in a closed circuit,

$$V_{emf} = \frac{-\partial \psi}{\partial t}$$

For transformer,

$$V_{emf} = -\int \frac{\partial \beta}{\partial t} . dS$$

and for motional emf,

$$V_{emf} = -\int (u \times B) \cdot dl$$

where u is called energy density

* Displacement Current

$$I_d = \int J_d dS$$

where

$$J_d = \frac{\partial D}{\partial t}$$

is known as displacement current density.

Reciprocal of attenuation constant is known as skin depth or penetration depth. It measures the depth at which field intensity reduces to the electron of the original value.

Skin depth

$$\delta = \frac{1}{\alpha}$$

The Poynting vector P is the power flow vector whose direction is the same as the direction of wave propagation.

P = E × H

$$and\ P_{av} = \frac{1}{2}\mathrm{Re}(E_s \times H_s^*)$$

Key Points

* Free space

$\sigma = 0, \varepsilon = \varepsilon_0, \mu = \mu_0$

* Lossless dielectrics

$\sigma = 0, \varepsilon = \varepsilon_r\varepsilon_0, \mu = \mu_r\mu_0$ or $\sigma << \omega\varepsilon$

* Lossy dielectrics

$\sigma \neq 0, \varepsilon = \varepsilon_r\varepsilon_0, \mu = \mu_r\mu_0$

* Good conductors

$\sigma = \infty, \varepsilon = \varepsilon_0, \mu = \mu_r\mu_0$ *or* $\sigma << \omega\varepsilon$

Where σ is conductivity, ε is permittivity and μ is the permeability of the medium.

Divergence is a dot product between the gradient operator (the flipped triangle) and the vector field, also the curl is a cross product of the gradient operator and the vector field.

Using the Biot-Savart law for a volume current we can calculate the divergence and curl of : $\overline{\nabla}\cdot\overline{B} = \square$

The curl of an electric field is zero. The electric field associated with a set of stationary charges has a curl of zero. In this situation, there is no magnetic field. Electric field lines curl up in opposition to a time-varying magnetic field (Faraday's law of induction). Magnetic field lines curl up in response to current flow, ie. moving charges (Ampere's law, first term).

All the Best.

Introduction to Boundary condition for the Magnetic field.

Lorentz force is the force on an electrically charged particle that moves through a magnetic plus an electric field. The Lorentz force has two vector components, one proportional to the magnetic field and one proportional to the electric field.

The magnetic field B is defined by Lorentz Force Law, and specifically force on a moving charge

$\mathbf{F} = q\,\mathbf{v} \times \mathbf{B}$

Force in an electrostatic field

$\mathbf{F} = Q\mathbf{E}$

Force in a magnetostatic field

$\mathbf{F} = Q\,(\mathbf{u} \times \mathbf{B})$

So, net force in electromagnetic fields

$\mathbf{F} = Q(\mathbf{E} + \mathbf{u} \times \mathbf{B})$

* Force experienced by a current element $Id\mathit{l}$ in magnetic field B is

$d\mathbf{F} = Id\mathit{l} \times \mathbf{B}$

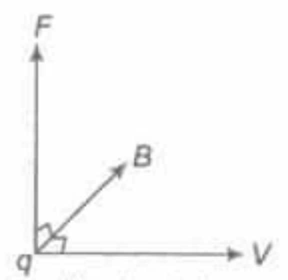
* Magnetic flux density can also be defined as the force per unit current element.

* Magnetic moment

$m = IS\mathbf{a}_n$

* Torque on a current loop T = m × B = *IS* an × B

* For linear materials, magnetization $M = \chi_m H$



Positive test charge

* Boundary conditions for magnetic fields

$B_{1n} = B_{2n}$

$(H_1 - H_2) \times a_{n12} = K$

And $H_{1t} = H_{2t}$ if K = 0

Magnetic Energy & Inductance relation.

- Induction of an electromotive force occurs in a circuit by varying the magnetic flux linked with the circuit.

- From Faraday's law, the concept of Inductance may be derived from the property of an electric circuit by which an electromotive force is induced in it as the result of a changing magnetic flux.
- Inductance L may be defined in terms of the electromotive force generated to oppose a change in current ΔI in the given time duration Δt.

Emf = -L (ΔI) / (Δt)

- Unit for L is Volt Second / Ampere = Henry.
- 1 henry is 1 volt-second / ampere.
- If the rate of change of current in a circuit ΔI / Δt is one ampere per second and the resulting electromotive force is one volt, then the inductance of the circuit is one henry.
- Since emf results due to the rate of change of magnetic flux , inductance L may also be defined as a measure of the amount of magnetic flux φ produced for a given electric current I as: L = φ/I, where the inductance L is one henry, if current I of one ampere, produces magnetic flux φ of one weber.
- For an inductor of inductance L,

$$W_m = \frac{1}{2}LI^2$$

- The energy in a magnetostatics field

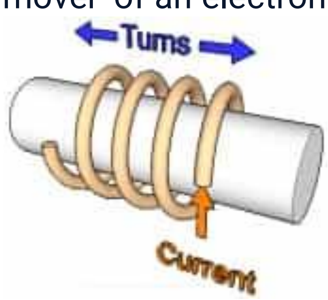
$$W_m = \frac{1}{2}\int B.H dV$$

- Inductance is defined as

$$L = \frac{\lambda}{I} = \frac{N\psi}{I}$$

Introduction to Magnetomotive Force

Magnetomotive Force (MMF) performs a similar role in a magnetic circuit to the electromotive force (EMF) of a battery in a basic electrical circuit, thus acting as the 'prime mover' of an electromagnetic system.



The MMF resulting from passing an electrical current through a coil is given by the electrical current flowing through the coil multiplied by the number of turns, as shown in the following equation:

Equation: MMF, $F_m = I \times N$.

MMF is measured in amperes (A) rather than ampere-turns, since 'turns' is not an SI unit.

Magnetomotive force is also used in the derivation of magnetic field strength (H), as shown below. $H = I N / l = F_m / l$

$F_m = Hl$

Inductance Introduction

Magnetic circuits are analogous to resistive electronic circuits if we define the magnetomotive force (MMF) analogous to the voltage (EMF). The flux then plays the same role as the current in electronic circuits so that we define the magnetic analogue to resistance as the reluctance (R).

MMF = ΦR

R = MMF/ Φ

R is proportional to the reciprocal of the inductance.

The reluctance of a uniform gap, without leakage, is, therefore:

$$R = \frac{1}{\mu_o} \frac{l_g}{A_g}$$

Calculation of the electrical resistance of a wire or bar of uniform cross-section:

$$R = \frac{1}{\sigma} \frac{l}{A}$$

where σ is the wire material conductivity, l the length and A is the wire cross-sectional area.

$R_{eq.} = R_1 + R_2 + R_3 + \ldots$

Reluctances in Series: The combination of reluctances in series, with a common flux-path, results in:

Reluctances in Parallel: Flux in a magnetic circuit may flow in various leakage paths, as well as the useful ones. It can be shown, by considering two parallel flux-paths, with flux driven by the same MMF, that the equivalent reluctance for two reluctances in parallel is:

$$R_{eq.} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots}$$

Maxwell's, Poisson's & Laplace equations

Maxwell's Equations in Time-Varying Field

- Maxwell's equations in point form
  1. (Faraday's Law)
     $$\nabla \times E = -\frac{\partial B}{\partial t}$$
  2. (Modified Ampere's circuital Law)
     $$\nabla \times H = J + \frac{\partial D}{\partial t}$$
  3. (Gauss's Law)
     $$\nabla \cdot D = \rho_V$$
  4. (Non-existence monopole or Gauss's law of magnetostatics)
     $$\nabla \cdot B = 0$$

Maxwell's equations in integral form

$$\oint E \cdot dI = -\oint_s \frac{\partial B}{\partial t} \cdot dS$$

$$\oint H \cdot d(u \times B) \cdot dI = I + \int_s \frac{\partial D}{\partial t} \cdot dS$$

$$\int_s D \cdot dS = \int_V \rho_V dV$$

$$\int_s B \cdot dS = 0$$

Key Points

For static fields (or non-time varying fields)

$$\frac{\partial B}{\partial t} = 0 \quad and \quad \frac{\partial D}{\partial t} = 0$$

The basic equations of electromagnetism are the four Maxwell Equations and the Lorentz force law. In principle, these, together with Newton's second law of motion are enough to completely determine the motion of an assembly of charges given the initial positions and velocities of all the charges. Maxwell's equations are,

$$\nabla \cdot \vec{e} = \rho / \varepsilon_0 \text{ ...(1)}$$

$$\nabla \cdot \vec{b} = 0 \text{ ...(2)}$$

$$\nabla \times \vec{e} = -\frac{\partial \vec{b}}{\partial t} \text{ ...(3)}$$

$$\nabla \times \vec{b} = \mu_0 \vec{j} + \mu_0 \varepsilon_0 \frac{\partial \vec{e}}{\partial t} \text{ ...(4)}$$

Here, $\vec{e}$ and $\vec{b}$ are the electric and magnetic fields respectively. The sources for the fields are the volume charge density *ρ* and the current density. The two parameters in these equations are the permittivity of free space $\varepsilon_0 = 8.85 \cdot 10^{-12}$ F/m and the permeability of free space $\mu_0 = 1.26 \cdot 10^{-6}$ H/m. The vector, differential-operator in these equations is defined as

$$\nabla = \hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}$$

these equations can be used to determine the fields if the charge and current densities are known. Once the fields are known, the force felt by a given charge *q* moving with velocity is given by the Lorentz force law,

$$\vec{F} = q\left(\vec{e} + \vec{v} \times \vec{b}\right) \text{ ....(5)}$$

In principle, Equations (1)-(5) constitute all electromagnetic. However, in practice, the charge and current densities associated with matter are too complicated to specify and so a phenomenological means of dealing with the matter is often introduced.

Maxwell Equation for Time-Varying Field :

S.N.	Differential form	Integral form	Name
1.	$\nabla \times E = -\frac{\partial B}{\partial t}$	$\oint_L E \bullet dL = -\frac{\partial}{\partial t}\int_S B \bullet dS$	Faraday's law of electromagnetic induction
2. □	$\nabla \times H = J + \frac{\partial D}{\partial t}$	$\oint_L H \bullet dL = \int_S \left(J + \frac{\partial D}{\partial t}\right) \bullet dS$	Modified Ampere's circuital law
3.	$\nabla \bullet D = \rho_v$	$\oint_S D \bullet dS = \int_v \rho_v dv$	Gauss' law of Electrostatics
4.	$\nabla \bullet B = 0$	$\oint_S B \bullet dS = 0$	Gauss' law of Magnetostatics (non-existence of magnetic mono-pole)

Maxwell Equation for Time-Varying Field in Free Space

S.N.	Differential Form	Integral Form	Name
1.	$\nabla \times E = -\frac{\partial B}{\partial t}$	$\oint_L E \bullet dL = -\frac{\partial}{\partial t}\int_S B \bullet dS$	Faraday's law of electromagnetic induction
2.	$\nabla \times H = -\frac{\partial D}{\partial t}$	$\oint_L H \bullet dL = \int_S \frac{\partial D}{\partial t} \bullet dS$	Modified Ampere's circuital law
3.	$\nabla \bullet D = 0$	$\oint_S D \bullet dS = 0$	Gauss' law of Electrostatics
4.	$\nabla \bullet B = 0$	$\oint_S B \bullet dS = 0$	Gauss' law of Magnetostatics (non-existence of magnetic mono-pole)