



JK Chrome

JK Chrome | Employment Portal



Rated No.1 Job Application of India

Sarkari Naukri
Private Jobs
Employment News
Study Material
Notifications



JOBS



NOTIFICATIONS



G.K



STUDY MATERIAL



JK Chrome

jk chrome
Contains ads



www.jkchrome.com | Email : contact@jkchrome.com

Circuit Theory

Index

| Topics | Page |
|-----------------------------|------|
| 1. Basics of Circuit Theory | 2 |
| 2. Kirchhoff's Laws | 13 |
| 3. Network Theorems | 17 |
| 4. Transient Analysis | 32 |
| 5. Steady-State Analysis | 38 |
| 6. AC Circuit Analysis | 46 |
| 7. Two Port Networks | 57 |

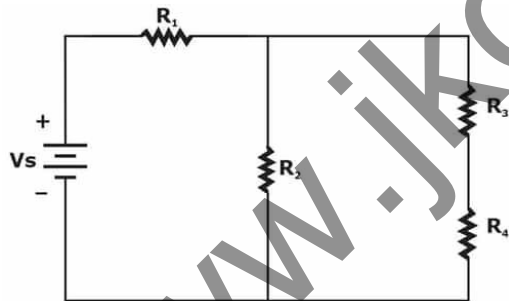
Basics of Circuit Theory

1. Basic Concepts

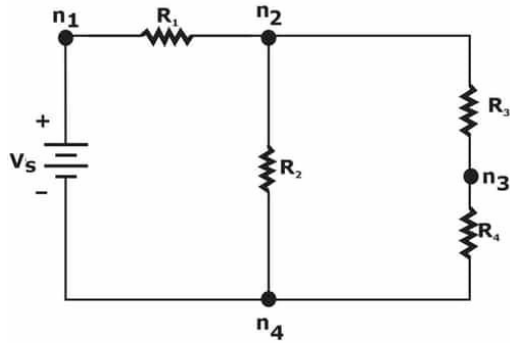
• □ NODAL ANALYSIS

- □ The procedure for analyzing a circuit with the node method is based on the following steps:
- □ Clearly, label all circuit parameters and distinguish the unknown parameters from the known.
- □ Identify all nodes of the circuit after that Select a node as the reference node also called the ground and assign to it a potential of 0 Volts. All other voltages in the circuit are measured with respect to the reference node.
- □ Label the voltages at all other nodes & Assign and label polarities.
- □ Apply KCL to each node and express the branch currents in terms of the node voltages.
- □ Solve the resulting simultaneous equations for the node voltages.
- □ Now that the node voltages are known, the branch currents may be obtained from Ohm's law.

Example: Find out the nodal voltage at each node & current in each loop by using the Nodal method?



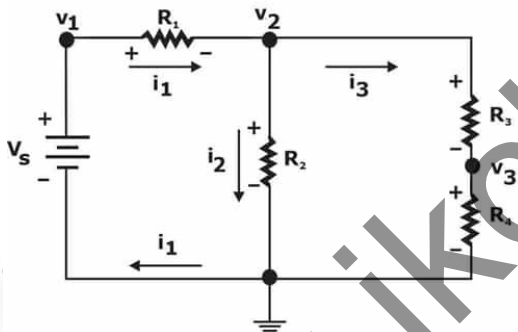
Solution: First of all we have labeled all elements and identified all relevant nodes in the circuit.



There are a few general guidelines that we need to remember as we make the selection of the reference node.

1. □ A useful reference node is one which has the largest number of elements connected to it.
2. □ A useful reference node is one which is connected to the maximum number of voltage sources.

For the next step we assign current flow and polarities



For node n_1 voltage of the voltage source is known so

$$v_1 = V_s \dots\dots\dots(1)$$

& KCL at node n_2 associated with voltage v_2 gives:

$$i_1 = i_2 + i_3 \dots\dots\dots(2)$$

The currents i_1, i_2, i_3 are expressed in terms of the voltages v_1, v_2, v_3 as follows

$$i_1 = \frac{V_s - v_2}{R_1} \quad i_2 = \frac{v_2}{R_2} \quad i_3 = \frac{v_2 - v_3}{R_3} \quad \dots\dots\dots$$

(3)

From the relation (1) (2) & (3) we get....

$$\frac{V_s - v_2}{R_1} - \frac{v_2}{R_2} - \frac{v_2 - v_3}{R_3} = 0 \quad \dots\dots\dots (4)$$

Rewrite the above expression as a linear function of the unknown voltages v_2 and v_3 gives.

$$v_2 \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right) - v_3 \frac{1}{R_3} = V_s \frac{1}{R_1} \quad \dots\dots\dots (5)$$

KCL at node n_3 associated with voltage v_3 gives:

$$\frac{v_2 - v_3}{R_3} - \frac{v_3}{R_4} = 0$$

or

$$-v_2 \frac{1}{R_3} + v_3 \left(\frac{1}{R_3} + \frac{1}{R_4} \right) = 0 \quad \dots\dots\dots (6)$$

Now we can write equation (5) & (6) in matrix form for the node voltage v_2 and v_3 .

$$\begin{bmatrix} 1 + \frac{R_1 + R_2}{R_3} & -\frac{R_1}{R_3} \\ -\frac{R_1}{R_3} & \frac{R_1 + R_4}{R_3} \end{bmatrix} \begin{bmatrix} v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} V_s \\ 0 \end{bmatrix}$$

Or

$$\overline{\overline{A}} \cdot \overline{v} = \overline{V} \dots\dots\dots (7)$$

In defining the set of simultaneous equations, we want to end up with a simple and consistent form. The simple rules to follow and check are

- Place all sources (current and voltage) on the right hand side of the equation, as inhomogeneous drive terms.
- The terms comprising each element on the diagonal of the matrix $\overline{\overline{A}}$ must have the same sign.
- If you arrange so that all diagonal elements are positive, then the off-diagonal elements are negative and the matrix is symmetric: $A_{ij} = A_{ji}$. If the matrix does not have this property there is a mistake somewhere.

Once we put the equations in matrix form and perform the checks detailed above the solutions

then there is a solution if the $\det \overline{\overline{A}} = 0$ the unknown voltage V_k is given by:

$$V_k = \frac{\det \overline{\overline{A}}_k}{\det \overline{\overline{A}}} \dots\dots\dots (8)$$

Where $\overline{\overline{A}}_k$ is the matrix $\overline{\overline{A}}$ with the kth column replaced by the vector \overline{V} .

So for our example the voltages v_2 and v_3 are given by:

$$v_2 = \frac{R_2(R_3 + R_4)V_s}{R_1R_2 + R_1R_3 + R_2R_3 + R_1R_4 + R_2R_4} \dots\dots\dots (9)$$

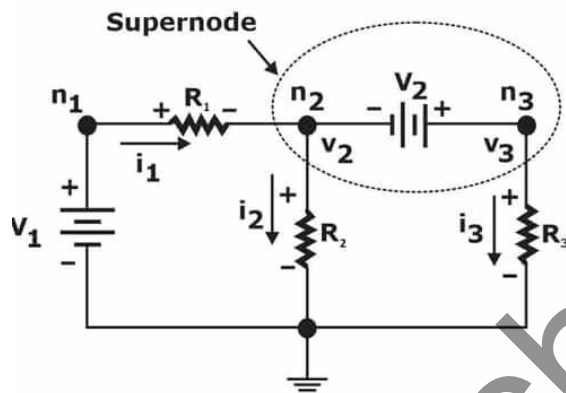
$$v_3 = \frac{R_2R_4V_s}{R_1R_2 + R_1R_3 + R_2R_3 + R_1R_4 + R_2R_4} \dots\dots\dots (10)$$

Nodal Analysis with Floating Voltage Sources. (THE SUPERNODE)

If a voltage source V_2 is not connected to the reference node it is called a floating voltage source and special care must be taken when performing the analysis of the circuit. In the circuit of given figure below the voltage source V_2 is not connected to the reference node and thus it is a floating voltage source. Here v_2 is the node voltage while V_2 is the source voltage between node n_2 & n_3 .

Circuit with a Supernode

The part of the circuit enclosed by the dotted ellipse is called a supernode. Kirchhoff's current law may be applied to a supernode in the same way that it is applied to any other regular node. This is not surprising considering that KCL describes charge conservation which holds in the case of the supernode as it does in the case of a regular node.



In our example application of KCL at the supernode gives

$$i_1 = i_2 + i_3 \quad \dots\dots\dots (11)$$

In term of the node voltages Equation (11) becomes:

$$\frac{V_1 - v_2}{R_1} = \frac{v_2}{R_2} + \frac{v_3}{R_3} \quad \dots\dots\dots (12)$$

The relationship between node voltages v_1 and v_2 is the constraint that is needed in order to completely define the problem. The constraint is provided by the voltage source V_2 .

$$V_2 = v_3 - v_2 \quad \dots\dots\dots (13)$$

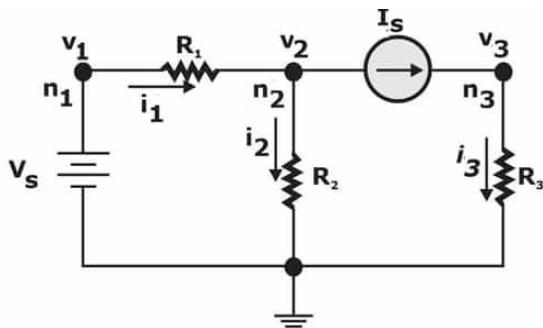
From equation (12) & (13),

$$v_2 = \frac{\frac{V_1}{R_1} - \frac{V_2}{R_3}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} \quad \text{and} \quad v_3 = \frac{\frac{V_1}{R_1} - \frac{V_2}{R_3}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} - V_2$$

.....(14)

Example- Nodal Analysis with supernode.

Determine the node voltages v_1 , v_2 , and v_3 of the circuit in Figure below?



Solution: We have applied the first five steps of the nodal method and now we are ready to apply KCL to the designated nodes. In this example, the current source I_s constrains the current i_3 such that i_3 .

KCL at node n_2 gives,

$$i_1 = i_2 + I_s \quad \text{..... (1)}$$

And with the application of Ohm's law

$$\frac{V_s - v_2}{R_1} = \frac{v_2}{R_2} + \frac{v_3}{R_3} \quad \text{.....}$$

(2)

Where we have used $v_1 = V_s$ at node n_1 .

The current source provides a constraint for the voltage v_3 at node n_3 .

$$V_3 = I_3 R_3 \quad \dots\dots\dots (3)$$

Now combining the equation (2) & (3).

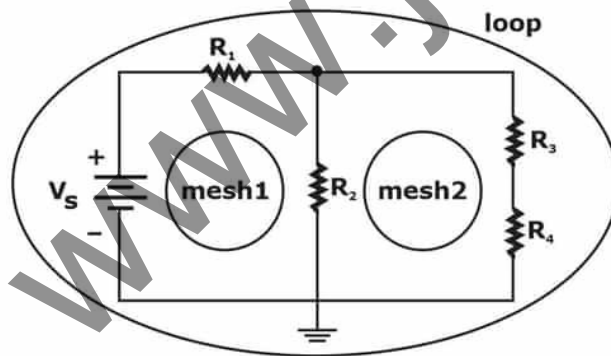
$$V_2 = \frac{\frac{V_s}{R_1} - I_3 R_3}{\frac{1}{R_1} + \frac{1}{R_2}} \quad \dots\dots\dots (4)$$

MESH ANALYSIS

A mesh is defined as a loop which does not contain any other loops. The procedure for obtaining the solution is similar to that followed in the Node method and the various steps are given below.

1. □ Clearly, label all circuit parameters and distinguish the unknown parameters from the known.
2. □ Identify all meshes of the circuit & assign mesh currents and label polarities.
3. □ Apply KVL to each mesh and express the voltages in terms of the mesh currents.
4. □ Solve the resulting simultaneous equations for the mesh currents.
5. □ Now that the mesh currents are known, the voltages may be obtained from Ohm's law.

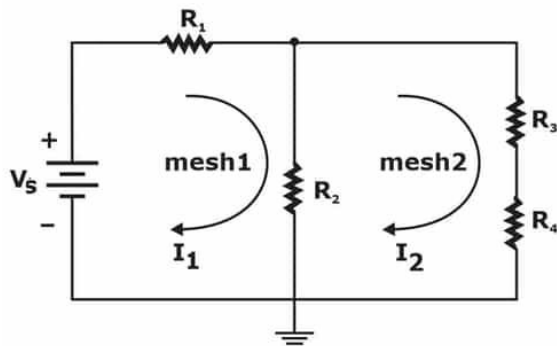
Example - Find out the mesh current i_1 & i_2 for mesh 1 & mesh 2?



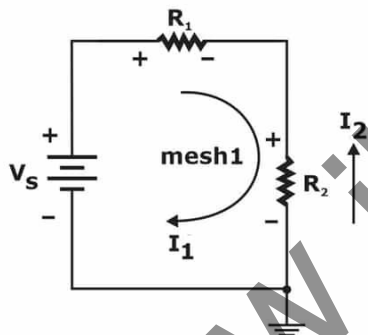
Solution: Our circuit example has three loops but only two meshes as shown, the meshes of interest are mesh 1 and mesh 2.

In the next step we will assign mesh currents, define current direction and voltage polarities. The direction of the mesh currents I_1 and I_2 is defined in the clockwise direction as shown in the next figure.

The branch of the circuit containing resistor R_2 is shared by the two meshes and thus the branch current (the current flowing through R_2) is the difference of the two mesh currents.



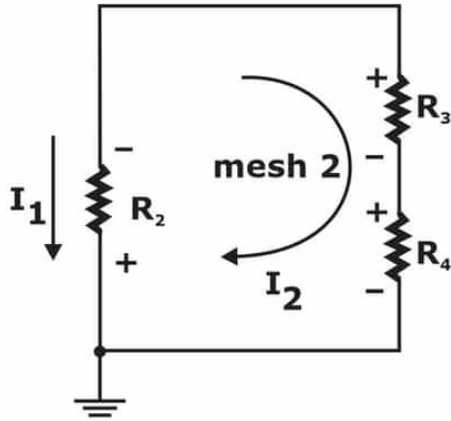
Considering mesh1. For clarity we have separated mesh1 from the circuit in doing this, care must be taken to carry all the information of the shared branches. Here we indicate the direction of mesh current I_2 on the shared branch.



Apply KVL to mesh1. Starting at the upper left corner and proceeding in a clockwise direction the sum of voltages across all elements encountered is

$$I_1 R_1 + (I_1 - I_2) R_2 - V_s = 0 \quad \dots\dots\dots (1)$$

Similarly, consideration of mesh2 : we have indicated the direction of the mesh current I_1 on the shared circuit branch.



Apply KVL to mesh2:

$$I_2 (R_3 + R_4) + (I_2 - I_1) R_2 = 0 \quad \dots\dots\dots (2)$$

From equation (1) & (2),

$$I_1 (R_1 + R_2) - I_2 R_2 = V_s \quad \dots\dots\dots (3)$$

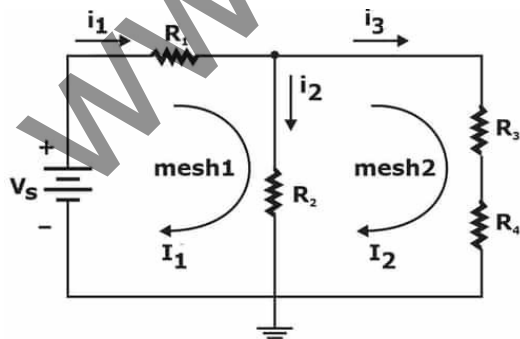
$$-I_1 R_2 + I_2 (R_2 + R_3 + R_4) = 0 \quad \dots\dots\dots (4)$$

In matrix form equations (3) & (4) becomes,

$$\begin{bmatrix} R_1 + R_2 & -R_2 \\ -R_2 & R_2 + R_3 + R_4 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_s \\ 0 \end{bmatrix} \quad \dots\dots\dots (5)$$

Equation (5) may now be solved for the mesh currents I_1 and I_2 .

Note: It is evident from Figure next figure below that the branch currents are i_1 , i_2 & i_3 are obtained from the mesh currents I_1 & I_2 such as.

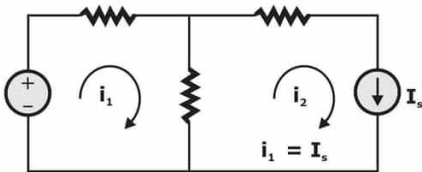


$$I_1 = i_1 \quad I_2 = I_1 - I_2 \quad I_3 = I_2$$

MESH ANALYSIS With CURRENT SOURCES:

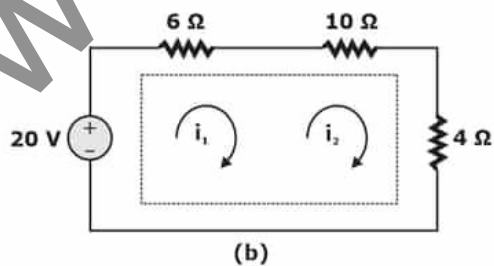
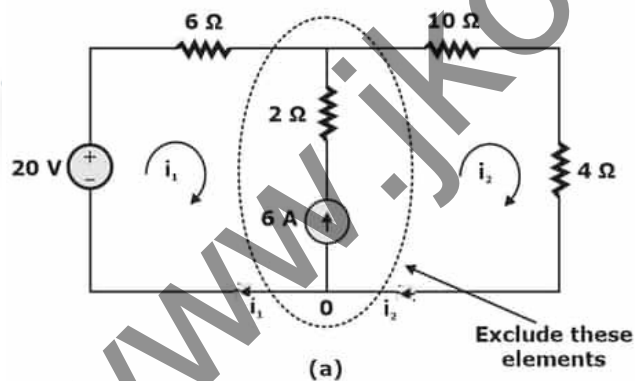
1. □ If a current source exists only in one mesh.

- (i) The mesh current is defined by the current source.
- (ii) Number of variables is reduced.

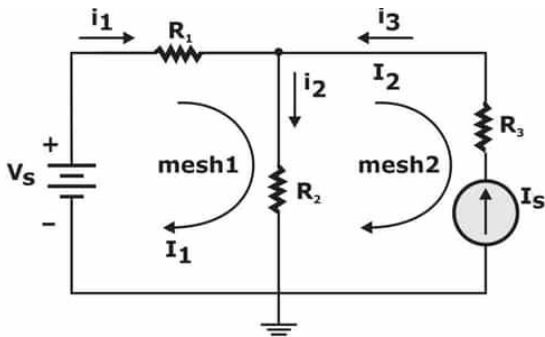


1. □ If a current source exists between two meshes.

- (i) The two nodes form a Supermesh.
- (ii) Use one current variable for both meshes. The current difference between these two meshes is known.
- (iii) Apply KVL to the Supermesh.



Example: Find out the unknown mesh current I_1 ?



Solution:

Consider the circuit in the figure which contains a current source. The application of the mesh analysis for this circuit does not present any difficulty once we realize that the mesh current of the mesh containing the current source is equal to the current of the current source:

$$\text{i.e.} \quad I_2 = I_s \dots\dots\dots (1)$$

In defining the direction of the mesh current, we have used the direction of the current I_s . We also note that the branch current $I_3 = I_s$.

Applying KVL around mesh1 we obtain

$$I_1 R_1 + (I_1 + I_s) R_2 = V_s \dots\dots\dots (2)$$

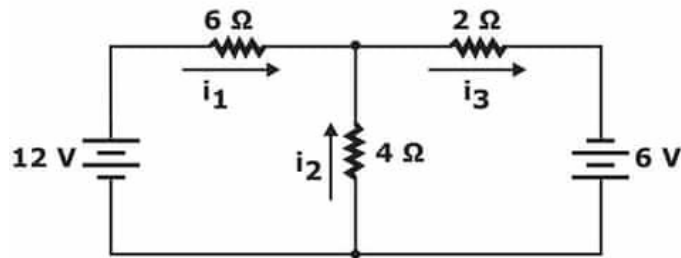
The above equation simply indicates that the presence of the current source in one of the meshes reduces the number of equations in the problem.

The unknown mesh current is:

$$I_1 = \frac{V_s - I_s R_2}{R_1 + R_2}$$

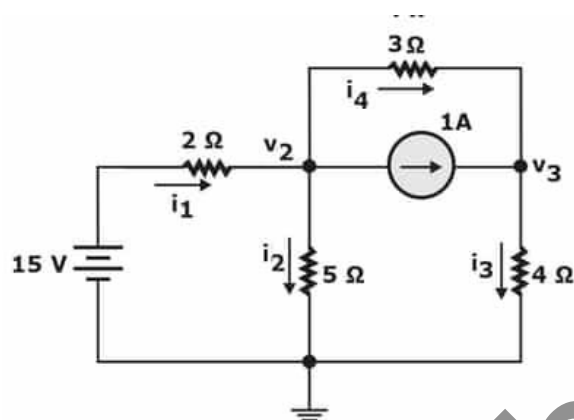
PRACTICE PROBLEMS WITH ANSWERS.

Q-1 Determine the currents in the given circuits with reference to the indicated direction?



Answer: $i_1 = 2.180\text{A}$, $i_2 = 0.270\text{A}$, $i_3 = 2.450\text{A}$

Q-2 Determine the currents in the given circuits with reference to the indicated direction?



Answer: $i_1 = 3.31\text{A}$, $i_2 = 1.68\text{A}$, $i_3 = 1.63\text{A}$, $i_4 = 0.627\text{A}$,
 $v_2 = 8.39\text{V}$, $v_3 = 6.51\text{V}$

Kirchhoff's Laws

1. KIRCHHOFF'S LAW

- **Kirchhoff's Voltage Law (KVL):** The sum of voltages around a closed loop circuit is equal to zero.
- Kirchhoff's Voltage Law (**KVL**) is Kirchhoff's second law that deals with the **conservation of energy** around a closed circuit path. ... This voltage law states that for a closed loop series path the algebraic sum of all the voltages around any closed loop in a circuit is equal to zero.

$$\sum_{k=1}^n V_k = 0$$

- **Kirchhoff's Current Law (KCL):** The algebraic sum of electrical current that merge in a common node of a circuit is zero.

- Kirchhoff's Current Law (**KCL**) is Kirchhoff's first law that deals with the **conservation of charge** entering and leaving a junction. This idea by Kirchhoff is commonly known as the **Conservation of Charge**, as the current is conserved around the junction with no loss of current.

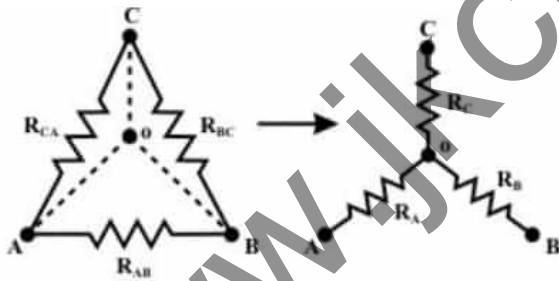
$$\sum I_{IN} = \sum I_{OUT}$$

2. STAR-DELTA TRANSFORMATION

- A part of a bigger circuit that is configured with the three-terminal network Y (or Δ) has to be converted into an equivalent Δ (or Y) through transformations.
- Applications of these transformations will be studied by solving the resistive circuits.

Delta (Δ) – Wye (Y) conversion:

Let us consider the network shown below and assumed the resistances (R_{AB} , R_{BC} , R_{CA}) in Δ network are known. Now the requirement is to measure the resistance values of the branches of the Wye (Y) network that would produce same resistances when measured across similar pairs of terminals of Δ network. For this We have to write the equivalence resistance between any two terminals in the following form.



Between A & C terminals:

$$R_A + R_C = \frac{R_{CA}(R_{AB} + R_{BC})}{R_{AB} + R_{BC} + R_{CA}}$$

Between C & B terminals:

$$R_C + R_B = \frac{R_{BA}(R_{AB} + R_{CA})}{R_{AB} + R_{BC} + R_{CA}}$$

Between B & A terminals:

$$R_B + R_A = \frac{R_{AB}(R_{CA} + R_{BC})}{R_{AB} + R_{BC} + R_{CA}}$$

By combining above three equations, one can write an expression as given below.

$$R_A + R_B + R_C = \frac{R_{AB}R_{BC} + R_{BC}R_{CA} + R_{CA}R_{AB}}{R_{AB} + R_{BC} + R_{CA}}$$

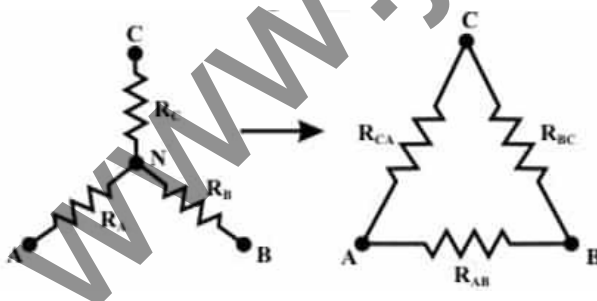
- on solving above equations, we get values for star network resistances

$$R_A = \frac{R_{AB}R_{CA}}{R_{AB} + R_{BC} + R_{CA}}$$

$$R_B = \frac{R_{AB}R_{BC}}{R_{AB} + R_{BC} + R_{CA}}$$

$$R_C = \frac{R_{BC}R_{CA}}{R_{AB} + R_{BC} + R_{CA}}$$

Conversion from Wye or Star (Y) to Delta (Δ):



- To convert a Wye (Y) to a Delta (Δ), the relationships R_{AB} , R_{BC} & R_3 must be obtained in terms of the Wye (Y) resistances R_A , R_B and R_C . Considering the Y connected network, we can write the current expression through R_A resistor as

$$I_A = \frac{(V_A - V_N)}{R_A} \quad (\text{for } Y \text{ network})$$

Applying KCL at 'N' for Y connected network (assume A, B, C terminals having higher potential than the terminal N) we have,

$$\frac{(V_A - V_N)}{R_A} + \frac{(V_B - V_N)}{R_B} + \frac{(V_C - V_N)}{R_C} = 0 \Rightarrow V_N \left(\frac{1}{R_A} + \frac{1}{R_B} + \frac{1}{R_C} \right) = \left(\frac{V_A}{R_A} + \frac{V_B}{R_B} + \frac{V_C}{R_C} \right)$$

$$\text{or, } \Rightarrow V_N = \frac{\left(\frac{V_A}{R_A} + \frac{V_B}{R_B} + \frac{V_C}{R_C} \right)}{\left(\frac{1}{R_A} + \frac{1}{R_B} + \frac{1}{R_C} \right)}$$

For Δ -network

Current entering at terminal A = Current leaving the terminal 'A'

$$I_A = \frac{V_{AB}}{R_{AB}} + \frac{V_{AC}}{R_{AC}} \quad (\text{for } \Delta \text{ network})$$

$$\frac{(V_A - V_N)}{R_A} = \frac{V_{AB}}{R_{AB}} + \frac{V_{AC}}{R_{AC}}$$

Using the V_N expression in the above equation, we get

$$\frac{\left(V_A - \frac{\left(\frac{V_A}{R_A} + \frac{V_B}{R_B} + \frac{V_C}{R_C} \right)}{\left(\frac{1}{R_A} + \frac{1}{R_B} + \frac{1}{R_C} \right)} \right)}{R_A} = \frac{V_{AB}}{R_{AB}} + \frac{V_{AC}}{R_{AC}} \Rightarrow \frac{\left(\frac{V_A - V_B}{R_B} + \frac{V_A - V_C}{R_C} \right)}{\left(\frac{1}{R_A} + \frac{1}{R_B} + \frac{1}{R_C} \right)} = \frac{V_{AB}}{R_{AB}} + \frac{V_{AC}}{R_{AC}}$$

$$\text{or } \frac{\left(\frac{V_{AB}}{R_B} + \frac{V_{AC}}{R_C} \right)}{\left(\frac{1}{R_A} + \frac{1}{R_B} + \frac{1}{R_C} \right)} = \frac{V_{AB}}{R_{AB}} + \frac{V_{AC}}{R_{AC}}$$

- After equating the coefficients of V_{AB} and V_{AC} in both sides, we get the following relationship

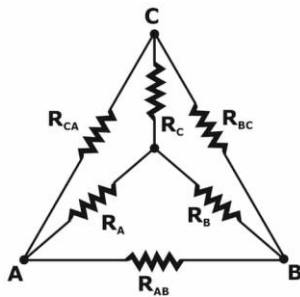
$$\frac{1}{R_{AB}} = \frac{1}{R_A R_B \left(\frac{1}{R_A} + \frac{1}{R_B} + \frac{1}{R_C} \right)} \Rightarrow R_{AB} = R_A + R_B + \frac{R_A R_B}{R_C}$$

$$\frac{1}{R_{AC}} = \frac{1}{R_A R_C \left(\frac{1}{R_A} + \frac{1}{R_B} + \frac{1}{R_C} \right)} \Rightarrow R_{AC} = R_A + R_C + \frac{R_A R_C}{R_B}$$

- similarly we can obtain for R_{BC} for equivalent delta configuration

$$\frac{1}{R_{BC}} = \frac{1}{R_B R_C \left(\frac{1}{R_A} + \frac{1}{R_B} + \frac{1}{R_C} \right)} \Rightarrow R_{BC} = R_B + R_C + \frac{R_B R_C}{R_A}$$

- **Observations:** With a view to the symmetry of the transformation equations, the Wye (Y) and Delta (Δ) networks have been superimposed on each other.
- The equivalent Wye (Y) resistance connected to a given terminal is equal to the product of the two Delta (Δ) resistances connected to that same terminal divided by the sum of all the Delta (Δ) resistances.
- The equivalent Delta (Δ) resistance between the two-terminals is the product of the two star (Y) resistances connected to those terminals divided by the third-star resistance plus the sum of the two same star (Wye) resistances.



Network Theorems

The fundamental theory on which many branches of electrical engineering, such as electric power, electric machines, control, electronics, computers, communications, and instrumentation are built is the Electric circuit theory. So here the network theorem helps us to solve any complex network for a given condition.

Note: All the theorems are only applicable to Linear Network only, according to the theory of Linear Network they follow the condition of Homogeneity & Additivity.

Homogeneity Principle:

A system is said to be homogeneous, for any input signal $x(t)$

If input $x(t)$ gives \rightarrow response $y(t)$

then, it must follow $\Leftrightarrow k x(t) \rightarrow k y(t)$

i.e scaling in any input signal scales the output signal by same factor.

Additivity Principle:

A system is said to be homogeneous, for any input signal $x(t)$

If two input $x_1(t) + x_2(t) \Leftrightarrow y_1(t) + y_2(t)$

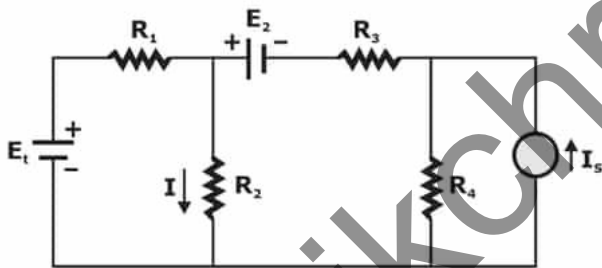
then, $k_1x_1(t) + k_2x_2(t) \Leftrightarrow k_1y_1(t) + k_2y_2(t)$

i.e the output corresponding to the sum of any two inputs is the sum of their respective outputs.

1. SUPERPOSITION THEOREM

Superposition theorem finds use in solving a network where two or more sources are present and connected not in series or in parallel.

Superposition theorem states that if a number of voltage or current sources are acting simultaneously in a linear bidirectional network, the resultant response in any branch is the algebraic sum of the responses that would be produced in it, when each source acts alone replacing all other independent sources by their internal resistances.



Procedure for using the superposition theorem:

Step-1: Retain one source at a time in the circuit and replace all other sources with their internal resistances.

Step-2: Determine the output (current or voltage) due to the single source acting alone.

Step-3: Repeat steps 1 and 2 for each of the other independent sources.

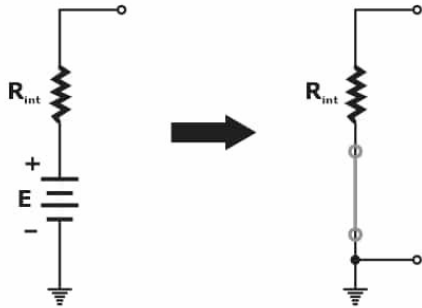
Step-4: Find the total contribution by adding algebraically all the contributions due to the independent sources.

So for above given circuit the total response or say current I through resistor R_2 will be equal to the sum of individual response obtained by each source.

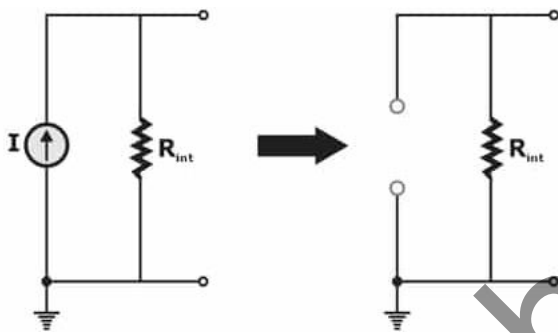
$$I = I'_{\text{due to } E_1(\text{alone})} + I''_{\text{due to } E_2(\text{alone})} + I'''_{\text{due to } I_s(\text{alone})}$$

Removing of Active Element in Superposition Theorem:

1. Ideal voltage source is replaced by short circuit.

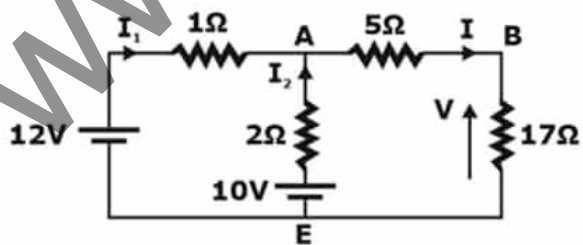


2. Ideal current source is replaced by open circuit.



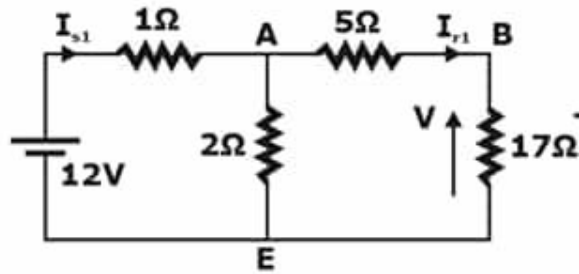
Limitation: Superposition cannot be applied to power calculation because the power is related to the square of the voltage across a resistor or the current through a resistor. The squared term results in a non-linear (a curve, not a straight line) relationship between the power and the determining current or voltage.

Example: Determine the value of current I in 17 ohm resistor by using superposition theorem.

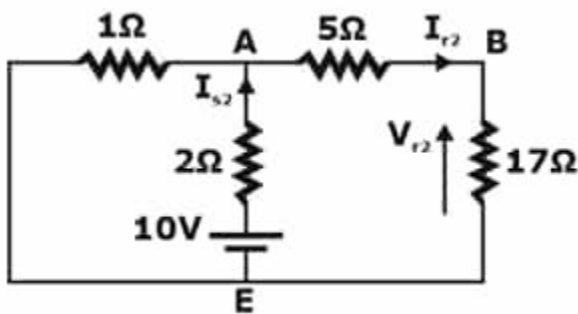


Solution:

Firstly, only 12 V source is active.



Now, only 10 V source is active.



From the sub circuit 1

$$I_{s1} = \frac{12}{1 + [2 \parallel (5 + 17)]} = 4.23529 A$$

$$\therefore I_{r1} = \frac{2}{1 + (5 + 17)} \times 4.23529 A = 0.35294 A$$

similarly

$$I_{s2} = \frac{10}{2 + [1 \parallel (5 + 17)]} = 3.38235 A$$

$$\therefore I_{r2} = \frac{1}{1 + (5 + 17)} \times 3.38235 A = 0.14706 A$$

\therefore From superposition theorem

$$I = I_{r1} + I_{r2}$$

$$I = 0.35294 + 0.14706$$

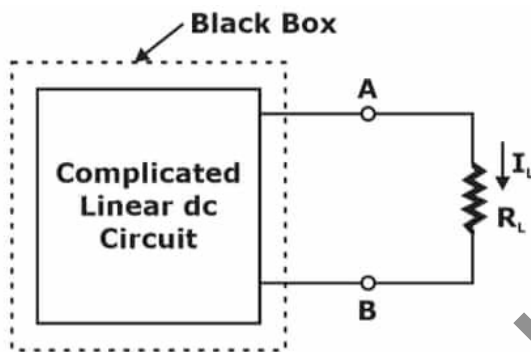
$$I = 0.5 A$$

2. THEVENIN'S THEOREM

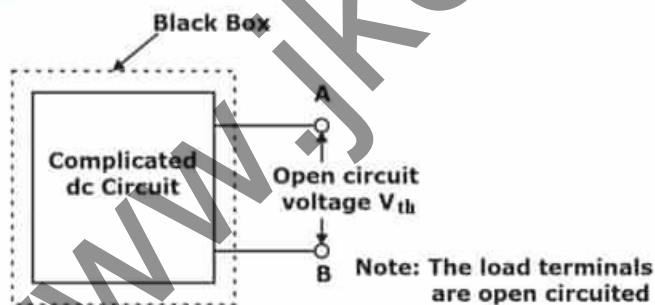
Thevenin's theorem states that any two output terminals of an active linear network containing independent sources (it includes voltage and current sources) can be replaced by a simple voltage source of magnitude V_{TH} in series with a single resistor R_{TH} where R_{TH} is the equivalent resistance of the network when looking from the output terminals A & B with all sources (voltage and current) removed and replaced by their internal resistances and the magnitude of V_{TH} is equal to the open circuit voltage across the A & B terminals.

The procedure for applying Thevenin's theorem

To find a current I_L through the load resistance R_L using Thevenin's theorem, the following steps are followed:



Step-1: Disconnect the load resistance (R_L) from the circuit,

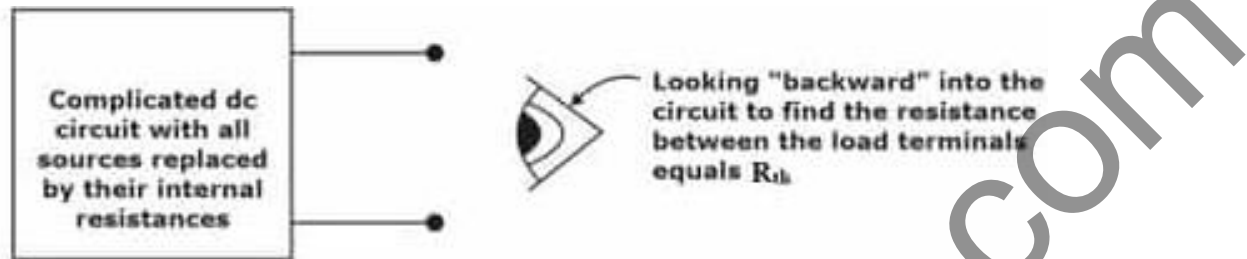


Step-2: Calculate the open-circuit voltage V_{TH} at the load terminals (A & B) after disconnecting the load resistance (R_L).

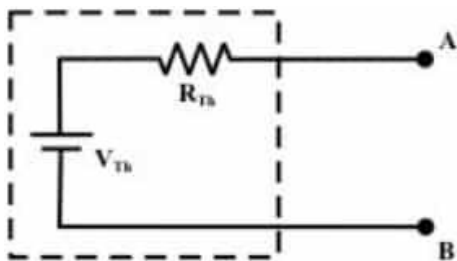
Step-3: Redraw the circuit with each independent source replaced by its internal resistance.

Note: Voltage sources should be short-circuited and current sources should be open-circuited.

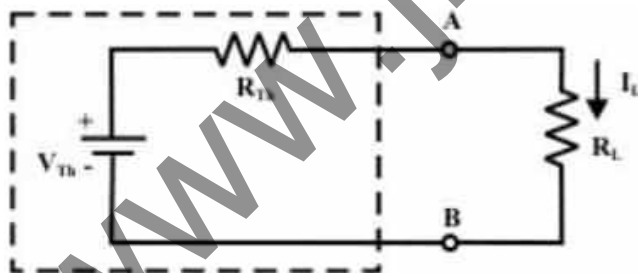
Step-4: Look backward into the resulting circuit from the load terminals (A & B). Calculate the resistance that would exist between the load terminals.



Step-5: Place R_{TH} in series with V_{TH} to form the Thevenin's equivalent circuit.



Step-6: Reconnect the original load to the Thevenin equivalent circuit as shown in the load's voltage, current and power may be calculated by a simple arithmetic operation only.



$$\text{Load current } I_L = \frac{V_{Th}}{R_{Th} + R_L}$$

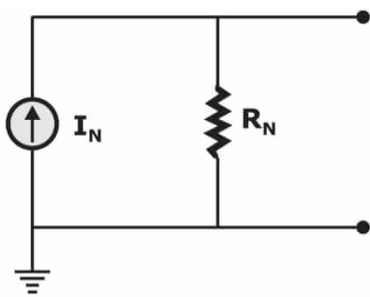
$$\text{Voltage across the load } V_L = \frac{V_{Th}}{R_{Th} + R_L} \times R_L = I_L \times R_L$$

$$\text{Power absorbed by the load } P_L = I_L^2 \times R_L$$

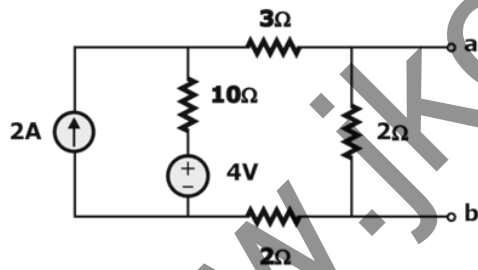
3. NORTON'S THEOREM

Norton's theorem states that any two output terminals of an active linear network containing independent sources (it includes voltage and current sources) can be replaced by a current source and a parallel resistor R_N . Where, R_N which is the equivalent resistance of the network when looking from the output terminals A & B with all sources (voltage and current) removed and replaced by their internal resistances and the magnitude of I_N is equal to the short-circuit current across the A & B load terminals.

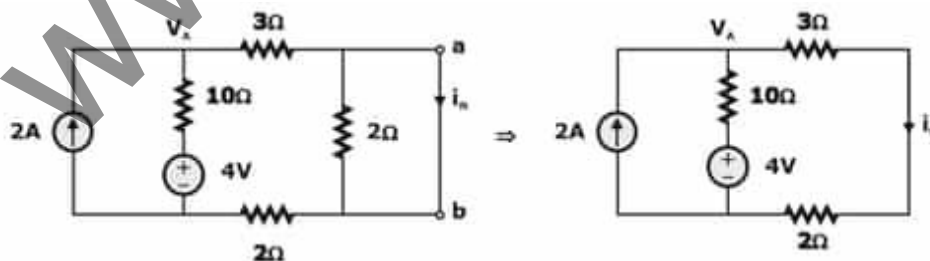
Norton Equivalent Circuit can be shown as:



Example: Find the Norton equivalent circuit of the following circuit at terminal a-b.



Solution: Circuit can be redrawn as:



Applying KCL,

$$\frac{V_A - 4}{10} + \frac{V_A}{5} = 2$$

$$\frac{V_A - 4 + 2V_A}{10} = 2$$

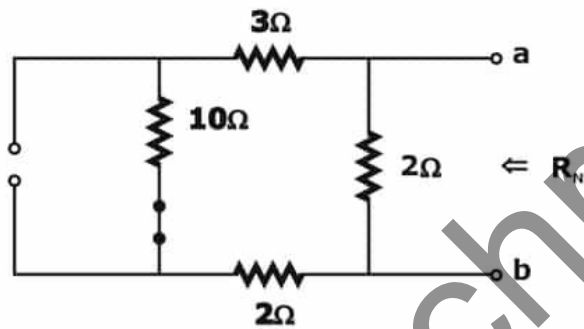
$$3V_A - 4 = 20$$

$$V_A = 8V$$

$$i_N = \frac{8}{5}$$

$$i_N = 1.6 \text{ A}$$

For calculating newton resistance R_N :

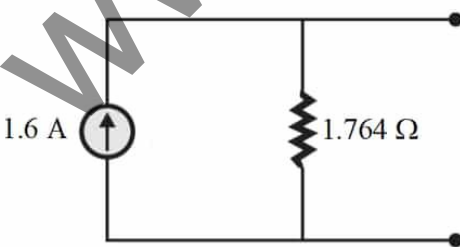


$$R_N = (10 + 2 + 3) || 2$$

$$R_N = 15 || 2$$

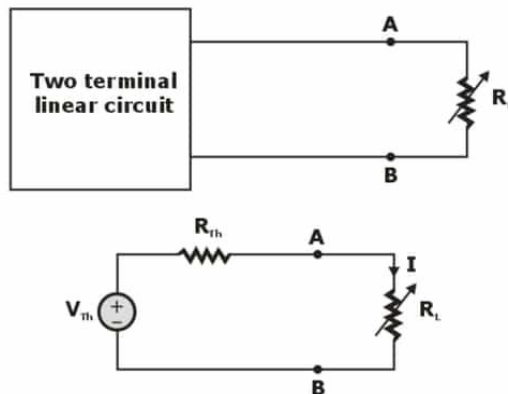
$$R_N = 1.764 \Omega$$

Norton Equivalent Circuit can be drawn as:



4. MAXIMUM POWER TRANSFER THEOREM

Maximum Power Transfer Theorem states a resistive load, being connected to a DC network, consumes maximum power when the load resistance is equal to the thevenin's equivalent resistance of the source network as seen from the load terminals.



A variable resistance R_L is connected to a dc source network as shown in figure above and the Thevenin's voltage V_{Th} and Thevenin's equivalent resistance R_{Th} of the source network. The aim is to determine the value of R_L such that it consumes maximum power from the DC source.

$$I = \frac{V_{Th}}{R_{Th} + R_L}$$

The power delivered to the resistive load is

$$P_L = I^2 R_L = \left(\frac{V_{Th}}{R_{Th} + R_L} \right)^2 \times R_L$$

P_L can be maximised by varying R_L and hence maximum power can be delivered when

$$\left(\frac{dP_L}{dR_L} \right) = 0$$

However,

$$\frac{dP_L}{dR_L} = \frac{1}{\left[(R_{Th} + R_L)^2 \right]^2} \left[(R_{Th} + R_L)^2 \frac{d}{dR_L} (V_{Th}^2 R_L) - V_{Th}^2 R_L \frac{d}{dR_L} (R_{Th} + R_L)^2 \right]$$

$$= \frac{1}{(R_{Th} + R_L)^4} \left[(R_{Th} + R_L)^2 V_{Th}^2 - V_{Th}^2 R_L \times 2(R_{Th} + R_L) \right]$$

$$= \frac{V_{Th}^2 (R_{Th} + R_L - 2R_L)}{(R_{Th} + R_L)^3} = \frac{V_{Th}^2 (R_{Th} - R_L)}{(R_{Th} + R_L)^3}$$

$$\frac{dP_L}{dR_L} = 0$$

$$\frac{V_{Th}^2 (R_{Th} - R_L)}{(R_{Th} + R_L)^3} = 0$$

$$(R_{Th} - R_L) = 0$$

$$\boxed{R_{Th} = R_L}$$

Hence, the maximum power transferred to the load resistance is

$$\boxed{P_{max} = \frac{V_{Th}^2}{4R_{Th}}}$$

Steps for Solution of a Network using Maximum Power Transfer Theorem:

Step 1: Remove the load resistance and find the Thevenin's resistance (R_{Th}) of the source network looking through the open circuited load terminals.

Step 2: As per maximum power transfer theorem, this R_{Th} is the load resistance of the network i.e. $R_L = R_{Th}$ that allows maximum power transfer.

Step 3: Find the Thevenin's voltage (V_{Th}) across the open circuited load terminals.

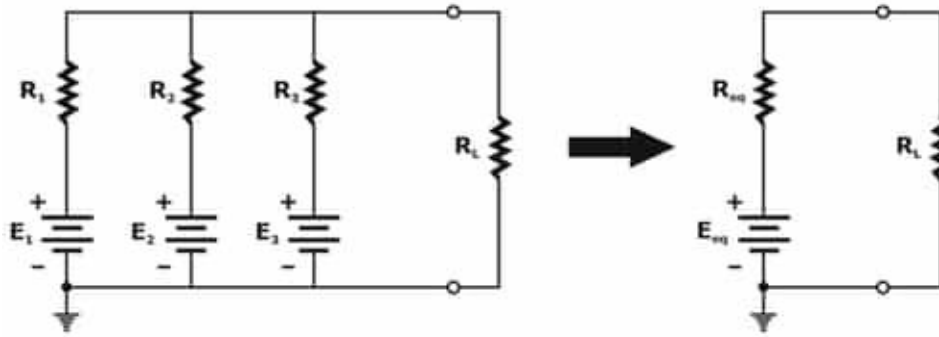
Step 4: Maximum Power Transfer is given by:

$$P_{max} = (V_{th})^2 / 4R_{th}$$

Note: Maximum power transfer condition results in **50 percent** efficiency in Thevenin equivalent, however much lower efficiency in the original circuit.

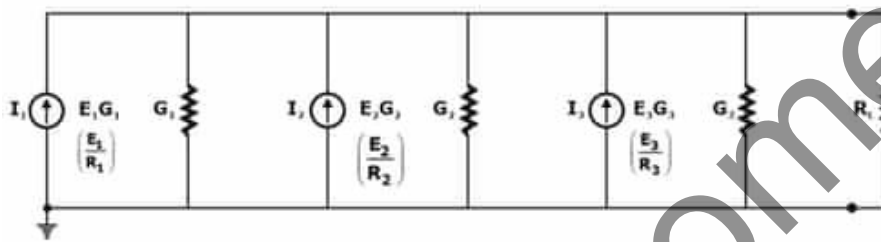
5. MILLMAN'S THEOREM

Millman's theorem helps to reduce 'N' number of parallel voltage sources to one. It can be observe in the given figure below. This technique permits finding the current through or voltage across R_L without applying any method such as mesh analysis, nodal analysis, superposition, and so on. For example, the given three voltage sources can be reduced to one voltage source.



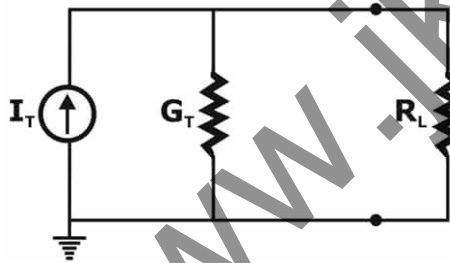
Generally, three steps are included in its application

Step 1: Convert all voltage sources into current sources.

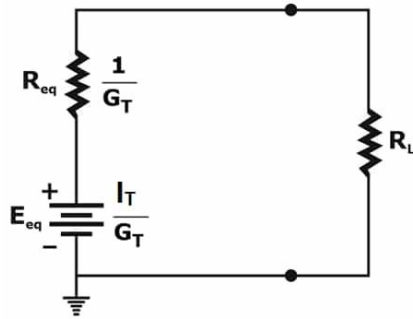


Step 2: Convert the parallel current source into resulting network as shown below

$$I_T = I_1 + I_2 + I_3 \quad \& \quad G_T = G_1 + G_2 + G_3$$



Step 3: Convert the resulting current source to a voltage source by transformation, and the desired single-source network will be shown as



$$E_{eq} = \frac{I_T}{G_T} = \frac{\pm I_1 \pm I_2 \pm I_3 \pm \dots \pm I_N}{G_1 + G_2 + G_3 + \dots + G_N}$$

$$E_{eq} = \frac{\pm E_1 G_1 \pm E_2 G_2 \pm E_3 G_3 \pm \dots \pm E_N G_N}{G_1 + G_2 + G_3 + \dots + G_N}$$

The plus-and-minus signs appears in the last equation to include those cases where the sources may not be supplying energy in the same direction.

$$R_{eq} = \frac{1}{G_T} = \frac{1}{G_1 + G_2 + G_3 + \dots + G_N}$$

In terms of resistances values:

$$E_{eq} = \frac{\pm \frac{E_1}{R_1} \pm \frac{E_2}{R_2} \pm \frac{E_3}{R_3} \pm \dots \pm \frac{E_N}{R_N}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots + \frac{1}{R_N}}$$

and

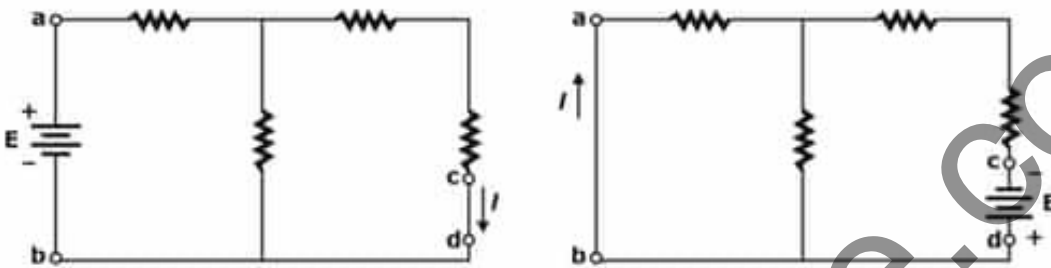
$$R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots + \frac{1}{R_N}}$$

6. RECIPROCITY THEOREM

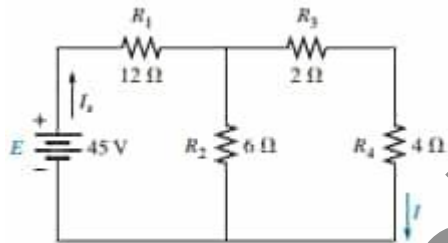
The reciprocity theorem is applicable to the single-source networks only. The theorem states that the current \$I\$ in any branch of a network due to a single voltage source \$E\$ present anywhere else in the network will be equal to the

current across the branch in which the source was originally placed if the source is placed in the branch in which the current I was originally (initially) measured.

In other words, the location of the voltage source and the resulting current may be interchanged without a change in magnitude of current. The theorem postulates that the polarity of the voltage source have the same adaptation with the direction of the branch current in each position.



- **Example:** Verify Reciprocity theorem.



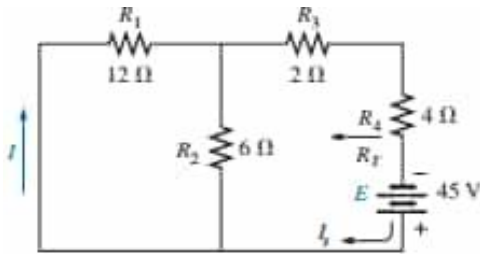
The total resistance is

$$R_T = R_1 + R_2 \parallel (R_3 + R_4) = 12 \Omega + 6 \Omega \parallel (2 \Omega + 4 \Omega) \\ = 12 \Omega + 6 \Omega \parallel 6 \Omega = 12 \Omega + 3 \Omega = 15 \Omega$$

and
$$I_s = \frac{E}{R_T} = \frac{45 \text{ V}}{15 \Omega} = 3 \text{ A}$$

with
$$I = \frac{3 \text{ A}}{2} = 1.5 \text{ A}$$

- Interchanging (or reciprocating) the location of I and E in the last figure to demonstrate the validity of the reciprocity theorem.



$$R_T = R_4 + R_3 + R_1 \parallel R_2 \\ = 4 \Omega + 2 \Omega + 12 \Omega \parallel 6 \Omega = 10 \Omega$$

and
$$I_4 = \frac{E}{R_T} = \frac{45 \text{ V}}{10 \Omega} = 4.5 \text{ A}$$

so that
$$I = \frac{(6 \Omega)(4.5 \text{ A})}{12 \Omega + 6 \Omega} = \frac{4.5 \text{ A}}{3} = 1.5 \text{ A}$$

which agrees with the above.

7. TELLEGEN'S THEOREM

- □ Tellegen's theorem is based upon two Kirchhoff's laws and is also applicable for any lumped network having elements which are linear or non-linear, active or passive, time-varying or time-invariant.
- □ For a lumped network whose element assigned by associate reference direction for branch voltage v_k and branch current j_k . The product $v_k j_k$ is the power delivered at time t by the network to the element k .
- □ If all branch voltages and branch currents satisfies KVL and KCL then

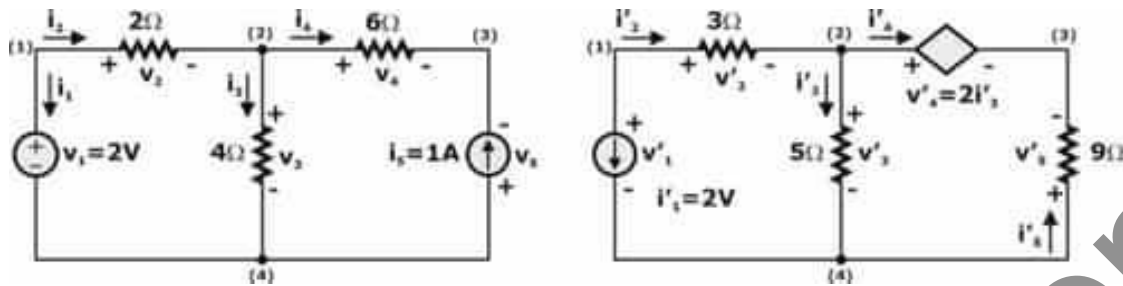
$$\sum_{k=1}^b v_k j_k = 0 \quad b = \text{number of branch}$$

Application of Tellegen's Theorem:

As seen from the last equation, the Tellegen's Theorem implies the law of energy conservation. **"The sum of power delivered by the independent sources to the network elements is equal to the sum of the power absorbed by all the branches of the network."** So, the application of Tellegen's theorem can be classified as

- □ Conservation of energy
- □ Conservation of complex power
- □ The real(or active) part and the phase of driving point impedance
- □ Driving point impedance

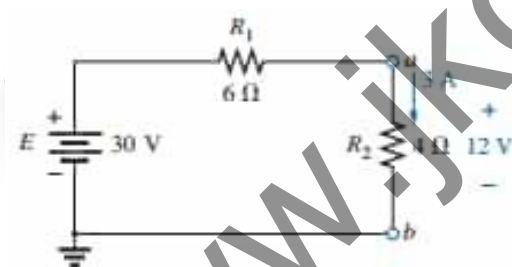
Example: Find all branch voltages and currents for both networks N1, N2, and then verify Tellegen's theorem.



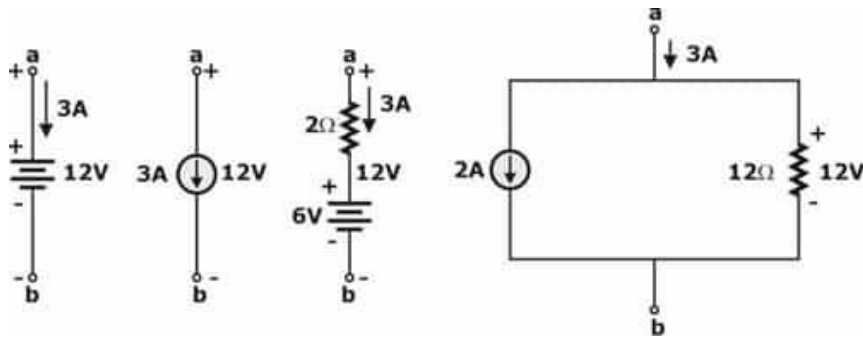
8. SUBSTITUTION THEOREM

The substitution theorem states that "If the voltage across any branch and the current flowing through that branch of a dc bilateral network are known, then that branch can be substituted by any one of the combinations that can consist of the same voltage and current through that chosen branch.

More simply, the theorem states that for branch equivalence, the voltage across the terminal and current through the terminal must be the same. Consider the circuit in which the voltage across and current through the branch a-b are determined. Through the use of the substitution theorem, few number of equivalent a-a' branches are shown. Note that for each equivalent circuit, the terminal voltage & the current remains same.

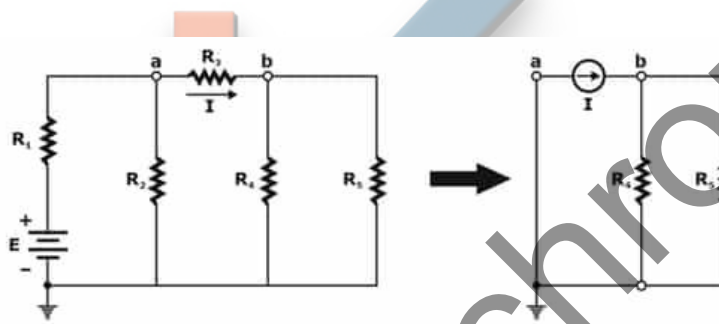


- By the use of the substitution theorem, the number of equivalent branches are:



Note : for each equivalent, the terminal voltage and current are the same and known potential difference and current in a network can be replaced by an ideal voltage source and current source respectively.

- Example: The current source equivalence where a known current is replaced by an ideal current source, permitting the isolation of R_4 and R_5 as shown below



Recall the discussion of bridge networks that $V = 0$ and $I = 0$ were replaced by a short-circuit and an open circuit respectively.

Transient Analysis

Time Domain

There are three basic components in Linear Circuits:

- Resistance (R)
- Capacitance (C) and
- Inductance (L)

The behavior of a circuit composed of R, C, and L elements and is modeled by differential equations with constant coefficients.

- An electrical system is said to be in **steady-state** when the variables describing its behavior (voltages, currents, etc.) are either invariant with time (d.c. circuits) or are periodic functions of time (a.c. circuits).
- The time-varying currents and voltages resulting from the sudden application of sources, usually due to switching, are called **transients**.
- An electrical system is said to be in a **transient state** when the variables are changed non-periodically, i.e., when the system is not in steady-state.
- The **transient response** is the fluctuation in current and voltage in a circuit (after the application of a step voltage or current) before it settles down to its steady state.

In a DC circuit, the electro-motive forces push the electrons along the circuit and resistors to remove that energy by conversion to heat. In AC circuits, currents vary in time, so we have to consider variations in the energy stored in electric and magnetic fields of capacitors and inductors, respectively.

Capacitance: the capacitance C between two oppositely charged surfaces is defined by:

$V = Q/C$, where Q is the magnitude of the charge distribution on either surface, and V is the potential difference between the surfaces.

Differentiate $V = Q/C$ using $I = dQ/dt$, We get $dV/dt = I/C$.

Inductance: The usual model for an inductor is a coil (solenoid). By Faraday's Law of self-inductance, a changing current in a coil induces a back electromotive force (emf) that opposes the change in current:

$V = L \cdot di/dt$, where V is the back emf across the inductor, di/dt is the derivative of the current through the inductor and L is the inductance.

A first-order circuit can only contain one energy storage element (a capacitor or an inductor). The circuit will also contain resistance. So there are two types of first order circuits: RC circuit, and RL circuit.

Voltage-Current Relationships for Passive Elements Note: V_R , V_L and V_C are the voltages across R , L and C respectively while i_R , i_L and i_C are the current through R , L and C respectively.

Element Transformations Resistor:

- Time Domain:
- s-Domain:

Inductor:

- Time Domain:
- s-Domain:

$$V(s) = L[sI(s) - i(0)]$$

Capacitor (C):

- Time Domain:
- s-Domain:

$$i(s) = sCV(s) - Cv$$

Steps for Finding Transient Response

- Identify the variable of interest (Inductor current for RL circuit, Capacitor voltage for RC circuit).
- Determine the initial value of the variable.
- Calculate the final value of the variable.
- Calculate the time constant for the circuit.

Transient Response of RL and RC Circuits

The analysis of circuits that contain combinations of resistors with capacitors or inductors follows the same general principles as for networks of resistors alone. Following figure shows a basic combination of a resistor with a capacitor, where a switch connected to a battery or ground allows the creation of voltage pulses at the input to the network.

We have 9 unknowns, using the above equations the following can be derived.

(We may assume the input voltage to be either 0 V or the battery emf E, and that it will be steady while the switch remains in a given position. During such periods, we may therefore treat V_{in} as a constant.)

Transient Analysis of R-L Circuit:

- When the switch is closed, current flows into the capacitor.
- Current flow ceases when charge collected on the capacitor produces a voltage equal and opposite to V .
- An equation describing the behavior is shown; it is both exponential and asymptotic.
- In the equation, the value RC is called the time constant (τ).
- As τ grows smaller, transient behavior disappears much faster.

The transient current through the inductor L at any time t

where, i_σ = Current through L at $t \rightarrow \sigma$ i.e., steady-state current through L i_0 = Current through L at $t = 0^-$ R_{eq} = Thevenin's equivalent resistance seen across L for $t > 0^-$

Transient Analysis of R-C Circuit:

- When the switch is closed, current flow is inhibited as the inductor develops an opposite voltage to the one applied.
- Current slowly begins to flow, as the inductor voltage falls toward 0.
- As the transient effect dies, current flow approaches V/R .
- The time constant τ in an RL circuit is defined as $\tau = L/R$.
- As τ grows smaller, transient behavior disappears much faster.

The transient voltage across capacitor C at any time

where, V_σ = Voltage across capacitor at $t \rightarrow \sigma$, i.e., steady state voltage across C V_0 = Voltage across C at $t = 0^-$ R_{eq} = Thevenin's equivalent resistance seen across C for $t > 0^-$.

Transient Analysis of RLC Circuit:

- A circuit with R , L , and C can exhibit oscillatory behavior if the components are chosen properly.
- For many values of R - L - C , there will be no oscillation.
- α is the damping factor, which determines the rate at which the oscillation dies out.
- The smaller L and C , the higher frequency of the oscillation.

- If R is too large the quantity under the square root is negative, which means there is no oscillation

Frequency Domain

- A periodic signal can be viewed as being composed of a number of sinusoids.
- Instead of specifying a periodic signal in terms of the time variable t , one can equivalently specify the amplitude and phase density of each sinusoid of frequency contained in the signal.
- It uses the frequency variable ω as an independent variable, and thus it is said to be the frequency domain of the given time-domain signal.

KCL in s-domain:

- t-domain (time domain): $i_1(t)+i_2(t)-i_3(t)+i_4(t)=0$
- s-domain (complex frequency domain): $I_1(s)+I_2(s)-I_3(s)+I_4(s)=0$

KVL in s-domain:

- t-domain (time domain): $-v_1(t)+v_2(t)+v_3(t) = 0$
- s-domain (complex frequency domain): $-V_1(s)+V_2(s)+V_3(s) = 0$

Signal Sources in s Domain: Voltage Source:

- t-domain: $v(t) = v_s(t)$, and $i(t)$ depends on circuit.
- s-domain: $V(s) = V_s(s)$, and $I(s)$ depends on circuit.

Current Source:

- t-domain: $i(t) = i_s(t)$, and $v(t)$ depends on circuit.
- s-domain: $I(s) = I_s(s)$, and $V(s)$ depends on circuit.

Resonance: The circuit is said to be in resonance if the current is in phase with the applied voltage. Power factor of the circuit at resonance is unity. At resonance, the circuit behaves like a resistive circuit. The frequency at which the resonance occurs is called the resonant frequency.

There are two types of Resonance circuits: 1. Series Resonance circuit, and 2. Parallel Resonance circuit.

Series Resonance

The series RLC can be analyzed in the frequency domain using complex impedance relations.

If the voltage source above produces a complex exponential waveform with complex amplitude $V(s)$ and angular frequency $s = \sigma + i\omega$, KVL can be applied:

At resonance

$|V_L| = |V_C|$ and these are 180° out of phase.

Z_{in} = Input impedance

The frequency at which the reactance of the inductance and the capacitance cancel each other is the resonant frequency (or the unity power factor frequency) of this circuit.

Conditions for ω and ω_0 in Series Resonance

Quality factor: The quality factor or Q-factor is basically an amplification factor for a resonant circuit.

Bandwidth: The bandwidth ($\omega_2 - \omega_1$) is called the half-power bandwidth or 3-dB bandwidth.

The bandwidth of the series circuit is defined as the range of frequencies in which the amplitude of the current is equal to or greater than $(1/1.414)$ times its maximum amplitude. This yields the bandwidth $B = \omega_2 - \omega_1 = R/L$.

The frequency at which voltage across an inductor is maximum

The frequency at which voltage across the capacitor is maximum

Selectivity It is defined as the ratio of resonant frequency to the bandwidth.

Key Points

- Selectivity of series R-L-C circuit with C variable is.
- Selectivity of series R-L-C circuit with L variable is also

- The higher the selectivity, the higher will be the quality factor.
- The higher the selectivity, the lesser will be the bandwidth.

Parallel Resonance

The parallel resonance circuit is also called an anti-resonance circuit. The complex admittance of this circuit is given by adding up the admittances of the components:

At resonance,

- $|i_L| = |i_C|$ and these are 180° out of phase

Conditions for ω and ω_0 in Series Resonance

A parallel RLC circuit is an example of a band-stop circuit response that can be used as a filter to block frequencies at the resonance frequency but allow others to pass.

Steady-State Analysis

1. Introduction

Many applications of control theory are to servomechanisms which are systems using the feedback principle designed so that the output will follow the input. Hence there is a need for studying the time response of the system. The time response of a system may be considered in two parts:

- Transient response: this part reduces to zero as $t \rightarrow \infty$
- Steady-state response: response of the system as $t \rightarrow \infty$

2. Response of the first order systems

- Consider the output of a linear system in the form $Y(s) = G(s)U(s)$ where $Y(s)$: Laplace transform of the output, $G(s)$: transfer function of the system and $U(s)$: Laplace transform of the input.
- Consider the first order system of the form $ay + \dot{y} = u$, its transfer function is

$$Y(s) = \frac{1}{as + 1} U(s)$$

- For a transient response analysis it is customary to use a reference unit step function $u(t)$ for which

$$U(s) = \frac{1}{s}$$

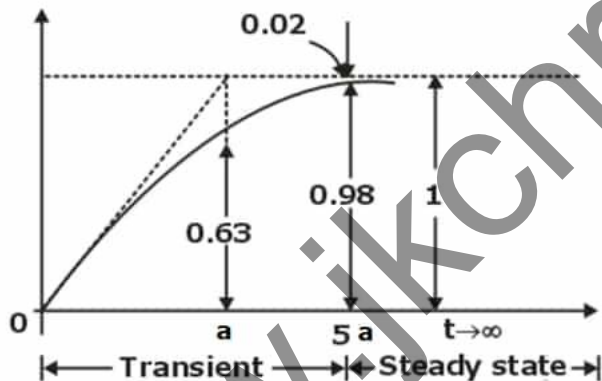
- It then follows that

$$Y(s) = \frac{1}{(as+1)s} = \frac{1}{s} - \frac{1}{s+1/a}$$

- On taking the inverse Laplace of equation, we obtain

$$y(t) = \underbrace{1}_{\text{steady-state part}} - \underbrace{e^{-t/a}}_{\text{transient part}} \quad (t \geq 0)$$

- The response has an exponential form. The constant 'a' is called the time constant of the system.



- Notice that when $t = a$, then $y(t) = y(a) = 1 - e^{-1} = 0.63$. The response is in two parts, the transient part $e^{-t/a}$, which approaches zero as $t \rightarrow \infty$ and the steady-state part 1 , which is the output when $t \rightarrow \infty$.
- If the derivative of the input are involved in the differential equation of the system, that is if $a\dot{y} + y = b\dot{u} + u$ then its transfer function is

$$Y(s) = \frac{bs+1}{(as+1)} U(s) = K \frac{s+Z}{s+p} U(s)$$

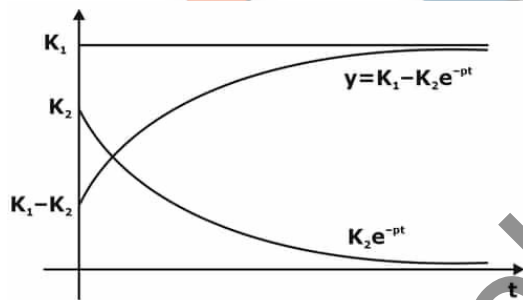
- where
 $K = b / a$
 $z = 1/ b$: the zero of the system
 $p = 1/ a$: the pole of the system
- When $U(s) = 1/ s$, Equation can be written as

$$Y(s) = \frac{K_1}{s} - \frac{K_2}{s+p}, \text{ Where } K_1 = K \frac{z}{p} \text{ and } K_2 = K \frac{z-p}{p}$$

- Hence,

$$y(t) = \underbrace{K_1}_{\text{steady-state part}} - \underbrace{K_2 e^{-pt}}_{\text{transient part}}$$

- With the assumption that $z > p > 0$, this response is shown in



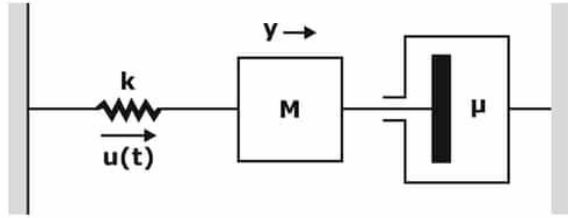
- We note that the responses to the systems have the same form, except for the constant terms K_1 and K_2 . It appears that the role of the numerator of the transfer function is to determine these constants, that is, the size of $y(t)$, but its form is determined by the denominator.

3. Response of second order systems

- An example of a second order system is a spring-dash pot arrangement, Applying Newton's law, we find

$$M\ddot{y} = -\mu \dot{y} - ky + u(t)$$

- where k is spring constant, μ is damping coefficient, y is the distance of the system from its position of equilibrium point, and it is assumed that $y(0) = y(0)' = 0$.



- Hence, $u(t) = M\ddot{y} + \mu\dot{y} + ky$
- On taking Laplace transforms, we obtain,

$$Y(s) = \frac{1}{Ms^2 + \mu s + k} U(s) = \frac{K}{s^2 + a_1 s + a_2} U(s)$$

- where $K = 1/M$, $a_1 = \mu/M$, $a_2 = k/M$.

Applying a unit step input, we obtain

$$Y(s) = \frac{K}{s(s + p_1)(s + p_2)}$$

- where $p_{1,2} = \frac{a_1 \pm \sqrt{a_1^2 - 4a_2}}{2}$, p_1 and p_2

are the poles of the transfer function

$$G(s) = \frac{K}{s^2 + a_1 s + a_2}$$

that is, the zeros of the denominator of $G(s)$.

- There are three cases to be considered:

over-damped system:

- In this case p_1 and p_2 are both real and unequal. Equation can be written as

$$Y(s) = \frac{K_1}{s} + \frac{K_2}{s + p_1} + \frac{K_3}{s + p_2}$$

$$\text{Where, } K_1 = \frac{K}{p_1 p_2} = \frac{K}{a_2}, K_2 = \frac{K}{p_1(p_1 - p_2)}, K_3 = \frac{K}{p_2(p_2 - p_1)}$$

critically damped system:

- In this case, the poles are equal: $p_1 = p_2 = a_1 / 2 = p$, and

$$Y(s) = \frac{K}{s(s+p)^2} = \frac{K_1}{s} + \frac{K_2}{s+p} + \frac{K_3}{(s+p)^2}$$

$$\text{Hence } y(t) = K_1 + K_2 e^{-pt} + K_3 t e^{-pt}$$

$$\text{Where, } K_1 = \frac{K}{p^2}, K_2 = \frac{-K}{p^2}, K_3 = \frac{-K}{p}$$

under-damped system:

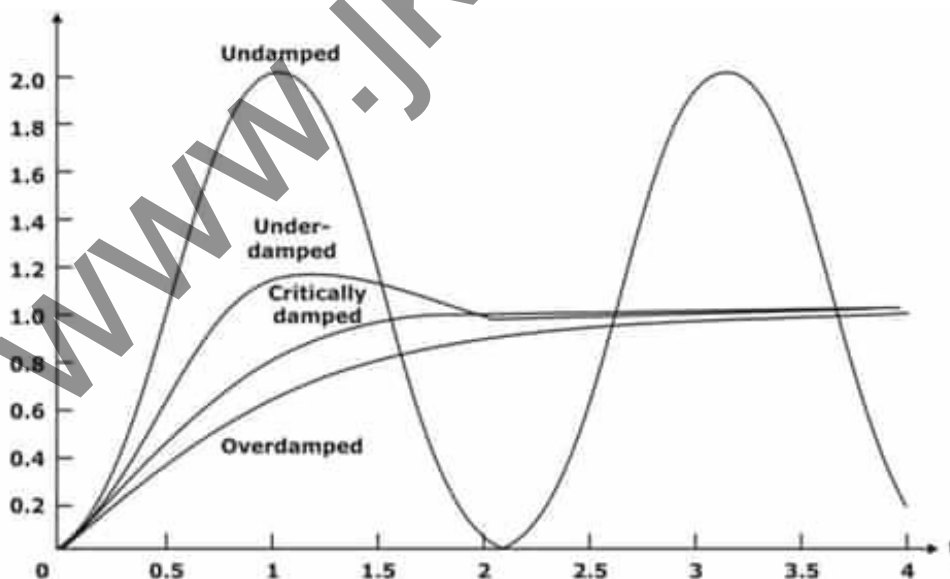
- In this case, the poles p_1 and p_2 are complex conjugate having the form

$$p_{1,2} = \alpha \pm i\beta \quad \text{Where } \alpha = a_1 / 2 \text{ and } \beta = \frac{1}{2} \sqrt{4a_2 - a_1^2}$$

$$Y(s) = \frac{K_1}{s} + \frac{K_2}{s+p_1} + \frac{K_3}{s+p_2}$$

$$\text{Where, } K_1 = \frac{K}{\alpha^2 + \beta^2}, K_2 = \frac{K(-\beta - i\alpha)}{2\beta(\alpha^2 + \beta^2)}, K_3 = \frac{K(-\beta + i\alpha)}{2\beta(\alpha^2 + \beta^2)}$$

The three cases discussed above are plotted as:



There are two important constants associated with each second order system:

- The undamped natural frequency ω_n of the system is the frequency of the response shown in Fig.

$$\omega_n = \sqrt{a_2}$$

- The damping ratio ξ of the system is the ratio of the actual damping $\mu (= a_1 M)$ to the value of the damping μ_c , which results in the system being critically damped.

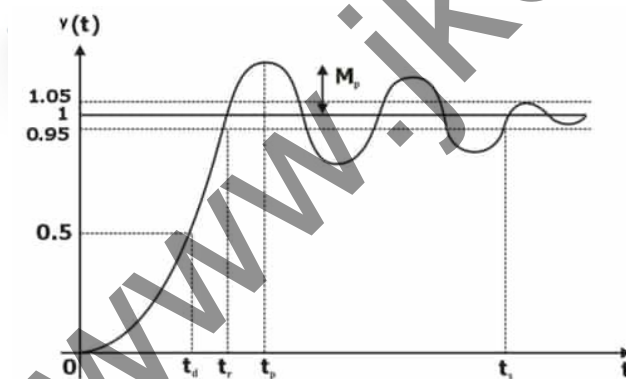
$$\text{Hence } \xi = \frac{\mu}{\mu_c} = \frac{a_1}{2\sqrt{a_2}}$$

- also,

$$y(t) = \frac{K}{\omega_n^2} \left(1 - \frac{1}{\sqrt{1-\xi^2}} e^{-\omega_n \xi t} \sin(\omega t + \varepsilon) \right)$$

$$\text{Where, } \omega = \omega_n \sqrt{1-\xi^2} \text{ and } \tan \varepsilon = \frac{\sqrt{1-\xi^2}}{\xi}$$

Some definitions:



- **Overshoot:** defined as

$$\frac{\text{Maximum overshoot}}{\text{Final desired value}} \times 100\%$$

- **Time delay τ_d** : the time required for a system response to reach 50% of its final value
- **Rise time**: the time required for the system response to rise from 10% to 90% of its final value
- **Settling time**: the time required for the eventual settling down of the system response to be within (normally) 5% of its final value
- **Steady-state error e_{ss}** : the difference between the steady state response and the input.

4. Steady state error

- Consider a unity feedback system



- where
 - $r(t)$: reference input
 - $c(t)$: system output
 - $e(t)$: error
- We define the error function as
- $e(t) = r(t) - c(t)$

hence,

$$e_{ss} = \lim_{t \rightarrow \infty} e(t)$$

Since $E(s) = R(s) - A(s)E(s)$, it follows that

$$E(s) = \frac{R(s)}{1 + A(s)}$$

and by the final value theorem

$$e_{ss} = \lim_{s \rightarrow 0} sE(s) = \lim_{s \rightarrow 0} \frac{sR(s)}{1 + A(s)}$$

- We now define three error coefficients which indicate the steady state error when the system is subjected to three different standard reference inputs $r(s)$.

step input:

$$r(t) = ku(t)$$

$$e_{ss} = \lim_{s \rightarrow 0} \frac{sk/s}{1+A(s)} = \frac{k}{1 + \lim_{s \rightarrow 0} A(s)}$$

$$\lim_{s \rightarrow 0} A(s) = K_p$$

called the position error constant, then

$$e_{ss} = \frac{k}{1 + K_p}$$

Ramp input: $r(t) = ktu(t)$

- In this case,

$$R(s) = \frac{k}{s^2}, \text{ so that } e_{ss} = \frac{k}{K_v} \text{ or } K_v = \frac{k}{e_{ss}}, \text{ where } K_v = \lim_{s \rightarrow 0} sA(s)$$

is called the velocity error constant.

Parabolic input: $r(t) = 1/2 kt^2 u(t)$

- In this case,

$$R(s) = \frac{k}{s^3}, \text{ so that } e_{ss} = \frac{k}{K_a} \text{ or } K_a = \frac{k}{e_{ss}}, \text{ where } K_a = \lim_{s \rightarrow 0} s^2 A(s)$$

- is called the acceleration error constant.

- From the definition of the error coefficients, it is seen that e_{ss} depends on the number of poles at $s = 0$ of the transfer function. This leads to the following classification. A transfer function is said to be of type N if it has N poles at the origin. Thus if

$$A(s) = \frac{K(s - z_1) \dots (s - z_m)}{s^j (s - p_1) \dots (s - p_n)}$$

- At $s = 0$, $A(s) = \lim_{s \rightarrow 0} \frac{K_1}{s^j}$ where $K_1 = \frac{K(-z_1) \dots (-z_m)}{(-p_1) \dots (-p_n)}$
- K_1 is called the gain of the transfer function.
- Hence the steady state error e_{ss} is summarized in Table

| Type of System | Unit Step Input | Unit Ramp Input | Unit Parabolic Input |
|-----------------|---------------------------------------|-------------------------------------|-------------------------------------|
| Type - 0 System | $K_p = K$ $e_{ss} = \frac{1}{1+K}$ | $K_p = 0$ $e_{ss} = \infty$ | $K_a = 0$ $e_{ss} = \infty$ |
| Type - 1 System | $K_p = \infty$ $e_{ss} = 0$ | $K_v = K$ $e_{ss} = \frac{1}{K}$ | $K_a = 0$ $e_{ss} = \infty$ |
| Type - 2 System | $K_p = \infty$ $e_{ss} = 0$ | $K_v = \infty$ $e_{ss} = 0$ | $K_a = K$ $e_{ss} = \frac{1}{K}$ |

AC Circuit Analysis

AC CIRCUIT ANALYSIS

ALTERNATING QUANTITY: An alternating quantity is that which acts in alternate directions and whose magnitude undergoes a definite cycle of changes in definite intervals of time.

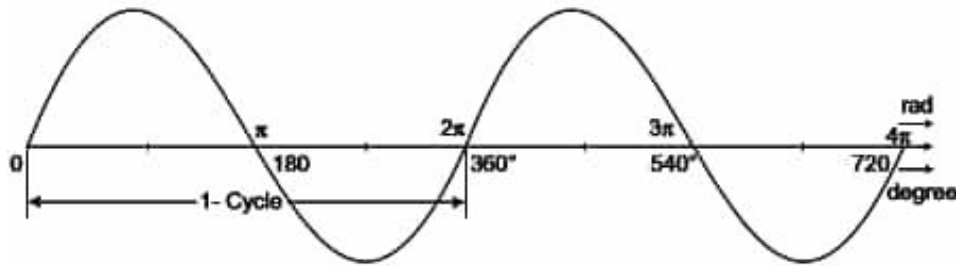
ALTERNATING VOLTAGE: Alternating voltage may be generated by

- ❑ By rotating a coil in a stationary magnetic field.
- ❑ By rotating a magnetic field within a stationary coil.

ADVANTAGES OF SINE WAVE

- ❑ Any periodic non-sinusoidal wave can be expressed as the sum of a number of Sine wave of different frequencies.
- ❑ A sine wave can be expressed in a simple mathematical form.
- ❑ The resultant of two or more quantities varying sinusoidally at the same frequency is another sinusoidal quantity of same frequency.
- ❑ The rate of change of any sinusoidal quantity is also sinusoidal.

CYCLE: A cycle may be defined as one complete set of positive and negative values of an alternating quantity repeating at equal intervals. Each complete cycle is spread over 360° electrical.



PERIODIC TIME: The time taken by an alternating quantity in seconds to trace one complete cycle is called periodic time or time-period. It is usually denoted by symbol T .

FREQUENCY: The number of cycles per second is called frequency and is denoted by symbol f .

$$f = 1/T$$

If the angular velocity ω is expressed in radians per second, then

$$\omega = 2\pi/T$$

$$\omega = 2\pi f$$

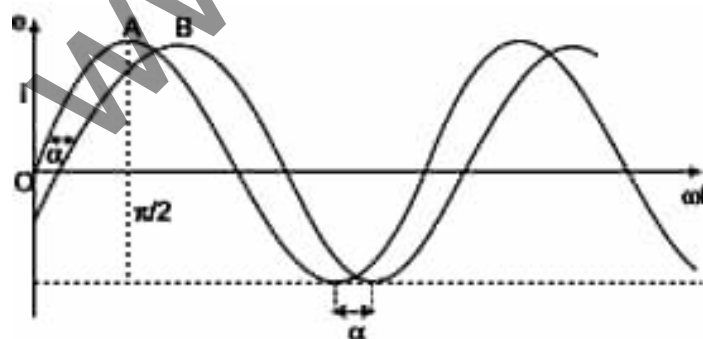
Peak Factor: Ratio of maximum value to the RMS value is known as crest or peak factor or amplitude factor.

Peak factor = Maximum Value / RMS Value

Form Factor: Ratio of RMS Value to average value is known as form factor.

Form Factor = RMS Value / Average Value

PHASE DIFFERENCE



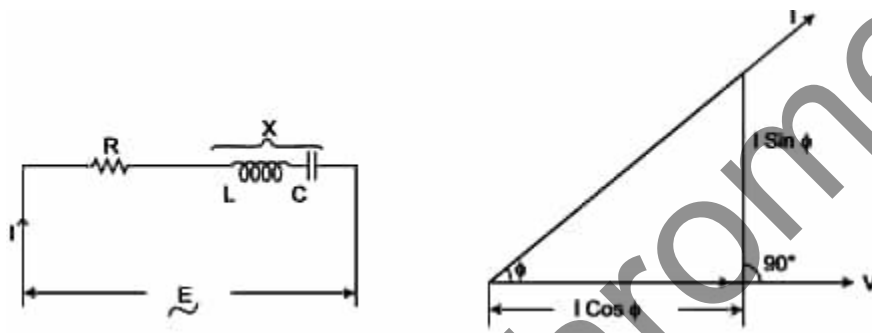
- Phase shift = α degrees
- A is ahead of B because A attains its maxima or minima before B
- A leads B
- B lags A

ACTIVE, REACTIVE & APPARENT POWER OF AC CIRCUIT

- When the current is out of phase with the voltage the power indicated by the product of the applied voltage and the total current gives only what is known as apparent power and measured in volt-amperes.
- The power that is returned to the source by the reactive components in the circuit is called reactive power and is measured in VAR.
- The power that actually used in the circuit (dissipated in resistance) is true or active power and is measured in watts or kW.

Impedance in AC circuit $Z = R \pm jX = |Z| \angle \Phi = |Z| \cos \Phi + j|Z| \sin \Phi$

where $X = j|(X_L - X_C)|$



where $|Z| = \sqrt{R^2 + X^2}$

$R = |Z| \cos \Phi$

$X = |Z| \sin \Phi$

Power Factor of the circuit $\Rightarrow \cos \Phi = R/Z$

Current in the circuit = E/Z

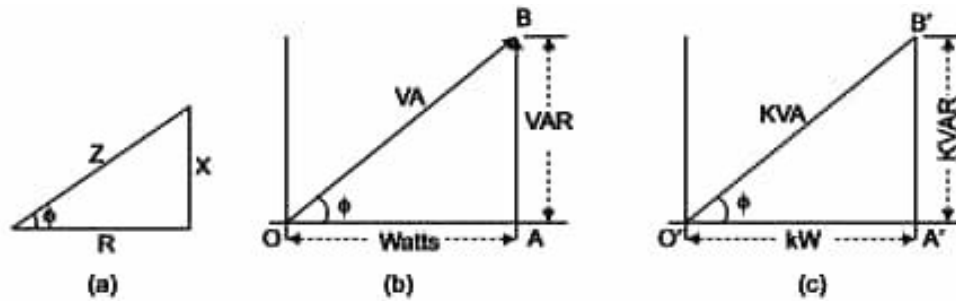
This current has two components $I \cos \Phi$ and $I \sin \Phi$. The component $I \cos \Phi$ is called in phase or watt full component and $I \sin \Phi$ is perpendicular to E and is called wattless component.

Active (Real) Power = Voltage \times Current $\times \cos \Phi$ watts

The total power EI in volt-amperes supplied to a circuit consists of two components:

(a) Active power = $EI \cos \Phi$ watts

(b) Reactive power = $EI \sin\Phi$ volt-amperes reactive or simply VAR.



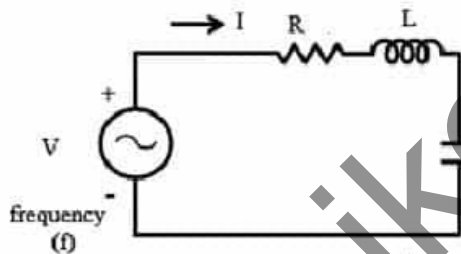
OA = Active power = $EI \cos\Phi$ presented by watts

AB = Reactive power = $EI \sin \Phi$ expressed by VAR

OB = Total power = EI expressed by VA

4. RESONANCE IN AC CIRCUIT

SERIES RESONANCE



The circuit, with resistance R , inductance L , and a capacitor, C in series is connected to a single-phase variable frequency (f) supply.

The total impedance of the circuit is

$$Z \angle \Phi = R + j(X_L - X_C)$$

where $X_L = j\omega L$

$$X_C = 1/j\omega C$$

The current is

$$I \angle -\phi = \frac{V \angle 0^\circ}{Z \angle \phi} = (V/Z) \angle -\phi$$

$$\text{where } I = \frac{V}{\left[R^2 + (\omega L - 1/\omega C)^2 \right]^{1/2}}$$

The current in the circuit is maximum, if $\omega L = \frac{1}{\omega C}$

The frequency under the above condition is

$$f_o = \frac{\omega_o}{2\pi} = \frac{1}{2\pi\sqrt{LC}}$$

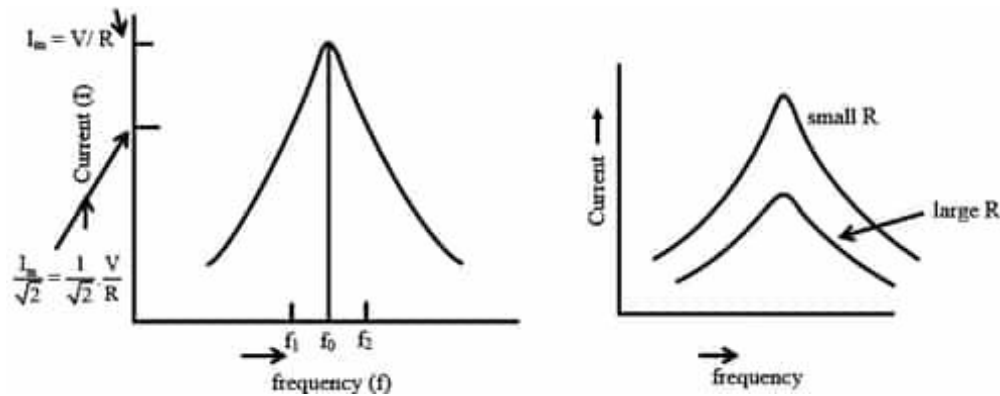
The magnitude of the impedance under the above condition is $|Z| = R$, with the reactance $X=0$, as the inductive reactance is equal to capacitive reactance. The phase angle is $\phi=0$, and the power factor is unity ($\cos \phi=1$), which means that the current is in phase with the input (supply) voltage. So, the magnitude of the current $I = V/R$.

The magnitude of the voltage drop in the inductance L/capacitance C, both are equal

as the reactance are equal is $\cdot = (I \cdot \omega_o L = I \cdot 1/(\omega_o C))$

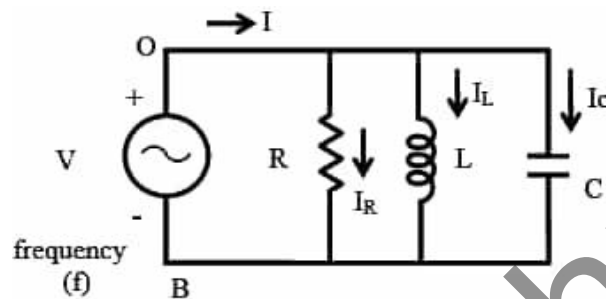
Quality Factor $Q = \omega_o L/R = 2\pi f_o L/R = 1/R \sqrt{L/C}$

The impedance of the circuit with the constant values of inductance L, and capacitance C is minimum at resonant frequency (f_o), and increases as the frequency is changed, i.e. increased or decreased, from the above frequency. The current is maximum at $f=f_o$, and decreases as frequency is changed ($f>f_o$ or $f<f_o$) i.e. $f \neq f_o$.



PARALLEL RESONANCE

The circuit, with resistance R , inductance L , and a capacitor, C in parallel is connected to a single-phase variable supply frequency (f).



The total admittance of the circuit is

$$Y \angle \phi = 1/R + j(\omega C - 1/\omega L)$$

$$Y = \sqrt{\frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L}\right)^2} \quad ; \quad \phi = \tan^{-1} \left[R \left(\omega C - \frac{1}{\omega L} \right) \right] \quad ; \quad \omega = 2\pi f$$

The impedance is $Z \angle -\phi = 1/Y \angle \phi$

The current is

$$I \angle \phi = V \angle 0^\circ / Y \angle \phi = (V/Y) \angle \phi = V \angle 0^\circ / Z \angle -\phi = (V/Z) \angle \phi$$

$$\text{where, } I = V \sqrt{\frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L}\right)^2}$$

The current in the circuit is minimum, if $\omega C = \frac{1}{\omega L}$

The frequency under the above condition is

$$f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi\sqrt{LC}}$$

The magnitude of the impedance under the above condition is ($|Z| = R$), while the magnitude of the admittance is ($|Y| = (1/R) = G$). The reactive part of the admittance is $B = 0$, as the susceptance (inductive) $B_L = (1/\omega L)$ is equal to the susceptance (capacitive) $B_C = \omega C$. The phase angle is $\varphi = 0$, and the power factor is unity ($\cos \varphi = 1$).

The input current increases as the frequency are changed, i.e. increased or decreased from the resonant frequency ($f > f_0$ or $f < f_0$) i.e. $f \neq f_0$.

POWER IN AC CIRCUIT

We saw in our tutorial about Electric Power that AC circuits which contain resistance and capacitance or resistance and inductance, or both, also contain real power and reactive power. So in order to calculate the total power consumed, we need to know the phase difference between the sinusoidal waveform of the voltage and current.

The phase angle is given by,

$$\varphi = \arg(V) - \arg(I)$$

i.e. the angle from the current to the voltage. Therefore, positive phase angles mean that the current lags the voltage, and thus are called lagging, and negative phase angles mean that the current leads the voltage, and are called leading.

The power factor is defined as,

$$\text{power factor} = P/S = \cos(\varphi)$$

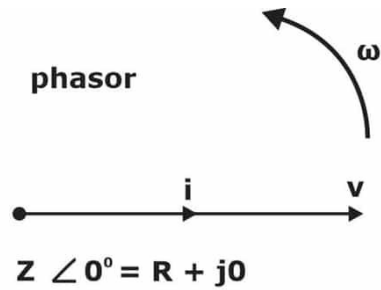
As this is always a positive number the tag 'leading' or 'lagging' is usually added to describe the phase difference.

where $P (=VI \cos \varphi)$ is the Real Power

$S (= VI)$ is the Apparent Power.

Real Power in AC Circuit

- Real power P , so called true or active power, **performs** the "**real work**" within an electrical circuit. Real power, measured in **watts**, defines the **power consumed by the resistive part** of a circuit.
- Then real power, P in an AC circuit is the same as power, P in a DC circuit. So just like DC circuits, it is always calculated as I^2R , where R is the total resistive component of the circuit.



- Since resistance does not produce any phase difference between voltage and current waveform, all the useful power is delivered directly to the resistance and converted to heat, light, and work.
- **Hence the power consumed by a resistance is the real power which is fundamentally the circuits average power.**

Real Power $P = I^2R = V \cdot I \cdot \cos(\theta)$ Watts, (W)

- since there is no phase difference the phase shift between the two waveforms will be zero (0). Then:

$$P = V_{\text{rms}} \times I_{\text{rms}} \times \cos\theta \Rightarrow \text{since } \cos 0^\circ = 1$$

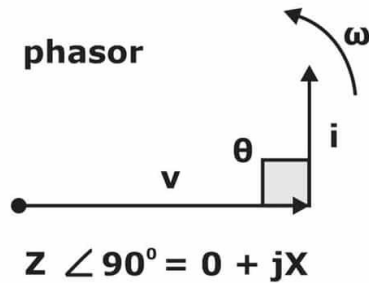
So $P = V_{\text{rms}} \times I_{\text{rms}} \times 1$

or $\{P = V_{\text{rms}} \times I_{\text{rms}}\}$

- Real power I^2R measured in watts, which we read on our utility energy meter and has units in Watts (W), Kilowatts (kW), and Megawatts (MW).
- Note that real power, P is always positive.

Reactive Power in AC Circuit:

- Reactive power Q, (also called wattless power) the power consumed in an AC circuit which does not perform any useful work but has a big effect on the phase shift between the voltage and current waveforms.
- Reactive power is linked to the reactance produced by inductors and capacitors and counteracts the effects of real power. Reactive power does not exist in DC circuits.



- Unlike real power (P), reactive power (Q) takes power away from a circuit due to the creation and reduction of both inductive magnetic fields and capacitive electrostatic fields, thereby making it harder for the true power to supply power directly to a circuit or load.
- The power stored by an inductor in its magnetic field tries to control the current, while the power stored by a capacitor's electrostatic field tries to control the voltage.
- The result is that capacitors “generate” reactive power and inductors “consume” reactive power. This means that they both consume and return power to the source so none of the real power is consumed.

Reactive Power $Q = I^2X = V \cdot I \cdot \sin(\theta)$ volt-amperes reactive, (VARs)

$$Q = V_{\text{rms}} \times I_{\text{rms}} \times \sin\theta \Rightarrow \text{since from phasor, } \sin 90^\circ = 1$$

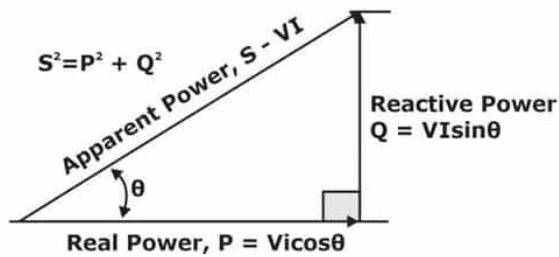
$$Q = V_{\text{rms}} \times I_{\text{rms}} \times 1$$

$$Q = V_{\text{rms}} \times I_{\text{rms}}$$

Thus, reactive power is the I^2X reactive element that has units in volt-amperes reactive (VAR), Kilovolt-amperes reactive (kVAR), and Megavolt-amperes reactive (MVAR).

Apparent Power in AC Circuit

- There is a relationship between the real power (P), and the reactive power (Q), called the complex power.
- The product V_{rms} & I_{rms} circuit is called the “volt-ampere product” (VA) given the symbol S and whose magnitude is known generally as **apparent power**.
- This complex Power is not equal to the algebraic sum of the real and reactive powers added together but is instead the vector sum of P and Q given in volt-amps (VA).



Where:

- ❑ **P** is the I^2R or Real power that performs work measured in watts, W.
- ❑ **Q** is the I^2X or Reactive power measured in volt-amperes reactive, VAR.
- ❑ **S** is the I^2Z or Apparent power measured in volt-amperes, VA.
- ❑ **θ** is the phase angle in degrees. The larger the phase angle, the greater the reactive power.
- ❑ $\cos\theta = P/S = W/VA =$ power factor, p.f.
- ❑ $\sin\theta = Q/S = VAR/VA$.
- ❑ $\tan\theta = Q/P = VAR/W$.

The power factor is calculated as the ratio of the real power to the apparent power because this ratio equals $\cos\theta$.

Power Factor in AC Circuit

- ❑ Power factor, $\cos\theta$, is an important part of an AC circuit, can express in terms of circuit impedance or circuit power.
- ❑ Power factor is defined as the ratio of real power (P) to apparent power (S).

$$\text{Power Factor} = \frac{\text{Watts}}{\text{Volt - Amperes}} = \frac{P}{S} = \frac{VI \cos\phi}{VI} = \cos\phi$$

- ❑ In a pure resistive circuit, the current and voltage are in-phase with each other so the real power consumed is the same as the apparent power as the phase difference is zero degrees (0°).

So the power factor will be:

$$\text{Power Factor, PF} = \cos 0^\circ = 1.0$$

Which means that the number of watts consumed is the same as the number of volt-amperes consumed producing a power factor of 1.0, or 100%. In this case, it is referred to a unity power factor.

- ❑ In a purely reactive circuit, the current and voltage out-of-phase with each other by 90° .

the power factor, in this case, will be:

$$\text{Power Factor, pf} = \cos 90^\circ = 0$$

Which means that the number of watts consumed is zero but there is still a voltage and current supplying the reactive load.

- ❑ Hence we concluded that reducing the reactive VAR component of the power triangle will cause θ to reduce improving the power factor towards one, unity.
- ❑ It is also desirable to have a high power factor as this makes the most efficient use of the circuit delivering current to a load.

$$\text{Real Power (P)} = \text{Apparent power (S)} \times \text{Power factor (p.f)}$$

The disadvantage of Low Power Factor:

Poor power Factor or Low Power Factor less than unity has the following disadvantage:

- ❑ **Effect on Transmission lines(Greater conductor size):** At low power factor, to transmit or distribute the power at a constant voltage requires more current. To transmit high current, higher conductor size transmission lines are needed. This increases the cost of transmission lines.
- ❑ **Effect on Generators, Transformers:** It is known that the electrical machinery (Generators, Transformers) etc are rated in kVA rather than kW.

Where $kVA = kW / \text{power factor}$

i.e The kVA is inversely proportional to kVA for a given kW. For lower power factor the electrical machinery should be having higher kVA rating to drive constant kW load. Then the size and cost of the electrical machine become expensive.

- **High losses in power system:** Due to the high current for low power factor, the copper losses increase in the transmission conductors and switch gear machinery.
- **High Voltage drops (poor voltage regulation):** The large current at low lagging power factor causes greater voltage drops in alternators, transformers, and transmission lines. This results in a decrease in voltage at the driving end and enables the use of extra equipment to counteract the voltage drop like voltage stabilizers. This increase the cost of the power supply system.

How to avoid low power factor? (or) What are the various methods to avoid low power factor?

- As far as possible, overexcited synchronous motors have to be used in place of induction motors.
- High-speed induction motors have better power factor than low-speed induction motors. They are smaller in frame size and economical too.
- The induction motors have maximum power factor when fully loaded. So we should try to use induction motors at full load condition.

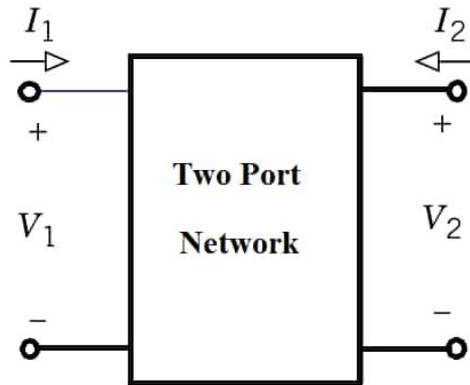
Various methods of power factor improvement are given below;

- Using Static capacitors
- Using synchronous motors
- Using synchronous condensers
- Using Phase advancers
- Using synchronous induction motors
- Using high power factor motors

Two Port Networks

TWO PORT NETWORK

- A pair of terminals through which a current may enter or leave a network is known as a port.
- Two-terminal devices or elements (such as resistors, capacitors, and inductors) result in one-port networks.
- The four-terminal or two-port circuits are used in op-amps, transistors, and transformers.
- Two Port Network circuit is shown below.



- ❑ The current entering one terminal leaves through the other terminal so that the net current entering the port equals zero.
- ❑ A two-port network consists of two pairs of terminals in which one pair of terminals is designated as input and other pair being output.
- ❑ Two port networks are useful in communications, control systems, power systems, and electronics.
- ❑ To characterise a two-port network requires that we relate the terminal quantities V_1 , V_2 , I_1 , and I_2 . The various terms that relate these voltages and currents are called parameters.
- ❑ When the voltage or current are compared at same port then the term defined the **Driving Imminence Function**, while on the other hand when the comparison is done at different port then the function will defined as **Transfer Imminence Function**.

1. (Z-parameters) Open circuit Impedance Parameters

Here, The voltages at input and output are expressed in the terms of input and output currents. The equations are given below.

$$V_1 = Z_{11}I_1 + Z_{12}I_2$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2$$

where, Z_{11} , Z_{12} , Z_{21} and Z_{22} are called the **Z-parameters**.

The Z-Parameter are found as follow:

$$z_{11} = \frac{V_1}{I_1} \Big|_{I_2=0}$$

$$z_{12} = \frac{V_1}{I_2} \Big|_{I_1=0}$$

$$z_{21} = \frac{V_2}{I_1} \Big|_{I_2=0}$$

$$z_{22} = \frac{V_2}{I_2} \Big|_{I_1=0}$$

The z-parameters are also called as open-circuit impedance parameters because they are obtained as the ratio of voltage and current when it is open-circuiting port 2 ($I_2 = 0$) or port 1 ($I_1 = 0$).

- □ Z_{11} is the driving Point impedance when output is open circuited.
- □ Z_{12} is the reverse transfer impedance when input is open-circuited.
- □ Z_{21} is the forward transfer impedance when output is open-circuit.
- □ Z_{22} is the driving Point impedance when input is open-circuited.

2. (Y-parameters) Short Circuit Admittance parameters

Y parameters are achieved by representing the currents at the two ports of the network in terms of voltages at two ports. Thus, voltages V_1 and V_2 are independent variables, while I_1 and I_2 are dependent variables.

The equations are given below.

$$I_1 = Y_{11}V_1 + Y_{12}V_2$$

$$I_2 = Y_{21}V_1 + Y_{22}V_2$$

Where, Y_{11} , Y_{12} , Y_{21} , Y_{22} are called the Y-parameters.

The y-parameters are also called short-circuit admittance parameters. These are obtained as a ratio of current and voltage. The parameters are calculated by short-circuiting port 2 ($V_2 = 0$) or port 1 ($V_1 = 0$).

So, the Y-Parameter can be found as follow:

$$y_{11} = \frac{I_1}{V_1} \Big|_{v_2=0}$$

$$y_{12} = \frac{I_1}{V_2} \Big|_{v_1=0}$$

$$y_{21} = \frac{I_2}{V_1} \Big|_{v_2=0}$$

$$y_{22} = \frac{I_2}{V_2} \Big|_{v_1=0}$$

- □ Y_{11} Short-circuit driving point input admittance
- □ Y_{12} Short-circuit reverse transfer admittance
- □ Y_{21} Short-circuit forward transfer admittance
- □ Y_{22} Short-circuit driving point output admittance

3. h-parameters: Hybrid Parameters

- □ These parameters are By expressing voltage at input port and the current at the output port, the h-parameters are obtained.
- □ Two-port network variables are selected as independent of the input current (I_1) and the output voltage (V_2).
- □ Here, the input voltage and the output current are the dependent variables of this model.

Equations for voltage at input port and current at the output port are given below.

$$V_1 = h_{11}I_1 + h_{12}V_2$$

$$I_2 = h_{21}I_1 + h_{22}V_2$$

The h-Parameter can be found as follow:

$$h_{11} = \frac{V_1}{I_1} \Big|_{V_2=0}$$

$$h_{12} = \frac{V_1}{V_2} \Big|_{I_1=0}$$

$$h_{21} = \frac{I_2}{I_1} \Big|_{V_2=0}$$

$$h_{22} = \frac{I_2}{V_2} \Big|_{I_1=0}$$

- h_{11} Short-circuit input impedance:
- h_{21} Forward short-circuit current gain it is dimensionless.
- h_{12} Reverse open-circuit voltage gain it is dimensionless:
- h_{22} Open-circuit output admittance:

The h-parameters are also called hybrid parameters since they consist of both open-circuit parameters (I_1) and short-circuit parameters (V_2)

4. (g-parameters) Inverse hybrid parameters

- These parameters are obtained by expressing voltage at output port and the current at the input port.
- Off-diagonal g-parameters are dimensionless, while diagonal members have dimensions the reciprocal of one another.

$$I_1 = g_{11}V_1 + g_{12}I_2$$

$$V_2 = g_{21}V_1 + g_{22}I_2$$

$$g_{11} = \frac{I_1}{V_1} \Big|_{I_2=0} \text{ (Open Circuit)}$$

$$g_{21} = \frac{V_2}{V_1} \Big|_{I_2=0} \text{ (Open Circuit)}$$

$$g_{12} = \frac{I_1}{I_2} \Big|_{V_1=0} \text{ (Short Circuit)}$$

$$g_{22} = \frac{V_2}{I_2} \Big|_{V_1=0} \text{ (Short Circuit)}$$

Here g_{12} and g_{21} are dimensionless coefficients, g_{22} is impedance and g_{11} is admittance.

5.(T-Parameters) or ABCD parameters Transmission Parameters

These parameters are generally used in the analysis of power transmission in which the input port is considered as the sending side while the output port is considered as receiving side. These parameters are calculated by expressing voltage and current at the output port.

ABCD parameters can be defined as following:

- A is the reverse voltage ratio with open output.
- B is the reverse transfer impedance with shorted output.
- C is the reverse transfer admittance with open output.
- D is the reverse current ratio with shorted output.

$$V_1 = AV_2 + B(-I_2)$$

$$I_1 = CV_2 + D(-I_2)$$

$$A = \left. \frac{V_1}{V_2} \right|_{I_2=0}$$

$$B = - \left. \frac{V_1}{I_2} \right|_{V_2=0}$$

$$C = \left. \frac{I_1}{V_2} \right|_{I_2=0}$$

$$D = - \left. \frac{I_1}{I_2} \right|_{V_2=0}$$

The transmission parameters express the primary (sending end) variables V_1 and I_1 by dependence of the secondary variables i.e., V_2 (receiving end) and $-I_2$. The negative sign of I_2 is used to mention that the current to enter the load at the receiving end.

- **Symmetry Condition:** if input impedance seen through both the port is same then both port are known as symmetric port. If port are symmetric then both port can be interchanged.

- **Reciprocity Condition:** If only single source is acting in the circuit then “By changing the position of response and excitation if ratio of response to excitation is constant then circuit is reciprocal”.

$$\frac{\text{Response}}{\text{Excitation}} = \text{Constant}$$

| Parameter | Symmetry | Reciprocity |
|-----------|-------------------|--------------------|
| Z | $Z_{11} = Z_{22}$ | $Z_{12} = Z_{21}$ |
| Y | $Y_{11} = Y_{22}$ | $Y_{12} = Y_{21}$ |
| h | $\Delta_h = 1$ | $h_{12} = -h_{21}$ |
| G | $\Delta_G = 1$ | $G_{12} = -G_{21}$ |
| T | $A = D$ | $\Delta_T = 1$ |

6. Conversion of Z-parameter in term of Y-parameter

$$z\text{-parameter } \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = [z] \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \rightarrow \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = [z]^{-1} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = [y] \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$\Rightarrow [y] = [z]^{-1} = \begin{bmatrix} \frac{z_{22}}{\Delta_z} & -\frac{z_{12}}{\Delta_z} \\ -\frac{z_{21}}{\Delta_z} & \frac{z_{11}}{\Delta_z} \end{bmatrix}, \Delta_z = |[z]| = z_{11}z_{22} - z_{12}z_{21}$$

$$[z] = [y]^{-1} = \begin{bmatrix} \frac{y_{22}}{\Delta_y} & -\frac{y_{12}}{\Delta_y} \\ -\frac{y_{21}}{\Delta_y} & \frac{y_{11}}{\Delta_y} \end{bmatrix}, \Delta_y = |[y]| = y_{11}y_{22} - y_{12}y_{21}$$

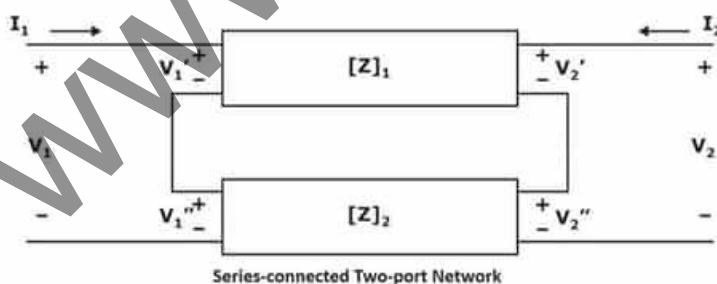
- In the similar fashion we can obtain the other relationship

| to/from | Z | Y | H | G | T |
|---------|--|--|--|--|--|
| Z | $\begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix}$ | $\begin{bmatrix} \frac{y_{22}}{\Delta_y} & \frac{-y_{12}}{\Delta_y} \\ \frac{-y_{21}}{\Delta_y} & \frac{y_{11}}{\Delta_y} \end{bmatrix}$ | $\begin{bmatrix} \frac{\Delta_h}{h_{22}} & \frac{h_{12}}{h_{22}} \\ \frac{-h_{21}}{h_{22}} & \frac{1}{h_{22}} \end{bmatrix}$ | $\begin{bmatrix} \frac{1}{g_{11}} & \frac{-g_{12}}{g_{11}} \\ \frac{g_{21}}{g_{11}} & \frac{\Delta_g}{g_{11}} \end{bmatrix}$ | $\begin{bmatrix} \frac{t_{11}}{t_{21}} & \frac{\Delta_t}{t_{21}} \\ \frac{1}{t_{21}} & \frac{t_{22}}{t_{21}} \end{bmatrix}$ |
| Y | $\begin{bmatrix} \frac{z_{22}}{\Delta_z} & \frac{-z_{12}}{\Delta_z} \\ \frac{-z_{21}}{\Delta_z} & \frac{z_{11}}{\Delta_z} \end{bmatrix}$ | $\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}$ | $\begin{bmatrix} \frac{1}{h_{11}} & \frac{-h_{12}}{h_{11}} \\ \frac{h_{21}}{h_{11}} & \frac{\Delta_h}{h_{11}} \end{bmatrix}$ | $\begin{bmatrix} \frac{\Delta_g}{g_{22}} & \frac{g_{12}}{g_{22}} \\ \frac{-g_{21}}{g_{22}} & \frac{1}{g_{22}} \end{bmatrix}$ | $\begin{bmatrix} \frac{t_{22}}{t_{12}} & \frac{-\Delta_t}{t_{12}} \\ \frac{-1}{t_{12}} & \frac{1}{t_{12}} \end{bmatrix}$ |
| H | $\begin{bmatrix} \frac{\Delta_z}{z_{22}} & \frac{z_{12}}{z_{22}} \\ \frac{-z_{21}}{z_{22}} & \frac{1}{z_{22}} \end{bmatrix}$ | $\begin{bmatrix} \frac{1}{y_{11}} & \frac{-y_{12}}{y_{11}} \\ \frac{y_{21}}{y_{11}} & \frac{\Delta_y}{y_{11}} \end{bmatrix}$ | $\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}$ | $\begin{bmatrix} \frac{g_{22}}{\Delta_g} & \frac{-g_{12}}{\Delta_g} \\ \frac{-g_{21}}{\Delta_g} & \frac{g_{11}}{\Delta_g} \end{bmatrix}$ | $\begin{bmatrix} \frac{t_{12}}{t_{22}} & \frac{\Delta_t}{t_{22}} \\ \frac{-1}{t_{22}} & \frac{t_{11}}{t_{22}} \end{bmatrix}$ |
| G | $\begin{bmatrix} \frac{1}{z_{11}} & \frac{-z_{12}}{z_{11}} \\ \frac{z_{21}}{z_{11}} & \frac{\Delta_z}{z_{11}} \end{bmatrix}$ | $\begin{bmatrix} \frac{\Delta_y}{y_{22}} & \frac{y_{12}}{y_{22}} \\ \frac{-y_{21}}{y_{22}} & \frac{1}{y_{22}} \end{bmatrix}$ | $\begin{bmatrix} \frac{h_{22}}{\Delta_h} & \frac{-h_{12}}{\Delta_h} \\ \frac{-h_{21}}{\Delta_h} & \frac{h_{11}}{\Delta_h} \end{bmatrix}$ | $\begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix}$ | $\begin{bmatrix} \frac{t_{21}}{t_{11}} & \frac{-\Delta_t}{t_{11}} \\ \frac{1}{t_{11}} & \frac{t_{12}}{t_{11}} \end{bmatrix}$ |
| T | $\begin{bmatrix} \frac{z_{11}}{z_{21}} & \frac{\Delta_z}{z_{21}} \\ \frac{1}{z_{21}} & \frac{z_{22}}{z_{21}} \end{bmatrix}$ | $\begin{bmatrix} \frac{-y_{22}}{y_{21}} & \frac{-1}{y_{21}} \\ \frac{y_{21}}{y_{21}} & \frac{y_{11}}{y_{21}} \end{bmatrix}$ | $\begin{bmatrix} \frac{-\Delta_h}{h_{21}} & \frac{-h_{11}}{h_{21}} \\ \frac{-h_{22}}{h_{21}} & \frac{-1}{h_{21}} \end{bmatrix}$ | $\begin{bmatrix} \frac{1}{g_{21}} & \frac{g_{22}}{g_{21}} \\ \frac{g_{11}}{g_{21}} & \frac{\Delta_g}{g_{21}} \end{bmatrix}$ | $\begin{bmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{bmatrix}$ |

7.INTERCONNECTION OF TWO-PORT NETWORKS

The two-port networks can be connected in many ways such as series, parallel or cascade. the configuration listed below:

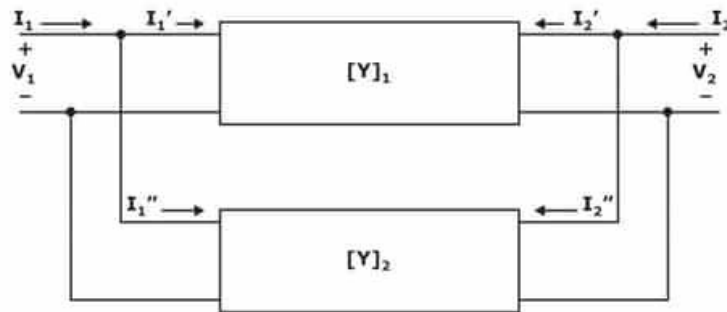
- Series Connection: When two 2-port network are connected in series configuration the z- parameter of each port will be directly added in the result of equivalent 2-port network.



This can be concluded that if two-port networks with Z-parameters $[Z]_1, [Z]_2, [Z]_3, [Z]_n$, are connected in **series**, then the equivalent two port-parameters are given as

$$[Z]_{eq} = [Z]_1 + [Z]_2 + [Z]_3 + [Z]_n$$

- **Parallel-connected Two-port Network:** When two 2-port network are connected in Parallel configuration the Y- parameter of each port will be directly added in the result of equivalent 2-port network.



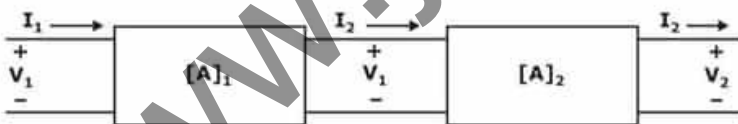
Parallel-connected Two-port Network

This can be concluded that if two-port networks with Y-parameters $[Y]_1, [Y]_2, [Y]_3, [Y]_n$, are connected in **Parallel**, then the equivalent two port-parameters are given as

$$[Y]_{eq} = [Y]_1 + [Y]_2 + [Y]_3 + [Y]_n$$

- **Cascade Connection of Two-port Network:**

When two 2-port network are connected in cascaded configuration then the T-parameter of equivalent two port network will be the Product of T-parameter of individual network.



Cascade Network

networks have transmission parameters $[A]_1, [A]_2, [A]_3, [A]_n$, then the equivalent two-port parameter will have a transmission parameter given as

$$[A]_{eq} = [A]_1 * [A]_2 * [A]_3 * [A]_n$$



JK Chrome

JK Chrome | Employment Portal



Rated No.1 Job Application of India

Sarkari Naukri
Private Jobs
Employment News
Study Material
Notifications



JOBS



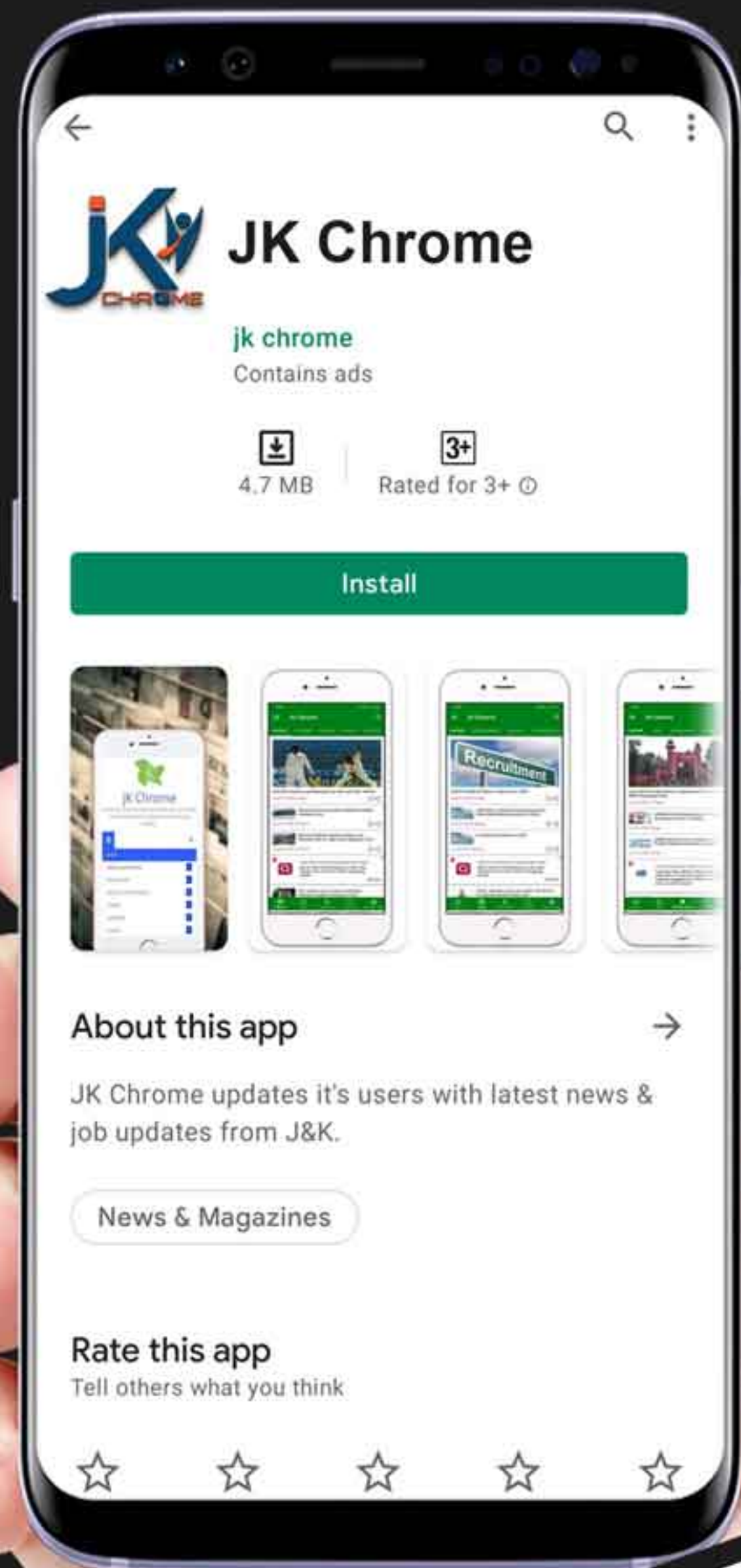
NOTIFICATIONS



G.K



STUDY MATERIAL



JK Chrome

jk chrome
Contains ads



www.jkchrome.com | Email : contact@jkchrome.com